

# Anomaly Mediated Neutrino-Photon Interactions at Finite Baryon Density

Jeffrey A. Harvey<sup>(a)</sup>, Christopher T. Hill<sup>(b)</sup>, and Richard J. Hill<sup>(b)</sup>

<sup>(a)</sup>*Enrico Fermi Institute and Department of Physics  
The University of Chicago,  
Chicago, Illinois, 60637, USA*

<sup>(b)</sup>*Fermi National Accelerator Laboratory  
P.O. Box 500, Batavia, Illinois, 60510, USA*

We propose new physical processes based on the axial vector anomaly and described by the Wess-Zumino-Witten term that couples the photon,  $Z$ -boson, and the  $\omega$ -meson. The interaction takes the form of a pseudo-Chern-Simons term,  $\sim \epsilon_{\mu\nu\rho\sigma}\omega^\mu Z^\nu F^{\rho\sigma}$ . This term induces neutrino-photon interactions at finite baryon density via the coupling of the  $Z$ -boson to neutrinos. These interactions may be detectable in various laboratory and astrophysical arenas. The new interactions may account for the MiniBooNE excess. They also produce a competitive contribution to neutron star cooling at temperatures  $\gtrsim 10^9$  K. These processes and related axion-photon interactions at finite baryon density appear to be relevant in many astrophysical regimes.

PACS numbers: 11.15.-q,12.15.-y,12.38.Qk,12.39.Fe,13.15.+g,13.40.-f,14.70.Hp,14.80.Mz,95.85.Ry,97.60.Jd

## I. INTRODUCTION

The axial vector anomaly plays a fundamental role in the structure of the Standard Model and describes many physical processes, including the classic decay  $\pi^0 \rightarrow 2\gamma$  [1, 2, 3]. One can summarize the traditional current algebra manipulations used to treat anomalous processes by an effective action. This is a functional,  $\Gamma(U, A_L, A_R)$ , of a chiral field of the pseudoscalar mesons,  $U = \exp(i\pi^a t^a / f_\pi)$ , and background gauge fields  $A_L, A_R$ , coupled to left- and right-handed chiral currents. It generates (consistent) anomalies under local gauge transformations and is known as the Wess-Zumino-Witten (WZW) term [4, 5].

The WZW term has been developed into a phenomenologically useful form by Kaymakçalan, Rajeev and Schechter (KRS) [6] following Witten's pioneering work. It can be understood as arising from a Chern-Simons term built of Yang-Mills gauge fields in  $D = 5$ , suitably compactified such that  $A_5$  zero modes emerge as the mesons [7]. It applies to any effective theory of pseudo-Nambu-Goldstone bosons (pNGB's) coupled to gauge fields, *e.g.*, as in the case of Little Higgs theories [8]. The WZW term arises naturally in connection with topological physics in extra dimensions, and occurs in both top-down [9] and bottom-up [10, 11] approaches to holographic QCD in order to correctly match the flavor anomalies of QCD.

The WZW term for spontaneously broken  $SU(3)_L \times SU(3)_R$  flavor symmetry describes anomalous processes involving the pseudoscalar pNGB's and fundamental gauge fields such as the photon. For example, we have  $\pi^0, \eta \rightarrow 2\gamma$  with  $A_L = A_R = A_Q$  and  $Q = \text{diag}(2/3, -1/3, -1/3)$ . However, the WZW term also summarizes processes containing effective vector mesons in pole approximation, such as  $\Phi \rightarrow 3\pi, \Phi \rightarrow K\bar{K}$ ,

where  $A_L = A_R = \Phi\lambda^8/2$ ; and  $\omega \rightarrow \rho\pi \rightarrow 3\pi, \omega \rightarrow \pi^0\gamma$ , *etc.*, where  $A_L = A_R = \omega B + \rho I_3$ , with  $I_3 = \text{diag}(1/2, -1/2, 0)$  and  $B = \text{diag}(1/3, 1/3, 1/3)$  [6].

To leading order in an expansion in  $\pi^a$  the WZW term is seen to contain "pseudo-Chern-Simons" terms (pCS) [25], such as  $\text{Tr}(\epsilon_{\mu\nu\rho\sigma} A_L^\mu A_R^\nu \partial^\rho A_L^\sigma)$ . Recently it has been proposed that terms involving  $A_1\omega d\rho$  may be of phenomenological interest [12], which has stimulated the present work.

Presently we note that the Standard Model implies a pCS term in the Lagrangian involving the photon, the  $Z$ -boson, and the isoscalar  $\omega$  vector meson of the form:

$$\frac{N_c}{48\pi^2} \frac{e g_\omega g_2}{\cos \theta_W} \epsilon_{\mu\nu\rho\sigma} \omega^\mu Z^\nu F^{\rho\sigma}. \quad (1)$$

The derivation of this result requires a careful accounting of anomalies. In the absence of  $\omega$ , gauge invariance is maintained by the combination of the WZW term and anomaly cancelling contributions from the lepton sector. In the presence of  $\omega$ ,  $SU(2)_L \times U(1)_Y$  gauge invariance must be enforced by including appropriate counterterms,  $\epsilon_{\mu\nu\rho\sigma} A_L^\mu A_R^\nu \partial^\rho A_L^\sigma$ , *etc.*. This uniquely specifies the coefficient of eq.(1), apart from coupling constant normalizations.  $Z_\mu$  in eq.(1) should be thought of as a gauge-invariant Stueckelberg field, *i.e.*, we are in unitary gauge for the broken  $SU(2)_L \times U(1)_Y$  generators. For simplicity, we restrict attention to two light flavors and the  $\omega$  is coupled to baryon number  $B = \text{diag}(1/3, 1/3)$ , inducing the coupling to the nucleon isodoublet  $N = (p, n)$ , as  $g_\omega \omega^\mu \bar{N} \gamma_\mu N$  [26]. We also note that the one-loop diagram responsible for the pCS term is closely related to the electroweak baryon number anomaly. The full details will be presented elsewhere [13].

Physically interesting effects arise when we integrate out the  $\omega$  and the  $Z$ , replacing them with the baryon current and neutrino current respectively. The new anoma-

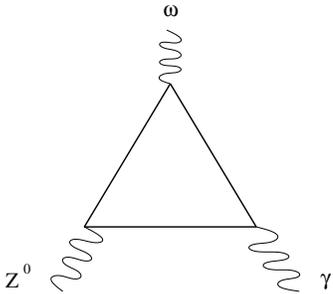


FIG. 1: Triangle diagram at chiral constituent quark level treating  $\gamma$ ,  $Z$  and  $\omega$  as background gauge fields. This is contained in the Wess-Zumino-Witten term,  $\Gamma(U, A_L, A_R)$ , which is an effective action expressing the full anomaly physics of pseudoscalars, axial- and vector mesons and fundamental gauge fields. Integrating out  $Z$  and  $\omega$  leads to a neutrino-photon interaction at finite baryon density.

lous interaction is then:

$$S_{\text{int}} = \sum_f \kappa \int d^4x \epsilon^{\mu\nu\rho\sigma} J_\mu \bar{\nu}_L^f \gamma_\nu \nu_L^f F_{\rho\sigma}, \quad (2)$$

where  $J_\mu = \bar{N} \gamma_\mu N$  is the isosinglet baryon current and we sum over all left-handed neutrino flavors. The normalization factor is:

$$\kappa = \frac{N_c}{12\pi^2} \frac{g_\omega^2}{m_\omega^2} \frac{eG_F}{\sqrt{2}}, \quad \nu_L = \frac{1 - \gamma^5}{2} \nu. \quad (3)$$

One could include similar terms for the  $\rho$  and  $\Phi$  mesons, but  $\omega$  is expected to dominate.

Interesting effects will arise in processes such as:

$$\nu + N \rightarrow \nu + \gamma + N \text{ (or } N') \quad (4)$$

$$\gamma + N \rightarrow \nu + \bar{\nu} + N \text{ (or } N'). \quad (5)$$

Coherent enhancements to these basic processes can occur in situations described by an ambient background baryon density.

We presently turn to two such classes of phenomena: neutrino scattering on nucleons; and the cooling of a young neutron star. We briefly mention that an analogous axion-photon-baryon current interaction arises with similarly important implications.

## II. NEUTRINO-NUCLEON COLLISIONS

We consider an incident neutrino of any flavor with 4-momentum  $p = (E_\nu, \vec{p})$  colliding with a stationary nucleon, producing an outgoing neutrino of momentum  $k$ , and a photon of momentum  $q = (E_\gamma, \vec{q})$ . We assume  $E_\nu \lesssim M$ , where  $M$  is the nucleon mass and recoil is treated as negligible. The action then becomes:

$$S_{\text{int}} \rightarrow \kappa \int d^4x \epsilon_{0ijk} \bar{\nu}_L \gamma^i \nu_L F^{jk} \delta^3(\vec{x}). \quad (6)$$

The resulting total cross-section is:

$$\begin{aligned} \sigma &= \frac{\alpha g_\omega^4 G_F^2}{480\pi^6 m_\omega^4} E_\nu^6 \\ &= 2.6 \times 10^{-41} (E_\nu/\text{GeV})^6 (g_\omega/10.0)^4 \text{ cm}^2. \end{aligned} \quad (7)$$

We remark that there is considerable uncertainty in the value of  $g_\omega$ , generally extracted from nuclear potential models [14], and the value we use here is ‘‘conservative.’’

To assess the possible experimental sensitivity to this effect we note that MiniBooNE observes an excess of  $\sim 10^2$  events with electromagnetic energy and with reconstructed mean neutrino energies of order  $\sim 400$  MeV [16, 17]. These events look like  $400 \text{ MeV } \nu_e + N \rightarrow e + N'$  charged current events, but the electron could be faked by the hard photon in our process.

The MiniBooNE  $\nu_\mu$  beam spectrum is fairly flat from 300 MeV to 1 GeV, peaking at  $\sim 700$  MeV. We focus on  $E_\nu \sim 700$  MeV  $\nu_\mu$ 's and assume that these produce photons with  $E_\gamma \sim 400$  MeV via our process. In this energy range MiniBooNE accepts  $\sim 2 \times 10^5$  charged current quasi-elastic (CCQE)  $\nu_\mu N \rightarrow \mu N'$  events with cross-section,  $\sigma_{QE} \sim 0.9 \times 10^{-38} \text{ cm}^2$  [18]. Thus we expect to produce  $\sim 2 \times (2 \times 10^5) \sigma(700 \text{ MeV}) / \sigma_{QE} \sim 140 (g_\omega/10)^4$  events. The extra factor of 2 arises from the fact that our process involves both  $n$  and  $p$  while CCQE involves only  $n$  in a carbon target. Our cross-section has a distinctly flat angular distribution,  $d\sigma/d\cos\theta_\gamma = \sigma(E_\nu)/2$  where  $\cos\theta_\gamma = \vec{p} \cdot \vec{q} / E_\nu E_\gamma$ . We note, however, that the photon will be pulled forward as form-factor effects, which we have ignored, set in.

While this estimate is extremely naive, it is encouraging that existing experiments may already have sensitivity to our effect. For a more refined analysis we require an improved  $\sigma(E_\nu)$  on nuclear (carbon) targets, including coherence and form-factor effects, and convolution of  $\sigma(E_\nu)$  with the beam spectrum.

Additional anomaly mediated interactions can also be studied systematically starting from the WZW term. For instance, a term  $\sim \epsilon_{\mu\nu\rho\sigma} \partial^\mu \pi^0 Z^\nu F^{\rho\sigma}$  involves pion exchange with the nucleus in place of  $\omega$  exchange. However, this process contains  $1/f_\pi^4 < g_\omega^4/m_\omega^4$ , and is not the dominant effect. Also, in contrast to (1), the amplitude for this process does not add coherently over individual nucleons at low energy [27].

We thus conjecture that the anomaly mediated process may be relevant to the MiniBooNE excess. Moreover, it could lead to multiple enhanced prospects for observing quasi-elastic neutrino scattering on nuclei. Modern neutrino experiments could ultimately provide a normalization of the uncertain input parameter,  $g_\omega$ . While the anomaly mediated neutrino process is higher order, it contains a hard photon. Even for relatively low energy reactor neutrinos,  $E_\nu \sim 3 \text{ MeV}$ , the enormous flux availability suggests that the anomaly process may be observable. The hard photon also provides a possible observable for a quartz-Cerenkov detector, or liquid halogen bubble detectors, *etc.*

### III. NEUTRON STAR COOLING

The anomaly mediated process is competitive with the conventional processes for neutron star cooling. For illustration, we consider a particular coherent subprocess in a superconducting neutron star core, but the anomaly mediated process will have a more general applicability. Neutron star cooling is reviewed in [19, 20, 21, 22].

The full set of processes contributing to neutron star cooling is complex. In the ultra-high density inner core the energy loss is dominated by the direct Urca process,

$$n \rightarrow p + e^- + \bar{\nu}_e, \quad e^- + p \rightarrow n + \nu_e. \quad (8)$$

Throughout most of star the nucleons are at lower densities,  $\lesssim 10^{15} \text{ g cm}^{-3}$ , and are Fermi degenerate. In this case, the direct Urca process is highly suppressed. Energy is then typically lost by the modified Urca (mUrca) process where a bystander nucleon is included to conserve energy and momentum. The mUrca process is affected by the superfluid phase, which appears below a critical temperature  $T_c \sim 10^{10} \text{ K}$ . This reduces the mUrca rate. However, new cooling mechanisms associated with Cooper pairing of nucleons may turn on at  $T \lesssim 10^9 \text{ K}$ , compensating the mUrca suppression.

We presently consider typical neutron star densities of  $2\rho_0 = 5.6 \times 10^{14} \text{ g cm}^{-3}$  (twice nuclear density  $\rho_0$ ) and assume that the regime  $10^9 \lesssim T \lesssim 10^{10} \text{ K}$  contains the standard mUrca processes. We summarize these effects by (see *e.g.*, Table 2 of [21]):

$$Q_\nu^{\text{mUrca}} = (10^{18} - 10^{21}) \times (T_9)^8 \text{ erg s}^{-1} \text{ cm}^{-3}, \quad (9)$$

at temperature  $T = T_9 \times 10^9 \text{ K}$ . This describes a collection of various processes, ignoring superfluidic suppression, and permits a naive comparison to our process.

We now want to estimate the cooling rate due to the anomaly mediated process. Our basic assumptions are:

1. The degenerate protons pair to make a superconductor. This spontaneously breaks  $U(1)_{EM}$  and gives a mass to the photon, also known as the inverse penetration length of the superconductor. The mass depends sensitively on strong interaction models, we take  $m_\gamma \sim 1 \text{ MeV}$  as a typical value.
2. The (massive) photons are in thermal equilibrium with the neutrons, protons and electrons in the superconducting interior of the neutron star and we work in the limit  $T \lesssim m_\gamma$ .

The photons are therefore characterized by the phase-space distribution function and number density,

$$f(p_\gamma) = \left[ e^{E_\gamma/T} - 1 \right]^{-1}, \quad n_\gamma = g \int \frac{d^3 p_\gamma}{(2\pi)^3} f(p_\gamma), \quad (10)$$

with  $g = 3$  for a massive spin-one particle. For  $T \lesssim 10^{10} \text{ K} \sim m_\gamma$ , we can approximate  $E_\gamma \sim m_\gamma + |\vec{p}_\gamma|^2/2m_\gamma$ . The emissivity of neutrinos due to  $\gamma \rightarrow \nu\bar{\nu}$  is

$$Q_\nu^{\text{anom}} = 3 \int \frac{d^3 p_\gamma}{(2\pi)^3} E_\gamma \Gamma_{p_\gamma}(\gamma \rightarrow \nu\bar{\nu}) f(p_\gamma), \quad (11)$$

thus requiring the decay rate of the photon,  $\Gamma_{p_\gamma}$ .

In the star rest frame the baryon number current is  $J_\mu = (n_B, \vec{0})$ , and the photon 4-momentum is  $p_\gamma \approx (m_\gamma, \vec{p}_\gamma)$ . However, it is convenient to work in the photon rest-frame, where  $p'_\gamma = (m_\gamma, \vec{0})$ . In this frame the nucleon current is slightly boosted:  $J'_\mu = n_B \eta_\mu$ , where  $\eta_\mu \approx (1, \vec{\beta})$  and  $\vec{\beta} \approx -\vec{p}_\gamma/m_\gamma$ . In the nonrelativistic limit we thus have:

$$\Gamma_{p_\gamma} = \frac{\kappa_a^2 n_B^2}{2m_\gamma} \int \frac{d^3 p_1}{2E_1(2\pi)^3} \frac{d^3 p_2}{2E_2(2\pi)^3} \times |M|^2 (2\pi)^4 \delta^4(p'_\gamma - p_1 - p_2), \quad (12)$$

and the invariant final-state spin-summed and initial state spin-averaged matrix element is:

$$|M|^2 = \frac{2m_\gamma^2}{3} \epsilon^{\mu\nu 0\rho} \epsilon^{\alpha\beta 0\rho} \eta_\mu \eta_\alpha \text{Tr}[(1 - \gamma^5) \gamma_\nu \not{p}'_1 \gamma_\beta \not{p}'_2]. \quad (13)$$

This results in the decay rate:

$$\Gamma_{p_\gamma} = \frac{2m_\gamma \kappa_a^2 n_B^2}{9\pi} |\vec{p}_\gamma|^2, \quad (14)$$

and emissivity:

$$Q_\nu^{\text{anom}} = C e^{-m_\gamma/T} (m_\gamma T)^{5/2} \frac{G_F^2 m_\gamma^2 n_B^2}{m_\omega^4}, \quad (15)$$

$$C = \frac{\sqrt{2}\pi\alpha g_\omega^4}{16\pi^6} = 0.012 - 0.96.$$

The range of  $C$  corresponds to a range of  $g_\omega = 10.0$  to  $g_\omega = 30.0$ . We note that both  $g_\omega$  and  $m_\omega$  run with nuclear density, *e.g.*, Refs. [15] obtain  $m_\omega$  reduced by a factor  $\sim 0.6 - 0.8$  at  $\rho \gtrsim \rho_0$ . Holding  $m_\omega$  fixed, larger values of  $g_\omega$  account for this effect, as well as other effects, *e.g.*, higher resonance contributions. For standard density  $2\rho_0 = 5.6 \times 10^{14} \text{ g cm}^{-3}$ , this yields an emissivity of

$$Q_\nu^{\text{anom}} = 2.31 \times 10^{22} \text{ erg s}^{-1} \text{ cm}^{-3} \times m_\omega^{9/2} (g_\omega/10)^4 e^{(-11.6m/T_9)} (T_9)^{5/2}, \quad (16)$$

where  $m = m_\gamma/(1 \text{ MeV})$ .

We see in Fig.(2) that this is competitive with the cooling rate from mUrca processes [28]. We note that in the early phase of neutron star formation with  $T \gtrsim 10^{11} \text{ K}$ , our process may actually dominate. This will be developed and reported elsewhere [13].

### IV. CONCLUSIONS

Anomaly mediated interactions between photons and neutrinos at finite baryon density may play an important role in laboratory neutrino experiments and astrophysical processes. The new interaction (1) may be relevant in accounting for the MiniBooNE low energy excess. It

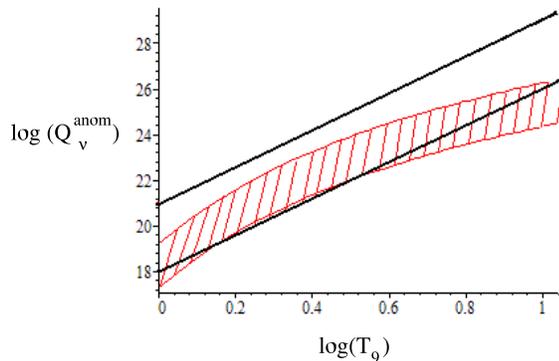


FIG. 2:  $\log(Q_v^{\text{anom}})$ , with  $Q$  measured in  $\text{erg s}^{-1} \text{cm}^{-3}$ , versus  $\log(T_9)$  for the range  $g_\omega = 10 - 30$  (hatched) compared to the range of standard mUrca processes of eq.(9). The curves for mUrca do not include superfluidic suppression factors.

may also play a significant role in neutron star cooling and early stage evolution. There are many potentially important applications in various other physical regimes. We will present a more detailed analysis and discussion elsewhere, including the detailed derivation of pCS and axion interactions from the WZW term [13][24].

We further remark that the axion will have a similar induced coupling to the photon and the  $\omega$ , leading to an interaction of the form:

$$c_{\text{axion}} \frac{eN_c}{24\pi^2} \frac{g_\omega^2}{m_\omega^2} \epsilon_{\mu\nu\rho\sigma} \frac{\partial^\mu a}{f_a} F^{\nu\rho} \bar{N} \gamma^\sigma N, \quad (17)$$

where  $c_{\text{axion}}$  is calculable from a given axion model. An important application is to consider axion emission and the resulting bounds on axion couplings from supernovae (SN1987A).

## Acknowledgements

We thank W. Bardeen, J. Conrad, and P. Cooper for helpful discussions. Research supported by the U.S. Department of Energy grant DE-AC02-76CHO3000 and by NSF Grants PHY-00506630 and 0529954. This work was supported in part by the Joint Theory Institute funded by Argonne National Laboratory and the University of Chicago. JH acknowledges the hospitality of the Aspen Center for Physics during part of this work.

- 
- [1] J. S. Bell and R. Jackiw, *Nuovo Cim. A* **60**, 47 (1969).
  - [2] S. L. Adler, *Phys. Rev.* **177**, 2426 (1969).
  - [3] W. A. Bardeen, *Phys. Rev.* **184**, 1848 (1969).
  - [4] J. Wess and B. Zumino, *Phys. Lett. B* **37**, 95 (1971).
  - [5] E. Witten, *Nucl. Phys. B* **223**, 422 (1983).
  - [6] O. Kaymakalan, S. Rajeev and J. Schechter, *Phys. Rev. D* **30**, 594 (1984).
  - [7] C. T. Hill and C. K. Zachos, *Phys. Rev. D* **71**, 046002 (2005); C. T. Hill, *Phys. Rev. D* **73**, 126009 (2006); *Phys. Rev. D* **73**, 085001 (2006)
  - [8] C. T. Hill and R. J. Hill, 0705.0697 [hep-ph]. *Phys. Rev. D* **75**, 115009 (2007)
  - [9] T. Sakai and S. Sugimoto, *Prog. Theor. Phys.* **113**, 843 (2005)
  - [10] J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, *Phys. Rev. Lett.* **95**, 261602 (2005)
  - [11] L. Da Rold and A. Pomarol, *Nucl. Phys. B* **721**, 79 (2005)
  - [12] S. K. Domokos and J. A. Harvey, 0704.1604 [hep-ph].
  - [13] J. A. Harvey, C. T. Hill and R. J. Hill, to appear.
  - [14] A standard reference for nucleon-meson couplings is: R. Machleidt, K. Holinde and C. Elster, *Phys. Rept.* **149**, 1 (1987).
  - [15] J. C. Caillon and J. Labarsouque, *J. Phys. G* **21**, 905 (1995). H. Kouno, Y. Horinouchi and K. Tuchitani, *Prog. Theor. Phys.* **112**, 831 (2004)
  - [16] A. A. Aguilar-Arevalo *et al.* [MiniBooNE Collaboration], *Phys. Rev. Lett.* **98**, 231801 (2007) and “Measurement of Muon Neutrino Quasi-Elastic Scattering on Carbon,” 0706.0926 [hep-ex].
  - [17] We thank Janet Conrad for suggesting this comparison.
  - [18] J. Monroe [MiniBooNE Collaboration], *Nucl. Phys. Proc. Suppl.* **139**, 59 (2005) [hep-ex/0408019].
  - [19] “Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects,” S. L. Shapiro and S. A. Teukolsky (John Wiley, New York, 1983).
  - [20] G. G. Raffelt, “Stars As Laboratories For Fundamental Physics: The Astrophysics Of Neutrinos, Axions, And Other Weakly Interacting Particles,” Chicago, USA: Univ. Pr. (1996).
  - [21] D. G. Yakovlev, A. D. Kaminker, O. Y. Gnedin and P. Haensel, *Phys. Rept.* **354**, 1 (2001)
  - [22] D. G. Yakovlev and C. J. Pethick, *Ann. Rev. Astron. Astrophys.* **42**, 169 (2004)
  - [23] E. Flowers, M. Ruderman and P. Sutherland, *Astrophys. J.* **205**, 541 (1976).
  - [24] Some implications of  $U(1)$  anomalies at constant baryon density have been studied in: D. T. Son and A. R. Zhitnitsky, *Phys. Rev. D* **70**, 074018 (2004) [hep-ph/0405216].
  - [25] We use the terminology “pseudo-Chern-Simons term” to distinguish from “Chern-Simons terms” which only occur in odd spacetime dimension.
  - [26] Upon introducing  $\omega$  in this way, the WZW term contains a term  $g_\omega \omega^\mu J_\mu$ , where  $J_\mu$  is the properly normalized Goldstone-Wilczek Skyrminion baryon current when  $N_c = 3$ .
  - [27] Coherent enhancements of neutrino-photon interactions were studied in a current algebra framework in: D. Rein and L. M. Sehgal, *Phys. Lett. B* **104**, 394 (1981) [Erratum-ibid. B **106**, 513 (1981)]. However, at the energies of interest, the assumption of coherence on the entire nucleus leads to only small effects confined to the forward direction.
  - [28] Although our effect is one-loop order, it is significantly enhanced relative to other loop processes, *e.g.*, to the electromagnetic penguin,  $\gamma \rightarrow \bar{\nu}\nu$ , by a factor of  $\sim Tg_\omega^4 n_B^2 / m_\omega^4 m_\gamma^3 (\ln(M_W/m_\gamma))^2 \sim 10^3$ .