Abstract

MANX is a 6-dimensional muon ionization-cooling experiment that has been proposed to Fermilab to demonstrate the use of a helical cooling channel (HCC) for muon beam emittance reduction for future muon colliders and neutrino factories. The HCC for MANX has solenoidal, helical dipole, and helical quadrupole magnetic components, which diminish as the beam loses energy as it slows down in the liquid helium absorber inside the magnet. The proposed magnet system design is comprised of coil rings positioned along a helical path, which will provide the desired solenoidal and helical dipole and quadrupole fields. Additional helical multipole coils discussed that provide matching 6D cooling conditions at short helix periods. The results of a magnetic field simulations and mechanical analysis are presented.

INTRODUCTION

MANX - a 6-dimensional muon ionization-cooling experiment was proposed to confirm the cooling efficiency of helical cooling channels described in [1]. The novel configurations of Helical Solenoids were investigated in [2] - [4]. It was shown that 0.5 m diameter superconducting Helical Solenoid with the period length of 1.6 m provides 6D cooling conditions. Nevertheless, further analysis showed that more effective cooling and the cooling channel transmission could be obtained with shorter periods: 1 m for the pre-cooling and 0.25 m for the final stage of muon cooling [5]. The goal of this paper is to investigate parameters of Helical magnet systems for the pre-cooler with the period of 1.0 m. It should be noted that the Helical Solenoid diameter should be large enough to accommodate in future the RF cavity.

HELICAL SOLENOIDS

Helical Solenoids with various parameters were investigated. There were used different magnetic field correction schemes to match optimal cooling conditions: a large bore correction solenoid, a helical multipole, and non-circular forms of Helical Solenoid coils. All these approaches are described in the sections below.

Helical Solenoids with 1 m Helix Period

Helical Solenoids with the period 1.0 m were investigated using TOSCA code. The Helical Solenoid main parameters are shown in the Table 1.

Table 1: Helical Solenoid Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coil inner diameter</td>
<td>mm</td>
<td>550</td>
</tr>
<tr>
<td>Helical reference beam orbit radius</td>
<td>mm</td>
<td>159.2</td>
</tr>
<tr>
<td>Helix period</td>
<td>m</td>
<td>1.0</td>
</tr>
<tr>
<td>Transverse field Bt on the reference orbit</td>
<td>T</td>
<td>1.64</td>
</tr>
<tr>
<td>Bz - field on the reference orbit</td>
<td>T</td>
<td>-5.35</td>
</tr>
<tr>
<td>Gradient dBz/dr on the reference orbit</td>
<td>T/m</td>
<td>9.4</td>
</tr>
<tr>
<td>Bzo – field in the magnet system center</td>
<td>T</td>
<td>-6.99</td>
</tr>
<tr>
<td>Bz = Bz/Bzo – on the reference orbit</td>
<td>T</td>
<td>0.765</td>
</tr>
<tr>
<td>Bt/Bz – on the beam reference orbit</td>
<td></td>
<td>-0.307</td>
</tr>
</tbody>
</table>

In the previous works [2], [3], when the beam orbit was about equal to the coil radius, the magnetic field was specified on orbit: Bt – transverse dipole field component, dBt/dr – transverse field gradient. Because the Bt has a 1/r dependence, it is more convenient specify the field gradient dBz/dr, which is about constant in aperture for such magnet systems but coupled with dBt/dr through Laplace equation in cylindrical coordinate system.

Figure 1: Helical Solenoid geometry and flux density.
The field simulation (see Fig. 1) using specified in the Table 1 parameters showed large misbalance between transverse $B_t = 1.64$ T and longitudinal $B_z = -8.4$ T field components at coil current 166.84 kA. The $B_t/B_z$ is only -0.195 instead of -0.307. So, the relation between transverse and longitudinal field should be 57% increased. At transverse field 1.64 T the coil peak field reaches 13 T.

**Helical Solenoid with Compensation Solenoid**

The discrepancy between the transverse and longitudinal fields could be compensated by a straight solenoid placed outside of the Helical Solenoid. This additional solenoid should generate 2.76 T field in the opposite to the helix direction. It produces positive demagnetization effect and reduces the coil peak field from 13 T down to 9.2 T (See Fig. 2).

One could see a linear behavior of $B_z$ field at the beam reference orbit: the gradient $dB_z/dr = 9.76$ T/m, $B_t = 1.64$ T, $B_z = -5.35$ T, $B_t/B_z = 0.307$ at 166.84 kA of Helical Solenoid coils and -54.96 kA for the straight solenoid. All circular coils in helical and straight solenoids have the coil length of 20 mm in Z – direction.

**Helical Solenoid with Helical Multipole Coils**

Another way to achieve the optimal field is to use the Helical Dipole and Helical Quadrupole coils wound on the cylindrical surface. This approach was investigated in [2]. Rather large field gradients produce large peak fields, Lorentz forces and substantially increase the energy of magnetic field. It is possible to reduce these effects by placing helical windings on the surface of Helical Solenoid. The Helical Dipole wound on the outer surface of Helical Solenoid is shown in Fig. 4 and has 10 sections per pole evenly distributed in the azimuth angle 60°.

The $B_z$ field component distribution in the Helical Solenoid aperture from radius -0.2 m to 0.5 m is shown in Fig. 3.

The coil peak field is 10.7 T on the inner surface of Helical Solenoid and 2.4 T in the Helical Dipole.

The $B_z$ field distribution from radius -0.2 m to 0.5 m.
The magnet system has parameters close to the optimal: gradient \( \frac{dBz}{dr} = 10.35 \, \text{T/m} \), \( B_t = 1.64 \, \text{T} \), \( B_z = -5.35 \, \text{T} \), \( \frac{B_t}{B_z} = 0.307 \) at 230.6 kA Helical Solenoid coil current, and 665.3 kA of Helical Dipole winding total current.

The Helical Quadrupole coils wound on the surface of Helical Solenoid could correct the transverse field gradient \( \frac{dB_t}{dz} \). The transverse field distribution in the aperture generated by the Helical Quadrupole is shown in Fig. 6.

![Helical Quadrupole transverse field Bt](image)

**Figure 6:** Helical Quadrupole transverse field \( B_t \).

The Helical quadrupole capable correct transverse field gradient 0.6 T/m at 200 kA total Helical Quadrupole winding current.

**Helical Solenoids with Non-Circular Coils**

The discrepancy between transverse and longitudinal fields could be reduced by changing the form of Helical Solenoid coils. Because the field should have larger gradient than in the case of circular coils, the coil should have trapezoidal form. The maximum gradient is achieved with the triangular coils, as shown in Fig. 7.

![Helical Solenoid with triangular coils Bmax = 8.5 T](image)

**Figure 7:** Helical Solenoid with triangular coils \( B_{max} = 8.5 \, \text{T} \).

This solenoid has large field gradient \( \frac{dBz}{dr} = 11.9 \, \text{T/m} \), \( B_z = -5.35 \, \text{T} \), \( B_t = 1.36 \, \text{T} \) at 136.4 kA coil current. The trapezoidal coil shape can reduce this gradient to the optimal 9.4 T/m and improve the balance between components.

**CONCLUSION**

- Magnet systems based on the Helical Solenoids are capable of generating fields required for the optimal muon cooling even at short helix periods.
- Large bore straight solenoids, helical multipole windings or trapezoidal coils can be used for eliminating of the misbalance between transverse and longitudinal fields.
- The best type of field compensation depends on the application. Demonstration models can use helical multipole windings for greater flexibility. The final design will be more efficient with non-circular shape coils.
- The high 8.5 T - 11 T peak fields drive the design to the use of Nb_3Sn superconductors.
- The presented results could be distributed for the higher fields and smaller orbit radii which used at the cooling channel end.

**REFERENCES**