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Topological physics in the standard model and beyond

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Abstract. Topological interactions are an essential ingredient for building consistent low-energy theories of fermions, gauge fields and Nambu-Goldstone bosons in the absence of explicit UV completions, such as in Little Higgs theories. These interactions are also a probe of UV completion physics that may be out of direct experimental reach. The technology of topological, or Wess-Zumino-Witten interactions is described, using explicit examples in the standard model and in Little Higgs models. The construction of a simple topological action on $SU(3)/SU(2)$ is described. Inconsistencies in some popular Little Higgs models are pointed out.

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1 Introduction

Topological interactions and anomaly physics are of basic importance in the standard model, in model-building applications beyond the standard model, and in formal studies of quantum field theory.

However, the methods used to study these interactions are not part of the everyday theoretical toolkit. This may be due partly to the perceived complexity of the mathematics involved, and partly to the perceived scarcity of relevant physics applications. My goal in this talk is to help dispel these notions, and to point out some new applications of these tools.

Much of this perceived complexity is associated with ancient notions of current algebra that still pervade many discussions of anomalies. Section 2 reviews some simple processes in the standard model from a modern effective field theory point of view. To illustrate the mathematical simplicity of this physics, a new and especially simple topological construction for $SU(3)/SU(2)$ is described in Section 3; the most complicated mathematical object needed here is a sphere. Section 4 describes the application of these tools to Little Higgs models, pointing out some confusions regarding gauge invariance, anomalies, and spurious parities that have afflicted the literature. Section 5 concludes by mentioning some more formal applications of the technology that was developed to study Little Higgs models.

2 Standard model examples

There are a number of applications, both new and old, of topological interactions in the standard model. We mention three examples here.

2.1 $\pi^0 \to \gamma\gamma$ and the QCD chiral lagrangian

Textbook treatments [1] of the famous $\pi^0 \to \gamma\gamma$ decay may lead the uninitiated reader to believe that computing anomalous divergences of axial vector currents, and working through subtle regularization schemes, are prerequisite to making rigorous predictions. In modern effective field theory language, the situation is much simpler. Once we know the fields in our effective theory (a matrix field $U(x)$ taking values in $SU(n_f) \times SU(n_f)/SU(n_f) = SU(n_f)$, with n_f the number of quark flavors) and the symmetries of our effective theory (global $SU(n_f) \times SU(n_f)$), we can write down the most general operator made from these fields, and obeying these symmetries. One such operator—the Wess Zumino Witten (WZW) term [2,3]—happens to have the remarkable property that it uniquely predicts the $\pi^0 \to \gamma\gamma$ rate. In detail, when expanded onto the relevant fields, this interaction reads [4,5,6]

$$\Gamma_{WZW} = -\frac{N_c}{96\pi^2 f_\pi} \int d^4x\, \epsilon^{\mu\nu\rho\sigma} \pi^0 F_{\mu\nu} F_{\rho\sigma} \,, \qquad (1)$$

where $N_c = 3$ is the number of quark colors, and $f_\pi \approx 93\,\mathrm{MeV}$ is the pion decay constant. From this interaction Lagrangian, we can simply write down the appropriate Feynman diagram and compute the decay rate.

2.2 Weak currents in the QCD chiral lagrangian

Although processes like $\pi^0 \to \gamma\gamma$ involving just pseudoscalars and photons provide the most familiar examples of gauged WZW terms, the low-energy standard model also contains the charged and neutral weak currents, vector mesons coupling to baryon number

and isospin, and the hypothetical axion. Incorporating these ingredients into the WZW structure leads to interesting effects. An example is the neutrino-photon interaction mediated by [7]

$$\Gamma_{WZW} = \frac{N_c}{48\pi^2}\frac{eg_\omega g_2}{\cos\theta_W}\int d^4x\,\epsilon^{\mu\nu\rho\sigma}\omega_\mu Z_\nu F_{\rho\sigma}\,, \qquad (2)$$

where ω is the isoscalar vector meson coupling to baryon number. The interaction (2) will lead to the detection mode $\nu + N \to \nu + N + \gamma$ in laboratory neutrino experiments; and to the cooling mechanism $\gamma \to \nu\bar{\nu}$ in neutron stars. [7].

2.3 WZW for the standard model Higgs

As a prelude to applications in Little Higgs models, it is instructive to consider the WZW term for the Higgs boson in the standard model. Consider, for example, the situation where the third generation (t, b) quarks are integrated out, leaving an effective Lagrangian involving just the (ν_τ, τ) leptons, and the complete set of first and second generation fermions. We would like to think that the result is a consistent and predictive effective field theory. In particular, the low-energy theory should be gauge invariant under $SU(2)_L \times U(1)_Y$. Naively, it appears that such an effective theory is not possible—an inspection of triangle diagrams for the remaining fermions shows that there are uncancelled gauge anomalies from the third-generation leptons.

Of course, the resolution to this paradox is clear—we've left out an operator. At low energies, the only other matter fields in the theory besides the fermions are the Goldstone modes of the Higgs, represented by a field ϕ in $SU(2) \times U(1)/U(1) \cong S^3$ (S^3 is the three-sphere):

$$\phi = \exp\left[\frac{i}{v}\begin{pmatrix} 0 & g^+ \\ g^- & g^0 \end{pmatrix}\right]\begin{pmatrix} 0 \\ v \end{pmatrix}\,. \qquad (3)$$

This field transforms as an electroweak doublet:

$$\delta\phi = i(\epsilon + \epsilon')\phi\,, \qquad (4)$$

where ϵ and ϵ' generate $SU(2)_L$ and $U(1)_Y$. What operators can we build out of ϕ ?

It turns out that up to normalization, there is an essentially unique operator that can be built out of ϕ and the gauge bosons W, B that: (i) is globally invariant under $SU(2) \times U(1)$; and (ii) generates a consistent anomaly in the spontaneously broken generators. As outlined in Section 3, the explicit form of this operator can be obtained from a topological construction. Explicit calculation shows that under a general $SU(2) \times U(1)$ gauge transformation, the operator produces an anomaly proportional to the uncancelled lepton anomaly. Enforcing gauge invariance (that is, anomaly cancellation) then fixes the overall normalization factor. [8]

This example illustrates an important point in building consistent models of fermions, gauge fields and pNGB's, for example Little Higgs models. In order to enforce gauge invariance, we must be able to find operators made out of the fields in our theory that cancel any left-over gauge anomalies from explicit fermion degrees of freedom. Such a construction is guaranteed in the present case—we started with the consistent standard model, and by integrating out heavy degrees of freedom we should end up with a consistent low-energy theory. In bottom-up approaches where we don't know the UV theory, the possibility of anomaly cancellation is not guaranteed. Additional degrees of freedom might be necessary for consistency.

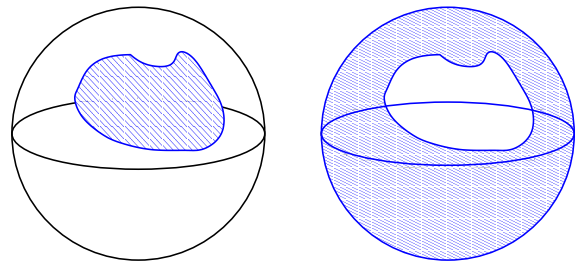

Fig. 1. Two possibilities for the surface whose boundary is the image of Φ. The WZW action is given by the area of this surface.

3 The simplest WZW term

I describe here the analog of Witten's construction [3] on $SU(3) \times SU(3)/SU(3) \cong SU(3)$, but with the simpler topology of $SU(3)/SU(2) \cong S^5$. This was outlined in [9] and details are presented in [10]. [1] The topological complexity in the former case comes from the need to identify a five-sphere inside of $SU(3)$. Since the field space in the latter case *is* a five sphere, the WZW term for $SU(3)/SU(2)$ has a particularly simple construction. Note that just as reducing $SU(3) \times SU(3)/SU(3)$ to $SU(2) \times SU(2)/SU(2)$ describes the pion sector in the QCD chiral Lagrangian, so reducing $SU(3) \times U(1)/SU(2) \times U(1)$ to $SU(2) \times U(1)/U(1)$ describes the SM Higgs sector. [2]

Fields on $SU(3)/SU(2) = S^5$ are represented by a vector $\Phi(x) = (\phi^1 + i\phi^2, \phi^3 + i\phi^4, \phi^5 + i\phi^6)^T$ with

$$\Phi^\dagger\Phi = \sum_{i=1}^{6}(\phi^i)^2 = 1\,. \qquad (5)$$

What are the globally $SU(3)$ invariant operators that we can write in terms of Φ ?

Let us focus on the ungauged action. There is of course the "ordinary" class of operators, such as the kinetic energy term

$$\mathcal{L}_K = \partial_\mu\Phi^\dagger\partial^\mu\Phi + \ldots\,. \qquad (6)$$

[1] Including the $U(1)$ factor in $SU(3) \times U(1)/SU(2) \times U(1)$ is straightforward and for simplicity it is not included here.

[2] In fact additional interesting subtleties come up in this case - it turns out that the reduction is only possible for an even number of "colors".

These operators are manifestly four-dimensional, local, and globally $SU(3)$ invariant.

Another operator that is not so obvious is constructed as follows. Note that $x \mapsto \Phi(x)$ is a mapping from spacetime [3] into S^5. Let the action be proportional to the area bounded by this mapping. Explicitly, we have

$$\Gamma_{WZW}(\Phi) = \frac{N}{\pi^2} \int_{M^5} \omega \,, \tag{7}$$

where M^5 is the surface with the image of spacetime as its boundary, and

$$\omega = \frac{-i}{8} \epsilon^{ABCDE} \Phi^\dagger \partial_A \Phi \partial_B \Phi^\dagger \partial_C \Phi \partial_D \Phi^\dagger \partial_E \Phi \,. \tag{8}$$

is the volume element (=surface area) on the sphere. In fact, as shown in Figure 1, there are two different surfaces with the image of spacetime as their boundary; for consistency, $e^{i\Gamma}$ should be independent of this choice. Since the difference is proportional to the total volume, [4] setting this difference equal to a multiple of 2π yields the displayed quantization condition, with N an even integer.

Having constructed the WZW term, it is straightforward to check that it is: (i) four dimensional (given $\Phi(x)$ defined on 4-d spacetime, we can compute $\Gamma(\phi)$); (ii) local (for a small change in $\Phi(x)$, the area defining the action changes by a small amount); and (iii) globally $SU(3)$ invariant ($SU(3)$ acts as a subgroup of rotations on the sphere, and the area is rotationally invariant). The action can be coupled to gauge fields for the $SU(3)$ generators, in which case it is still globally invariant, but generates an anomaly under local $SU(3)$ transformations. This anomaly corresponds to that of N left-handed fermions with global $SU(3)$ flavor symmetry.

This construction can be formalized in terms of the homotopy groups describing the topology of the sphere: [5] $\pi_4(S^5) = 0$, meaning that the construction is possible (for a given $\Phi(x)$, there is a surface with the image of $\Phi(x)$ as its boundary); and $\pi_5(S^5) = Z$, meaning that the construction is nontrivial (the difference of the mappings in Figure 1 wraps the sphere nontrivially, and the action must be quantized). This simple construction on the sphere carries over to more complicated spaces such as $SU(n) \times SU(n)/SU(n) = SU(n)$, $SU(n)/SO(n)$ and $SU(2n)/Sp(2n)$.

4 Little Higgs models

The starting point for so-called "composite" [12] and "Little" [13,14] Higgs models is summarized by the formula for one-loop radiative corrections to pNGB masses. Suppose that we weakly gauge a collection of symmetry generators,

$$\Lambda = \Lambda_V + \Lambda_A \,, \tag{9}$$

where Λ_V and Λ_A are the unbroken and broken components of the generator. [6] Then the mass-matrix for pNGB's is [15]

$$m^2_{ab} = M^2 \sum_{\Lambda} \mathrm{Tr} \Big\{ \, [\Lambda_V, [\Lambda_V, t^a_A]] \, t^b_A - [\Lambda_A, [\Lambda_A, t^a_A]] \, t^b_A \Big\} \,, \tag{10}$$

where M^2 is a nonperturbative mass scale set by underlying strong dynamics. If we suppose that an axial generator is gauged strongly enough so that $m^2 < 0$ for the physical Higgs, then electroweak symmetry will be broken "by vacuum misalignment" [12]. Alternatively, suppose that the gauged generators are arranged so that, for the physical Higgs, $m^2 = 0$ through one-loop corrections. Electroweak symmetry can then be broken by higher-order loop corrections, and contributions from the top-quark sector [13,16]. Assuming that the full electroweak symmetry breaking potential can be tuned or engineered in a plausible way, these models give a mechanism for a weakly coupled Higgs boson to leak down to the electroweak scale, along with extra particles such as heavy partners of the top quark, and partners of the $SU(2)_L \times U(1)_Y$ gauge bosons, that are involved in stabilising the Higgs mass against radiative corrections.

The anomaly structure of such theories provides an important probe of the underyling UV completion physics. The situation is analogous to probing QCD if we only had access to sub-GeV experiments. Anomaly physics enters in two ways. First, there are consistency conditions on the low-energy theory. For example, suppose we were able to deduce the electroweak gauge structure of a complete first-generation standard model—the electron and its neutrino (ν_e, e), and the pions, coupled to $SU(2)_L \times U(1)_Y$. For a consistent gauge theory, we would find that the only possible value of the coefficient multiplying the WZW term is $N_c = 3$. This provides an important clue to the UV completion, namely a theory of fundamental $SU(N_c = 3)$ quarks. We would also know that interactions such as $\pi^0 \to \gamma\gamma$ are not only possible in some UV completions, but required in any UV completion.

Anomaly physics can also enter in a second way. Focusing just on the pseudoscalar + gauge boson sector, we can look for reactions such as $\pi^0 \to \gamma\gamma$, and count $N_c = 3$. [7] We then know that whatever other light fermion content exists must be consistent with this value.

Let us look at a few simple examples to see how these arguments can be used to construct consistent Little Higgs models, and to constrain their UV completions.

[3] To be precise, we consider Euclidean spacetime compactified onto S^4.

[4] Note the minus sign coming from the relative orientation of the two surfaces. The total volume of S^5 is π^3.

[5] An equivalent statement can be made in terms of de-Rham cohomology describing the classes of differential forms that can be defined on the sphere [11].

[6] The subscripts denote "vector" and "axial" components in analogy to QCD-like symmetry-breaking patterns.

[7] We of course need an independent measurement of f_π.

4.1 $SU(3)/SU(2)$ Little Higgs

Consider first the implementation of the Little Higgs idea based on two copies of $SU(3) \times U(1)/SU(2) \times U(1)$, introduced by Kaplan and Schmaltz [17,18]. The model is described by two distinct "condensate" fields Φ_1, Φ_2, coupled to a single copy of $SU(3)_W \times U(1)_X$ gauge fields.

When three fermion generations are present, and treated on the same footing, there are uncancelled $SU(3)_W^3$, $SU(3)_W^2 \times U(1)_X$ and $U(1)_X^3$ gauge anomalies ("Model 1" of [18]). This is not necessarily a bad thing. In fact, if we let N_1 and N_2 be the coefficients of the WZW terms for the Φ_1 and Φ_2 sectors [9], then we can enforce anomaly cancellation as long as

$$N_1 + N_2 = 12\,, \tag{11}$$

or more generally $N_1 + N_2 = 4N_g$, with N_g the number of generations. Without adding additional fields into the low-energy theory, there is no freedom to speculate on the absence of the WZW term. Any candidate UV completion of this model must have exactly six "colors" per sector. [8] This is like the first application to the standard model discussed in the introduction to this section—before doing any measurements, consistency places tight constraints on the form of the UV completion.

By assigning different quantum numbers to the three generations, it is possible to cancel anomalies of the fermions amongst themselves ("Model 2" of [18]). In this case, we must have

$$N_1 = -N_2\,. \tag{12}$$

As we saw in Section 3, the "number of colors" must be even, so that we have the possibilities $N_1 = -N_2 = 0, 2, 4, \ldots$. An especially interesting scenario is where the two sectors are identical apart from the chirality of the underlying condensates, represented by the relative sign in front of the WZW term. Then apart from Yukawa couplings to fermions, an exact exchange symmetry exists between the sectors, and is broken only by topological interactions. In terms of the physical Higgs and W boson, and the extra isosinglet scalar field η and isodoublet vector field C_μ appearing in the model, we have [9]

$$\Gamma_{WZW} = \int d^4x\, \epsilon_{\mu\nu\rho\sigma} \Big\{ \frac{-2N}{8\pi^2\sqrt{3}F} \eta \mathrm{Tr}\big[F_W^{\mu\nu} F_W^{\rho\sigma}\big] + \frac{2N}{16\pi^2 F}\big[D^\mu H^\dagger F_W^{\nu\rho} C^\sigma + h.c.\big] + \ldots \Big\}. \tag{13}$$

This is like the second application to the standard model— if we can identify and measure an anomaly-mediated interaction, we acquire a discrete and powerful probe of the UV completion.

This example also illustrates a deficiency of the "moose" language of links and sites that is sometimes used to describe Little Higgs models. In this language it appears obvious that when the couplings in both sectors are identical, the reflection $\Phi_1 \leftrightarrow \Phi_2$ must be an exact symmetry. This led to the interesting proposal of an exact "T" parity [19] reflection symmetry, with implications for dark matter and missing energy collider signatures [20,21]. However, the parity is broken once anomalies are taken into account, since Γ_{WZW} in (13) is odd under this exchange. These considerations apply to models invoking T parity as a reflection symmetry in $[SU(3) \times SU(3)/SU(3)]^4$ [19]. Similarly, the T-parity, or "Goldstone boson parity" appearing in the kinetic terms of symmetric-space cosets like $SU(3) \times SU(3)/SU(3)$, $SU(5)/SO(5)$ and $SU(6)/Sp(6)$ is violated by anomalies (in the QCD chiral Lagrangian π^0 is odd under T parity, the photon is even, yet $\pi^0 \to \gamma\gamma$ is allowed!). [9]

4.2 $SU(5)/SO(5)$ Little Higgs

The usefulness of anomaly constraints apply more generally. Here we discuss briefly two implementations of the Little Higgs idea based on $SU(5)/SO(5)$.

In the so-called "Littlest Higgs without T parity" model [23], the extended-SM fermions couple only to one of the two gauged $SU(2)$ groups. This gauging can be arranged to be anomaly free, however two subtle problems arise. First, the fact that there is no left-over anomaly tells us that without extending the low-energy theory, the coefficient of the WZW term must be zero. In the absence of additional fields, this rules out a technicolor-like UV completion [23] (either fundamental or composite fermions) that could have explained the origin of $SU(5)/SO(5)$ symmetry breaking. A more detailed study reveals another difficulty [24]. The naive basis of operators in the theory specified by this gauging is not closed under renormalization.[10] This limits the predictive power of the theory, even when restricted to low-energy observables.

In the so-called "Littlest Higgs with T parity" [25, 20], apart from the breaking of T parity by anomalies, we also run into problems of consistency. The fermion content is anomalous (uncancelled $SU(2)^2 \times U(1)$ and $U(1)^3$ anomalies with the quantum numbers in [25, 20]). The form of these anomalies is such that they cannot be cancelled by the globally $SU(5)$ invariant WZW term, indicating that additional structure is required for gauge invariance. As a low energy theory, the model is inconsistent!

[8] For example, $SU(3)/SU(2)$ can be embedded inside $SU(3) \times SU(3)/SU(3)$, and a potential UV completion consists of two triplets of "techniquarks" transforming under $\mathbf{6}$ and $\bar{\mathbf{6}}$ of $SU(6)$. [9]

[9] Some prospects for identifying an alternate "T" parity were considered in [22]. This will generically require the existence or introduction of multiple condensate fields.

[10] For example, both $\bar{\psi}\,(i\partial\!\!\!/ + V\!\!\!/ + A\!\!\!/)\,\psi$, and $\bar{\psi} U\,(i\partial\!\!\!/ + V\!\!\!/ - A\!\!\!/)\,U^\dagger \psi$ are gauge invariant kinetic terms for the fermions. Here V and A denote broken and unbroken components of the gauge field, and U is a unitary matrix of pNGB's transforming as $U \to e^{i\epsilon} U e^{-i\tilde{\epsilon}}$, where the tilde changes the sign of broken generators.

Concentrating just on the scalar + gauge sector, it is possible to look for anomaly-mediated interactions that could guide us to a more complete model. Some work along these lines is presented in [27].

4.3 Little Higgs Summary

It is essential in any low-energy effective theory to be able to write down the most general operator consistent with an assumed field content and symmetry. This is especially important in bottom-up models such as the Little Higgs, where no particular UV completion is specified. Topological interactions represent one such class of operators. The anomaly structure encoded by these interactions is truly an IR probe of UV physics, providing consistency conditions on the low-energy theory, and constraints that any proposed UV completion must obey [26].

A perplexing folklore has developed in the Little Higgs literature, whereby gauge invariance of the low-energy theory is considered optional. This is sometimes justified by the misleading argument that extra heavy fermions might exist that cancel anomalies. Of course, if the heavy fermions are truly heavy, they should be integrated out of the low-energy theory, generating new operators that maintain gauge invariance; if they are not heavy, then they should be present in the low-energy theory, again maintaining gauge invariance. Such fermions will in general either be directly observable if they are light; or break global symmetries and affect the dynamics of electroweak symmetry breaking if they are heavy and transform under an incomplete representation of the global symmetry group; or be a candidate to identify with underlying "techniquarks" of strong dynamics if they transform under the complete representation of the global symmetry.

Far from being a nuisance, anomalies and consistency conditions of the Little Higgs are one of the few handles we have to constrain the low-energy theory, and to probe UV completion physics that is out of direct experimental reach.

5 Conclusion

We can apply the technology developed for phenomenological applications in the standard model and Little Higgs theories to more general problems. For instance, in the study of formal large-N equivalences between different fermion field theories [28], or between four-dimensional field theories and their conjectured holographic duals [29], the discrete nature of the WZW term can provide exact relations that are independent of N, or that are independent of wavefunction profiles in the extra dimension. As a practical matter, these equivalences can be used as an efficient calculational tool. For instance, the WZW term constructed directly on $SU(n)/SO(n)$ [30] can be derived immediately, including general gauge fields, from the WZW term for $SU(n) \times SU(n)/SU(n)$ [9].

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