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# Charm decay constants and semileptonic form factors from lattice QCD

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I review the current status of lattice QCD calculations of $D$- and $D_s$-meson decay constants and semileptonic form factors and discuss prospects for improvements in future determinations of these quantities. Agreement between lattice calculations and experimental measurements of $D$-decays demonstrates the reliability of lattice methods for heavy-light mesons, and lends credence to similar calculations of $B$-decays.

## I. INTRODUCTION AND MOTIVATIONS

Leptonic and semileptonic decays of $B$- and $D$-mesons provide critical inputs to the unitarity triangle analysis. In particular, the branching fractions of the $D$- and $D_s$-mesons are related to $|V_{cd}|$ and $|V_{cs}|$ as follows:

$$\begin{aligned} \text{B.R.}(D \to \ell\nu) &= (\text{k.f.}) \times f_D^2 |V_{cd}|^2 \\ \text{B.R.}(D_s \to \ell\nu) &= (\text{k.f.}) \times f_{D_s}^2 |V_{cs}|^2, \end{aligned}$$

for the case of leptonic decays, and

$$\begin{aligned} \text{B.R.}(D \to \pi\ell\nu) &= |V_{cd}|^2 \int_0^{q^2_{max}} dq^2 f_+^{D\to\pi}(q^2)^2 \times (\text{k.f.}) \\ \text{B.R.}(D_s \to K\ell\nu) &= |V_{cs}|^2 \int_0^{q^2_{max}} dq^2 f_+^{D\to K}(q^2)^2 \times (\text{k.f.}), \end{aligned}$$

where "k.f." indicates a known factor. In both cases, experiments can only determine the product of a hadronic matrix element (either the decay constant, $f_D$, or the form factor, $f^+(q^2)$) and a CKM matrix element. Thus lattice QCD calculations of the decay constants and form factors are needed in order to extract $|V_{cd}|$ and $|V_{cs}|$.

Conversely, experimental measurements of branching fractions can be combined with values of $|V_{cd}|$ and $|V_{cs}|$ from elsewhere to "experimentally" determine the decay constants or form factors, which can then be compared with lattice QCD calculations. This approach provides a test of lattice QCD methods such as the incorporation of dynamical quark effects, the light quark formalism, the the heavy quark formalism, and the chiral and continuum extrapolations. Furthermore, correct lattice QCD results for $D$-mesons, where this check is possible, build confidence in similar lattice calculations with $B$-mesons. In this talk I will emphasize the use of $D-$decays to test the understanding and control of systematic uncertainties associated with lattice QCD calculations.

Currently two groups are calculating heavy-light meson quantities with three dynamical quark flavors: the Fermilab/MILC Lattice Collaboration and the HPQCD Collaboration.[1] Both use the publicly available "2+1 flavor" MILC configurations which have three flavors of improved staggered quarks: one heavy quark flavor that has a mass close to that of the strange quark ($m_s$) and two degenerate light quarks with masses ranging from $m_s/10 \le m_l \le m_s$ [1]. The two groups, however, use different heavy quark discretizations; Fermilab/MILC uses "Fermilab" quarks, while HPQCD uses nonrelativistic (NRQCD) heavy quarks. I will briefly describe the differences between the two methods, and how this translates to the systematic errors quoted by the two groups, in the next section.

## II. SYSTEMATICS IN LATTICE CALCULATIONS

Lattice calculations typically quote the following sources of error:

1. Monte carlo statistics and fitting,
2. Tuning the lattice spacing, $a$, and quark masses,
3. Matching lattice gauge theory to continuum QCD,
4. Extrapolation to continuum,
5. Chiral extrapolation to physical light quark masses.

Errors 3 and 5 in the above list are the dominant sources of uncertainty in current heavy-light lattice calculations, and I will discuss them in turn.

Because a generic lattice quark action will have discretization errors $\propto (am_q)^n$, one cannot use most light-quark actions to make heavy quarks ($\sim m_c, m_b$), for which $am_q$ is of $\mathcal{O}(1)$ at currently available lattice spacings. One solution is to use knowledge of the heavy quark or nonrelativistic quark limits of QCD to systematically eliminate heavy-quark discretization errors order-by-order. Both the "Fermilab" method and lattice NRQCD are variants of this general approach. The Fermilab method for lattice heavy quarks uses Heavy Quark Effective Theory to match continuum QCD directly to lattice gauge theory [2]. In contrast, lattice NRQCD requres a two step matching process: first one matches continuum QCD to continuum nonrelativistic QCD, then one matches continuum NRQCD to lattice gauge theory [3]. Both methods, however, require tuning parameters of the lattice action, and, for heavy-light decays,

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[1] Because of limited time, I will restrict the contents of this talk to three-flavor unquenched lattice calculations.

matching lattice currents to the continuum. Typically the matching coefficients are calculated using lattice perturbation theory [4].

In principle, one can remove errors of any order in the heavy quark mass. In practice, however, this becomes increasingly difficult at each higher order, and there remains a systematic error in lattice calculations due to inexact matching. Because the Fermilab method and lattice NRQCD use different matching procedures, Fermilab/MILC and HPQCD often divide their matching errors in different ways and give them different names, thereby making it difficult for those unfamiliar with the details and nomenclature to compare results between groups. In particular, with the Fermilab approach, one can combine all matching errors into "heavy quark discretization effects" and estimate their size using knowledge of short-distance coefficients and power-counting. With lattice NRQCD, however, one typically identifies errors from matching continuum QCD to NRQCD as "relativistic errors" [*e.g.* $\mathcal{O}(\alpha_s \Lambda_{QCD}/m_Q)$ and $\mathcal{O}(\Lambda^2_{QCD}/m_Q^2)$] and errors from the second step of matching NRQCD to lattice gauge theory as "perturbation theory erorrs" [*e.g.* $\mathcal{O}(\alpha_s^2)$]. The size of these errors can also be estimated using power-counting.

The second significant source of error in most heavy-light lattice calculations is the chiral extrapolation. Because current lattice simulations are restricted to quark masses heavier than $\approx m_s/10$, one must extrapolate lattice results to the physical values of the up and down quark masses. For simulations using staggered quarks, one must extrapolate with functional forms from staggered chiral perturbation theory (S$\chi$PT), which accounts for both the next-to-leading order light quark mass dependence and light quark discretization effects through $O(\alpha_S^2 a^2 \Lambda^2_{QCD})$[5–7]. Although the use of S$\chi$PT has been extremely successful for light-light meson quantities such as $m_\pi$ and $f_\pi$, it is important to keep in mind that staggered lattice results agree with experimental values after chiral extrapolation in large part because the simulated quark masses are light and the lattice calculations are already close to the correct answer.

## III. LATTICE RESULTS FOR $f_D$, $f_{D_s}$

The determination of $f_{D_s}$ by the HPQCD collaboration agrees well with experiment [9]:

$$\begin{aligned} f_{D_s} &= 290 \pm 20 \pm 29 \pm 29 \pm 6 \text{ MeV} \quad \text{lattice QCD} \\ &= 279 \pm 17 \pm 20\text{MeV} \quad \text{BaBar [10]} \end{aligned}$$

where the errors in the lattice result are from statistics and fitting, perturbation theory, relativistic corrections, and generic discretization effects, respectively. Because lattice NRQCD cannot be reliably used at the charm quark mass, the statistical error is dominated by the extrapolation of lattice data near the $b$-quark mass to charm. An example of this extrapolation is shown in

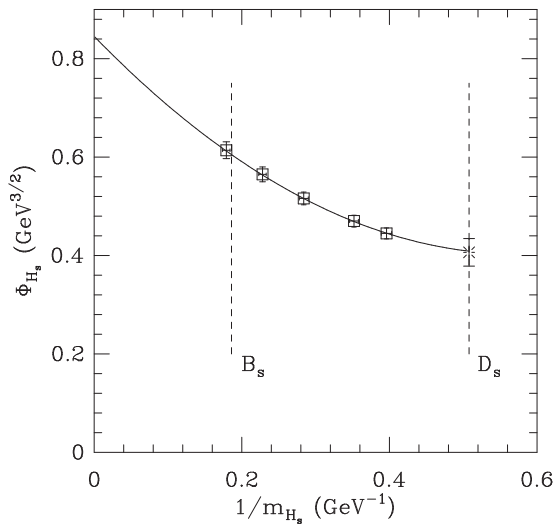


FIG. 1: $\Phi_{H_s} \equiv f_{H_s}\sqrt{m_{H_s}}$ vs. $1/m_{H_s}$. The solid curve shows the fit to the heavy-quark mass-dependence of the lattice data (squares) and the asterix denotes the extrapolated value of $\Phi$ at the $D_s$ mass [9].

Fig. 1. The perturbation theory error comes from 1-loop lattice-to-continuum operator matching.

The Fermilab/MILC lattice collaboration has calculated both $f_D$ and $f_{D_s}$ [11]:

$$\begin{aligned} f_{D^+} &= 201 \pm 3 \pm 6 \pm 9 \pm 13 \text{ MeV} \\ f_{D_s} &= 249 \pm 3 \pm 7 \pm 11 \pm 10 \text{ MeV}, \end{aligned}$$

where the errors are from statistics, lattice spacing and charm mass tuning, heavy quark discretization effects, and chiral extrapolation, respectively. (Note that these results have been finalized since CKM 2005.) These errors are smaller than those of HPQCD primarily for two reasons: with the Fermilab method one can simulate directly at the charm quark mass, and Fermilab/MILC uses partly nonperturbative current matching. Because these calculations preceded precise measurements by Cleo-c, and because they agree with experiment (see Fig. 2), they are successful lattice predictions.

The HPQCD collaboration can improve their calculation of the $D_s$-meson decay constant by using 2-loop perturbative (or nonperturbative) matching and by avoiding an extrapolation in the heavy-quark mass. They are currently developing a highly-improved staggered quark action which will allow them to simulate directly at the charm quark mass [13]. Similarly, the Fermilab/MILC collaboration can use 2-loop matching for the heavy-light current $\rho$-factor. They can also improve their heavy quark action by nonperturbatively determining the clover coefficient in the heavy-quark action as presented in Ref. [14], by extending the Fermilab method to include higher-dimension operators in the action as outlined in Ref. [15], or by some combination of these two techniques. In general, both groups could benefit from lighter quark masses and finer lattice spacings. Further-

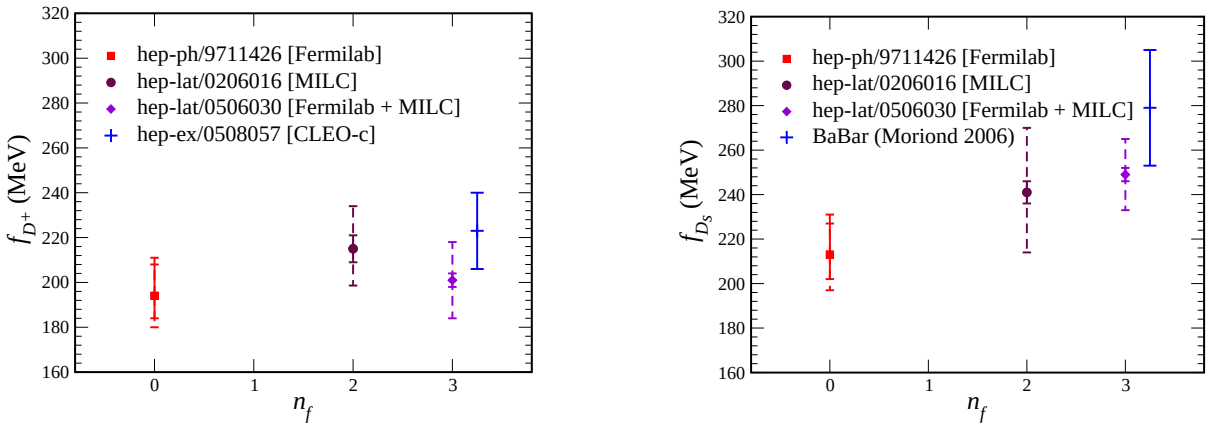


FIG. 2: Dependence of $f_D$ and $f_{D_s}$ on the number of dynamical quark flavors, $n_f$.

more, heavy-light calculations using different light-quark actions such as domain-wall, overlap, or improved Wilson fermions and using different heavy-quark formalisms such as the step-scaling method of Ref. [16] would provide valuable cross-checks on the light quark and heavy quark discretization errors, respectively.

Successful predictions of $f_D$ and $f_{D_s}$ lend confidence in lattice methods for quantities involving heavy-light mesons. The ratio of decay constants, in which several lattice uncertainties cancel, is particularly compelling:

$$\begin{aligned} R_{d/s} &= 0.786 \pm 0.043 \quad \text{lattice QCD [11]} \\ &= 0.779 \pm 0.093 \quad \text{Cleo-c/BaBar [10, 12]}, \end{aligned}$$

where $R_{d/s} \equiv f_D\sqrt{m_D}/f_{D_s}\sqrt{m_{D_s}}$. The HPQCD Collaboration's determination of $f_B$ is better than that of $f_D$ because they can simulate directly at the $b$-quark mass [17]:

$$\begin{aligned} f_B &= 216(9)(19)(4)(6)\ \text{MeV} \\ f_{B_s}/f_B &= 1.20(3)(1), \end{aligned}$$

whereas the Fermilab/MILC determinaton of $f_B$ is comparable to that of $f_D$, but with somewhat smaller heavy quark discretization errors [18]:

$$\begin{aligned} f_{B_s}/f_{D_s} &= 0.99(2)(6) \quad \text{(preliminary)} \\ f_B/f_D &= 0.95(3)(6) \quad \text{(preliminary)}. \end{aligned}$$

## IV. LATTICE RESULTS FOR $D \to \pi, K$

The Fermilab/MILC lattice collaboration has calculated both the $D \to \pi$ and $D \to K$ form factors [19]:

$$\begin{aligned} f_+^{D\to\pi}(0) &= 0.64(3)(6) \\ f_+^{D\to K}(0) &= 0.73(3)(7), \end{aligned}$$

where the errors are statistical and systematic. In combination with experiment, these can be used to determine the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$:

$$\begin{aligned} |V_{cd}| &= 0.239(10)(24)(20) \\ |V_{cs}| &= 0.969(39)(94)(24), \end{aligned}$$

where the third error is due to the experimental input. Given a value for $|V_{cd}|$, the result for $f_+^{D\to\pi}(0)$ is consistent with experiment, as shown in Fig. 3. Conversely, it enables a 14% measurement of $|V_{cd}|$, where the error dominated by discretization effects: 5% from momentum-dependent errors in the lattice data that impact the $q^2$ extrapolation and 7% from the lattice action for the heavy quarks. Figure 4 shows that the $D \to K$ form factor shape and normalization are also consistent with experiment. Furthermore, because the $D \to K$ form factor calculation preceded the Focus, Belle, and BaBar measurements, it is another lattice prediction, and thus provides further evidence that calculations of heavy-light meson decays are well-understood.

Both HPQCD and Fermilab/MILC have calculated the $B \to \pi$ semileptonic form factor, although the Fermilab/MILC result is still preliminary [22, 23]. There are two essential differences in the Fermilab/MILC lattice collaboration's error budgets for the $D \to \pi$ and $B \to \pi$ form factors: the discretization error decreases from $\sim 9\% \to 7\%$ because the $b$-quark is heavier, but the $q^2$ extrapolation error increases from $\sim 2\% \to 11\%$ for

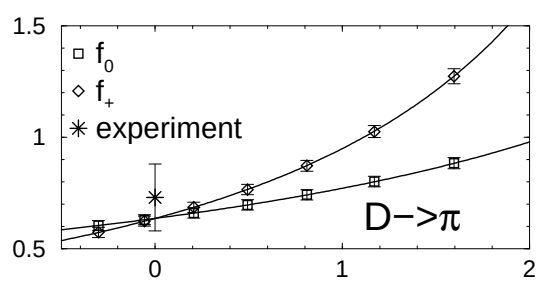


FIG. 3: $D \to \pi$ form factors vs. $q^2$ [19]. Experimental value of $f(0)$ from BES [20].

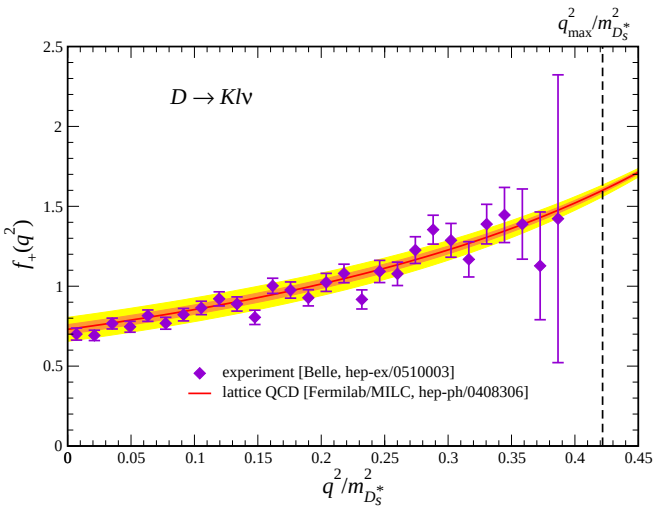


FIG. 4: $D \to K$ form factor vs. $q^2$. Figure from Ref. [21].

$f(0)$ because the minimum momentum transfer accessible using standard lattice techniques is $q^2_{\rm min} = 16$ GeV$^2$ [24]. Thus the dominant source of error changes from $D$-decays to $B$-decays, and, although most of the same lattice methods can be used, each calculation needs improvement in different areas

The Fermilab/MILC calculation of the $D$-meson form factors can be improved in the same manner as the decay constants – use of higher-order matching and an improved action. It is important, however, to have additional lattice calculations as a cross-check. Lattice calculations of the $B \to \pi$ form factor could be greatly improved with data at additional $q^2$ points, and there are currently two promising methods under study. "Moving NRQCD" allows one to generate lattice data at low $q^2$ (high pion momentum) while keeping statistical errors under control [25, 26]. Use of "twisted" boundary conditions allows one to generate high $q^2$ data points with pion momenta at noninteger values of $2\pi/L$, where $L$ is the spatial lattice size [27, 28].

## V. SUMMARY AND OUTLOOK

Leptonic and semileptonic $D$-decays allow for $\sim 10\%$ determinations of the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$, as well as provide important tests of lattice QCD methods. In particular, lattice QCD had made successful predictions for the leptonic decay constants $f_D$ and $f_{D_s}$ and for the shape of the $D \to K$ form factor. These results give confidence in similar lattice calculations of $B$-meson quantities. Lattice QCD calculations of $D$- and $B$-meson decays can still be improved, however, and effort is ongoing both to improve lattice heavy-quark actions and to increase the amount of lattice data for form factors at nonzero $q^2$, the latter which may be essential for a better than 10% determination of $|V_{ub}|$ exclusive.

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