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Lattice QCD results for the $B \to D^{(*)}\ell\nu$ form factors: $F(1)$ and $G(1)$

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I review the current status of lattice QCD calculations of the $B \to D$ and $B \to D^*$ form factors and discuss prospects for their improvement. Successful calculations within the quenched approximation demonstrate the power of lattice methods for calculating $F(1)$ and $G(1)$, and the unquenched calculations in progress should soon allow for a $2-3\%$ exclusive determination of $|V_{cb}|$.

I. INTRODUCTION AND MOTIVATIONS

Experimental measurements of the exclusive decays $B \to D\ell\nu$ and $B \to D^*\ell\nu$, in combination with theoretical input, allow for a precise measurement of the CKM matrix element $|V_{cb}|$. The $B \to D^{(*)}$ branching fractions are proportional to $|V_{cb}|^2$:

$$\begin{aligned}\frac{d\Gamma(B \to Dl\nu)}{dw} &= \text{known factor} \times |V_{cb}|^2 |\mathcal{F}_{B\to D}(w)|^2 \\ \frac{d\Gamma(B \to D^*l\nu)}{dw} &= \text{known factor} \times |V_{cb}|^2 |\mathcal{F}_{B\to D^*}(w)|^2,\end{aligned}$$

where $w \equiv v \cdot v'$ depends upon the $B-$ and $D-$ meson velocities, v and v', and the form factor $\mathcal{F}(w)$, parameterizes the nonperturbative contribution to the decay. Thus experiments can only measure the product $\mathcal{F}(w) \times |V_{cb}|$, and lattice QCD calculations of the form factor are needed to extract the CKM matrix element. Because a single q^2 point determines the normalization, one typically chooses the zero recoil point ($w = 1$) where the lattice calculation is easiest.

The $B \to D$ and $B \to D^*$ form factors were calculated by the Fermilab Lattice collaboration using the quenched approximation (neglecting dynamical quark effects) in Refs [1, 2]. Unquenched calculations are in progress by the Fermilab/MIlC Lattice Collaboration, and a preliminary result for the $B \to D$ form factor was presented at Lattice 2004 [4]. While the quenched calculations used Wilson light quarks, the unquenched calculations use the publicly available "2+1 flavor" MILC configurations which have three flavors of improved staggered quarks: one heavy quark flavor that has a mass close to that of the strange quark (m_s) and two degenerate light quarks with masses ranging from $m_s/10 \leq m_l \leq m_s$ [3]. Both sets of calculations use the Fermilab formulation for the b and c quarks.

II. OVERVIEW OF $B \to D$ AND $B \to D^*$ FORM FACTOR CALCULATIONS

In this section I will review the method used by the Fermilab Collaboration to calculate $\mathcal{F}_{B\to D}(1)$ [also referred to as $G(1)$] and $\mathcal{F}_{B\to D^*}(1)$ [also called $F(1)$]. This approach is the only one currently being used in unquenched simulations.

The $B \to D$ decay amplitude depends upon two form factors, h_+ and h_-:

$$\mathcal{F}_{B\to D}(w) = h_+^{B\to D}(w) - \frac{m_B - m_D}{m_B + m_D}\, h_-^{B\to D}(w).$$

Although h_+ and h_- could in principle be extracted directly from the lattice $B \to D$ correlation function, the errors resulting from this method would be large. Thus one instead constructs double ratios of correlation functions which isolate the form factors of interest:

$$\begin{aligned}\frac{\langle D|\bar{c}\gamma^4 b|\bar{B}\rangle\langle\bar{B}|\bar{b}\gamma^4 c|D\rangle}{\langle D|\bar{c}\gamma^4 c|D\rangle\langle\bar{B}|\bar{b}\gamma^4 b|\bar{B}\rangle} &= |h_+(1)|^2 \\ \frac{\langle D|\bar{c}\gamma_i b|\bar{B}\rangle\langle D|\bar{c}\gamma_0 c|D\rangle}{\langle D|\bar{c}\gamma_0 b|\bar{B}\rangle\langle D|\bar{c}\gamma_i c|D\rangle} &= 1 - \frac{h_-(w)}{h_+(w)},\end{aligned}$$

thereby canceling out the bulk of statistical fluctuations from Monte Carlo simulations as well as many systematic errors.

In contrast, the $B \to D^*$ decay amplitude only depends upon one form factor, h_{A_1}:

$$\mathcal{F}_{B\to D^*}(1) = h_{A_1}(1).$$

Because, however, h_{A_1}, cannot be determined from a single double ratio, one must use Heavy Quark Effective Theory (HQET) as an intermediary. When expanded in powers of the heavy-quark mass to $\mathcal{O}(1/m_Q^2)$, $h_{A_1}(1)$ depends upon three different HQET matrix elements :

$$h_{A_1}(1) = \eta_A\left[1 - \frac{\ell_V}{(2m_c)^2} + \frac{2\ell_A}{2m_c m_b} - \frac{\ell_P}{(2m_b)^2}\right].$$

The HQET matrix elements ℓ_V, ℓ_A, and ℓ_P can in turn be determined from the heavy quark mass dependence of three different double ratios. For example, the heavy-quark mass dependence of R_+ allows the extraction of ℓ_p:

$$\begin{aligned}\mathcal{R}_+ &= \frac{\langle D|\bar{c}\gamma^4 b|\bar{B}\rangle\langle\bar{B}|\bar{b}\gamma^4 c|D\rangle}{\langle D|\bar{c}\gamma^4 c|D\rangle\langle\bar{B}|\bar{b}\gamma^4 b|\bar{B}\rangle} = |h_+(1)|^2 \\ h_+(1) &= \eta_V\left[1 - \ell_P\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right]\end{aligned}$$

Once ℓ_V, ℓ_A, and ℓ_P have all been determined in this manner, they can be combined to form $h_{A_1}(1)$

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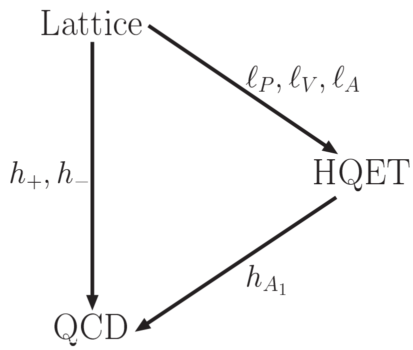


FIG. 1: Schematic of $B \to D$ and $B \to D^*$ form factor calculations.

The calculations of $F(1)$ and $G(1)$ share certain essential features: both use the same code (although with different operators) to generate correlation functions and take advantage of double ratios to reduce statistical and systematic errors. The calculation of $G(1)$, however, utilizes significantly less computing resources than that of $F(1)$. This is because $G(1)$ is extracted directly from lattice data at the tuned bottom and charm masses, whereas $F(1)$ is reconstituted from HQET matrix elements, the determination of which requires lattice data at multiple heavy quark masses. The difference in between the $F(1)$ and $G(1)$ calculations is illustrated schematically in Figure 1.

III. SYSTEMATICS IN LATTICE CALCULATIONS

Lattice calculations typically quote the following sources of error:

1. Monte carlo statistics and fitting,
2. Tuning the lattice spacing, a, and quark masses,
3. Matching lattice gauge theory to continuum QCD,
4. Extrapolation to continuum,
5. Chiral extrapolation to physical light quark masses.

Some lattice simulations also neglect dynamical quark loops – this is known as the "quenched approximation". Errors 3 and 5 in the above list are the dominant sources of uncertainty in current heavy-light lattice calculations, and I will discuss them in turn.

Because a generic lattice quark action will have discretization errors $\propto (am_q)^n$, one cannot use most light-quark actions to make heavy quarks ($\sim m_c, m_b$), for which am_q is of $\mathcal{O}(1)$ at currently available lattice spacings. The "Fermilab" method addresses this problem by using HQET to match continuum QCD directly to lattice gauge theory, thereby allowing systematic elimination of heavy quark discretization errors order-by-order [5–7]. This requires tuning the parameters of the lattice action and lattice currents to the continuum, and typically the matching coefficients are calculated using lattice perturbation theory [8]. Within the Fermilab formalism, one can combine all of the errors associated with discretizing the action into "heavy quark discretization errors" and estimate their size using knowledge of short-distance coefficients and power-counting.

The second significant source of error in most heavy-light lattice calculations is the chiral extrapolation. Because current lattice simulations are restricted to quark masses heavier than $\approx m_s/10$, one must extrapolate lattice results to the physical values of the up and down quark masses. For simulations using staggered quarks, one must extrapolate with functional forms from staggered chiral perturbation theory (SχPT), which accounts for both the next-to-leading order light quark mass dependence and light quark discretization effects through $O(\alpha_S^2 a^2 \Lambda_{QCD}^2)$[9–11]. Although the use of SχPT has been extremely successful for light-light meson quantities such as m_π and f_π, it is important to keep in mind that staggered lattice results agree with experimental values after chiral extrapolation in large part because the simulated quark masses are light and the lattice calculations are already close to the correct answer.

Figures 2 (a) and (b) show extrapolations with SχPT of the $B \to D$ and $B \to D^*$ form factors, respectively. The $B \to D$ form factor does not depend significantly on the light quark mass, and the systematic error associated with the chiral extrapolation is correspondingly small [12]. For the $B \to D^*$ form factor, however, extrapolation using the correct SχPT expression is essential. This is because the cusp located at $m_\pi^2 = (m_{D^*} - m_D)^2$, whose depth is proportional to the $D - D^* - \pi$ coupling in continuum χPT, becomes washed out by the presence of additional "tastes" of pions in the staggered lattice theory [13].

IV. QUENCHED RESULTS FOR $F(1)$ AND $G(1)$

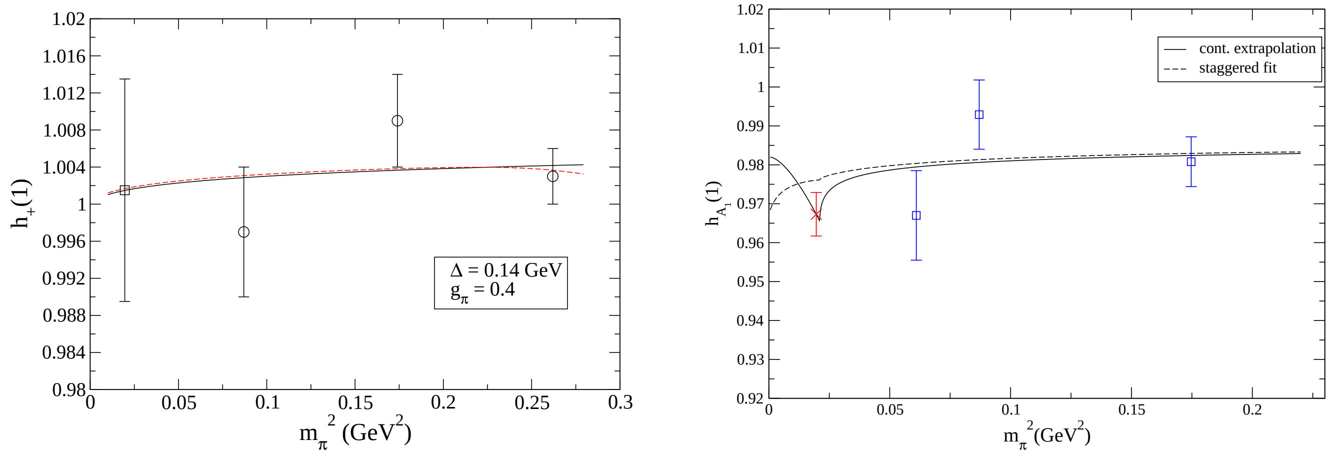
Using the methods described in the previous two sections, the Fermilab Collaboration determined $G(1)$ to 2% accuracy [1]:

$$\mathcal{F}_{B\to D}(1) = 1.058 \pm 0.016 \pm 0.003^{+0.014}_{-0.005},$$

where the errors are from statistics and chiral extrapolation, m_Q tuning, and perturbative matching, respectively. (Note that this calculation was only performed at a single lattice spacing.) They also determined $F(1)$ to 4% accuracy [2]:

$$\mathcal{F}_{B\to D^*}(1) = 0.9130 \pm^{+0.0238+0.0156+0.0032+0.0000}_{-0.0173-0.0157-0.0141-0.0163},$$

where the errors are from statistics, m_Q tuning and matching, lattice spacing dependence, and chiral extrapolation, respectively. Although these results do not reflect the systematic uncertainty due to quenching, they



(a) (b)

FIG. 2: Chiral extrapolations of the $B \to D$ [plot (a)] and $B \to D*$ [plot (b)] form factors [12, 13]. The dashed curves are fits to the lattice data using the appropriate staggered χPT expressions and the solid curves are the continuum extrapolations. Note that the $B \to D$ form factor has a mild light quark mass dependence, whereas the $B \to D^*$ extrapolation depends strongly on the value of the the $D - D^* - \pi$ coupling, g_π, which determines the depth of the continuum cusp.

show that all other sources of uncertainty are under control, and thus demonstrate the viability of the double ratio method for precise calculations of the $B \to D^{(*)}$ form factors at zero recoil.

The above results can, however, be improved in many ways. Unquenched calculations that include the effects of three light quark flavors are in progress [4, 13]. These will have lighter quark masses and increased statistics, and eventually finer lattice spacings, as compared to the quenched calculations. Heavy-quark discretization errors can be reduced through the use of 2-loop perturbative (or fully nonperturbative) matching. The heavy-quark action can also be improved by nonperturbatively determining the clover coefficient in the heavy-quark action as presented in Ref. [14], by extending the Fermilab method to include higher-dimension operators in the action as outlined in Ref. [15], or by some combination of these two approaches. Finally, calculations of the $B \to D^{(*)}$ form factors with different light quark actions (*e.g* domain-wall, overlap, or improved Wilson) or different heavy quark actions (*e.g* NRQCD) would provide valuable cross-checks.

One promising alternative to the Fermilab formulation of heavy quarks is the step-scaling method of Guagnelli *et. al.*, introduced in Ref. [16]. In this approach one calculates the quantity of interest on at least two lattices of different volumes. The lattices must have sufficiently fine lattice spacings that heavy quark discretization errors are under control [$(am_Q) < 1$] and perturbation theory is valid. One can then use knowledge of the scaling behavior with volume to extrapolate the two finite-volume results to infinite volume. A preliminary determination of the $B \to D$ form factor at $w = 1.05$ in the quenched approximation using the step-scaling method was presented at this workshop by Tantalo [17]:

$$\mathcal{F}_{B\to D}(1.05) = 0.986(30) \quad \text{(preliminary)}.$$

Neglecting quenching errors, the above result has a 3% uncertainty, and is therefore competitive with the Fermilab approach.

V. PROGRESS IN UNQUENCHED CALCULATIONS OF $F(1)$ AND $G(1)$

The Fermilab Collaboration presented a preliminary unquenched result for the $B \to D$ form factor in 2004 [4]:

$$\begin{aligned} \mathcal{F}^{n_f=2+1}_{B\to D}(1) &= 1.074\,(18)_{\rm sta}(16)_{\rm sys} \qquad \text{(preliminary)} \\ |V_{cb}| &= 3.8\,(1)_{\rm sta}(6)_{\rm sys} \times 10^{-2} \quad \text{(preliminary)}. \end{aligned}$$

As shown in Fig. 3, it is consistent with the earlier quenched value. Although the above result quotes a 2% error, it was determined with a single lattice spacing. This calculation, however, is still in progress, and the final result will have many improvements as compared to the preliminary number. It will have four times the statistics, an additional data point at an even lighter quark mass, and an additional (larger) lattice spacing to estimate the lattice spacing dependence. In addition, while the above calculation performed a simple linear extrapolation in the light quark mass, the improved calculation will use the correct SχPT expression [12].

Although there has not yet been an unquenched calculation of the $B \to D^*$ form factor, one can make a reasonable "prediction" of what the errors will be based on the quenched calculation and other factors. Recall that the total error in the quenched value of $F(1)$ is 4 %. In addition to removing the quenching uncertainty, the unquenched calculation will have increased statistics and a

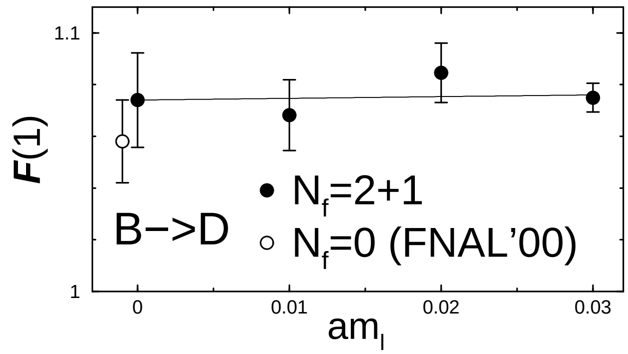


FIG. 3: Comparison of quenched and unquenched determinations of the $B \to D$ form factor. [4]

chiral extrapolation using the appropriate SχPT expression. Furthermore, because Sχ PT includes light quark discretization effects, this will not only reduce the chiral extrapolation error but also reduce the lattice spacing uncertainty. Thus the total error in the unquenched result for $F(1)$ will likely be 2-3%. A significant source of error in the calculation of $F(1)$ turns out to be the uncertainty in the $D - D^* - \pi$ coupling, g_π; this is because the size of the continuum cusp in Fig. 2(b) varies by ± 1% when g_π is varied within its experimental uncertainty. Thus, in order to get below the 2-3% level, a better experimental (or lattice) determination of g_π may be necessary.

VI. SUMMARY AND PROSPECTS FOR EXCLUSIVE DETERMINATION OF $|V_{ub}|$

The CKM matrix element $|V_{cb}|$ is currently known to 4% from exclusive decays, and this error is limited by the theoretical uncertainty in the $B \to D^{(*)}$ form factors at zero recoil [18]. Quenched results demonstrate the capability of lattice QCD calculations to determine $F(1)$, $G(1)$ to a few percent accuracy, and unquenched calculations are in progress using the same methodology. Although the quoted errors will likely only go from 4%→2-3% between the quenched and unquenched calculations, the results will be on a stronger theoretical footing, and hence more reliable. Reducing the errors to below 2-3%, however, will require additional work, such as higher-order matching and a better determination of g_π.

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