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Higgs Triplets and Limits from Precision Measurements

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In this letter, we present our results on a global fit to precision electroweak data in a Higgs triplet model. In models with a triplet Higgs boson, a consistent renormalization scheme differs from that of the Standard Model and the global fit shows that a light Higgs boson with mass of $100-200$ GeV is preferred. Triplet Higgs bosons arise in many extensions of the Standard Model, including the left-right model and the Little Higgs models. Our result demonstrates the importance of the scalar loops when there is a large mass splitting between the heavy scalars. It also indicates the significance of the global fit.

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1. Introduction: One of the major goals of the LHC is uncovering the mechanisms of electroweak symmetry breaking and the generation of fermion masses. In the Standard Model (SM) of particle physics, the masses of gauge bosons and fermions are generated by the interactions with a single scalar field. After spontaneous symmetry breaking, a neutral CP-even Higgs boson, h, remains as a physical particle and the fermion and gauge boson masses arise through couplings to the Higgs boson. Discovering the Higgs particle and measuring its properties is central to an understanding of electroweak symmetry breaking.

Measurements at LEP, SLD, and the Tevatron have been extensively used to restrict the parameters of the Standard Model. In the SM, the mass of the Higgs boson is strongly constrained by precision electroweak measurements. If there are new particles or new interactions beyond those of the SM, a global fit to the experimental data can yield information about the allowed parameters of the model.

In models which contain more Higgs bosons than the $SU(2)_L$ doublet of the SM, there are more parameters in the gauge/Higgs sector than in the Standard Model. If these additional Higgs bosons are in $SU(2)_L$ representations other than singlets and doublets, the SM relation, $\rho = M_W^2/(M_Z^2 \cos^2\theta_W) = 1$ does not hold at tree level. This has the implication that when the theory is renormalized at one-loop, extra input parameters beyond those of the SM are required [1–6].

In this letter we consider a simple model with $\rho \neq 1$ at tree level, the Standard Model with a Higgs doublet and an additional Higgs triplet. Higgs triplets are an essential ingredient of the Little Higgs (LH) class of models and so have received significant attention recently [7].

LH models [7] are a new approach to understanding the hierarchy between the TeV scale of possible new physics and the electroweak scale, $v = 246\ GeV = (\sqrt{2}G_F)^{-1/2}$. These models have an expanded gauge structure at the TeV scale which contains the Standard Model $SU(2)\times U(1)$ electroweak gauge groups. The LH models are constructed such that an approximate global symmetry prohibits the Higgs boson from obtaining a quadratically divergent mass until at least two loop order. The Higgs boson is a pseudo-Goldstone boson resulting from the spontaneous breaking of the approximate global symmetry and so is naturally light. The Standard Model then emerges as an effective theory which is valid below the scale f associated with the spontaneous breaking of the global symmetry. LH models contain weakly coupled TeV scale gauge bosons from the expanded gauge structure, which couple to the Standard Model fermions. In addition, these new gauge bosons typically mix with the Standard Model W and Z gauge bosons. Modifications of the electroweak sector of the theory, however, are severely restricted by precision electroweak data and require the scale of the little Higgs physics, f, to be in the range $f > 1-6\ TeV$ [8], depending on the specifics of the model. The LH models also contain expanded Higgs sectors with additional Higgs doublets and triplets, as well as a new charge $2/3$ quark, which have important implications for precision electroweak measurements [5]. In Ref. [5] we found that by including the one-loop contributions from the heavy scalars, the scale f can be lowered. We have also observed the non-decoupling behaviour of the triplet. The non-decoupling of the scalar fields in models with additional scalar fields that acquire electroweak breaking VEV was first pointed out in Ref. [9].

The effects of the prediction for the $W-$boson mass in a Higgs triplet model were considered in Ref. [6] and here we present a global fit to the electroweak observables in this model.

2. The Triplet Model: We consider the Standard Model with an additional Higgs boson which transforms as a real triplet under the $SU(2)_L$ gauge symmetry. The $SU(2)_L$ Higgs doublet is identical to that of the SM,

$$H = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v+\eta^0+i\phi^0) \end{pmatrix} , \tag{1}$$

while the real triplet is

$$\Phi = \begin{pmatrix} \eta^+ \\ v'+\eta^0 \\ \eta^- \end{pmatrix} . \tag{2}$$

The W boson mass is given by,

$$M_W^2 = \frac{g^2}{4}(v^2 + v'^2) \ , \tag{3}$$

leading to the relationship $v_{SM}^2 = (246 \text{ GeV})^2 = v^2 + v'^2$.

There are four physical Higgs bosons in the spectrum: two neutral Higgs bosons, H^0 and K^0, and a charged Higgs boson, $H^\pm$. The mixing between the two neutral Higgs bosons is described by an angle γ,

$$\begin{pmatrix} H^0 \\ K^0 \end{pmatrix} = \begin{pmatrix} c_\gamma & s_\gamma \\ -s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} \phi^0 \\ \eta^0 \end{pmatrix} . \tag{4}$$

The charged Higgs bosons $H^\pm$ are linear combinations of the charged components in the doublet and the triplet, with a mixing angle δ,

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} c_\delta & s_\delta \\ -s_\delta & c_\delta \end{pmatrix} \begin{pmatrix} \phi^\pm \\ \eta^\pm \end{pmatrix} , \tag{5}$$

where $G^\pm$ are the Goldstone bosons corresponding to the longitudinal components of $W^\pm$.

In terms of the custodial symmetry violating parameter, ρ, the relation between the W and Z boson masses is modified from the SM relationship, $\rho = 1$, to be,

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{1}{\cos^2\delta} . \tag{6}$$

where $v' = \frac{1}{2} v \tan\delta$.

The symmetry breaking in this model is described by the following scalar potential,

$$\begin{aligned} V(H, \Phi) \ = \ & \mu_1^2 \left|H\right|^2 + \frac{1}{2}\mu_2^2 \Phi^2 + \lambda_1 \left|H\right|^4 + \frac{1}{4}\lambda_2 \Phi^4 \\ & + \frac{1}{2}\lambda_3 \left|H\right|^2 \Phi^2 + \lambda_4 H^\dagger \sigma^\alpha H \Phi_\alpha \ , \end{aligned} \tag{7}$$

where σ^α denotes the Pauli matrices. This model has six parameters in the scalar sector, $(\mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4)$. Equivalently, we can choose $(M_{H^0}, M_{K^0}, M_{H^\pm}, v, \tan\delta, \tan\gamma)$ as the independent parameters. Two of these six parameters, v and $\tan\delta$, contribute to the gauge boson masses. The six independent parameters in the scalar sector, along with the gauge couplings, g and g' completely describe the theory. We can equivalently choose the muon decay constant, G_μ, the Z-boson mass, M_Z, the effective leptonic mixing angle, $s_\theta \equiv \sin^2\theta_{\text{eff}}$, and the fine structure constant evaluated at M_Z, $\alpha(M_Z)$, as our input parameters, along with M_{H^0}, M_{K^0}, and $M_{H^\pm}$ and $\tan\gamma$ and the fermion masses, m_f.

From the minimization conditions, we obtain,

$$4\mu_2^2 t_\delta + \lambda_2 v^2 t_\delta^3 + 2\lambda_3 v^2 t_\delta - 4\lambda_4 v = 0 \tag{8}$$

$$\mu_1^2 + \lambda_1 v^2 + \frac{1}{8}\lambda_3 v^2 t_\delta^2 - \frac{1}{2}\lambda_4 v t_\delta = 0 \ , \tag{9}$$

where $t_\delta \equiv \tan\delta$. Consider the case when there is no mixing in the neutral sector,

$$\frac{\partial^2 V}{\partial\phi^0 \partial\eta^0} = \frac{1}{2}\lambda_3 v^2 t_\delta - \lambda_4 v = 0 \ , \tag{10}$$

the condition $\tan\delta = (2\lambda_4/\lambda_3 v)$ then follows. In the absence of the neutral mixing, $\gamma = 0$, in order to take the charged mixing angle δ to zero while holding λ_4 fixed, one thus has to take λ_3 to infinity. In other words, for the triplet to decouple requires a dimensionless coupling constant λ_3 to become strong, leading to the breakdown of the perturbation theory. Alternatively, the neutral mixing angle γ can approach zero by taking $\mu_2^2 \to \infty$ while keeping λ_3 and λ_4 fixed. In this case, the minimization condition implies that the charged mixing angle δ has to approach zero. This corresponds to the case where the custodial symmetry is restored, as the triplet VEV vanishes, $v' = 0$. However, severe fine-tuning is needed to satisfy the minimization condition. Another way to get $\delta \to 0$ is to have $\lambda_4 \to 0$. This corresponds to a case in which the model exhibits tree level custodial symmetry. So unless one imposes by hand a symmetry to forbid λ_4, four input parameters are always needed in the renormalization. As the neutral mixing angle, γ, does not contributes to the gauge boson masses, it is assumed to be zero hereafter and thus the scalar sector consists only five parameters. Having a non-zero value for γ does not change our conclusions. The effects on the heavy scalar masses in the presence of a non-zero γ can be found in Ref. [10].

The effective leptonic mixing angle is defined through the vector and axial vector parts of the effective 1-loop $Ze\overline{e}$ coupling, g_V^e and g_A^e, as,

$$1 - 4\sin^2\theta_{\text{eff}} = \frac{Re(g_V^e)}{Re(g_A^e)} \ , \tag{11}$$

while the counter term for $\sin^2\theta_{\text{eff}}$ is given by [16],

$$\begin{aligned} \frac{\delta s_\theta^2}{s_\theta^2} \ = \ & \text{Re}\left\{ \frac{c_\theta}{s_\theta} \left[\frac{g_{V0}^{e\,2} - g_{A0}^{e\,2}}{2 s_\theta c_\theta g_{A0}^e} \Sigma_A^e(m_e) + \frac{\Sigma^{\gamma Z}(M_Z)}{M_Z^2} \right.\right. \\ & \left.\left. - \frac{g_{V0}^e}{2 s_\theta c_\theta} \left(\frac{\Lambda_V^{Zee}(M_Z)}{g_{V0}^e} - \frac{\Lambda_A^{Zee}(M_Z)}{g_{A0}^e} \right) \right] \right\} . \end{aligned} \tag{12}$$

Here g_{V0}^e and g_{A0}^e are the tree level vector and axial vector parts of the $Ze\overline{e}$ coupling, Σ_A^e is the axial part of the electron self-energy, $\Lambda_{V,A}^{Zee}$ are the un-renormalized $Ze\overline{e}$ vertex corrections, and $\Sigma^{\gamma Z}$ is the $\gamma - Z$ two point mixing function. Experimentally, the measured values for these input parameters are [12], $G_\mu = 1.16637(1) \times 10^{-5}$ GeV^{-2}, $M_Z = 91.1876(21)$ GeV, $M_W = 80.410(32)$ GeV, $\sin^2\theta_{\text{eff}} = 0.2315(3)$ and $\alpha(M_Z) = 1/128.91(2)$.

We emphasize that the case considered here is different from the model considered by Chankowski *et al.* in [13]. In the present example, a triplet Higgs which has a VEV that breaks the electroweak symmetry is present, while in the model of Ref. [13], additional scalar fields except the

Observable	Experimental Value
M_W	80.410 ± 0.032 GeV
Γ_Z	2.4952 ± 0.0023 GeV
R_Z	20.767 ± 0.025
R_b	0.21629 ± 0.00066
R_c	0.1721 ± 0.0030
A_{LR}	0.1465 ± 0.0032
A_b	0.923 ± 0.020
A_c	0.670 ± 0.027
$A_{FB}^{0,l}$	0.01714 ± 0.00095
$A_{FB}^{0,b}$	0.0992 ± 0.0016
$A_{FB}^{0,c}$	0.0707 ± 0.0035

TABLE I: Precision data included in the global fit [11].

SM Higgs boson acquire VEVs that break the electroweak symmetry. In their case the effects of the additional Higgs multiplets can be decoupled from the SM predictions.

3. Global fit: Here we present a global fit to the suite of electroweak precision measurements shown in Table 1. The observables in the triplet Higgs model are calculated at 1-loop order using the results of Refs. [2] and [6]. Due to the presence of the extra Higgs bosons, the theoretical predictions are different from those of the Standard Model.

The effective vector and axial vector couplings of the fermion f to the Z boson, g_V^f and g_A^f, are determined at one loop in the triplet model,

$$g_V^f = \left(\rho \frac{1-\Delta\tilde{r}}{1+\hat{\Pi}^Z(M_Z^2)}\right)^{1/2} \cdot \left[g_{V0}^f + 2 s_\theta c_\theta Q_f \hat{\Pi}^{\gamma Z}(M_Z^2) + F_V^{Zf}(M_Z^2)\right] \tag{13}$$

$$g_A^f = \left(\rho \frac{1-\Delta\tilde{r}}{1+\hat{\Pi}^Z(M_Z^2)}\right)^{1/2} \cdot \left[g_{A0}^f + F_A^{Zf}(M_Z^2)\right] \tag{14}$$

They completely determine the observables of Table 1.

Using the one-loop corrected effective couplings g_V^f and g_A^f, we can then calculate various Z-pole observables. The on-resonance asymmetries A_f are determined by the effective coupling constants via the following relation,

$$A_f = \frac{2 g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2} . \tag{15}$$

Specifically, the left-right asymmetry is defined as $A_{LR} = A_e$, and the forward-backward asymmetry of the fermion as $A_{FB}^f = \frac{3}{4} A_e A_f$. The dependence of the asymmetries on the scalar masses and m_t appears only at $\mathcal{O}((\frac{1}{16\pi^2})^2)$ in the Higgs triplet model. As a result, predictions for various asymmetries, A_{LR}, A_b, A_c, $A_{FB}^{0\ell}$, $A_{FB}^{0,b}$ and $A_{FB}^{0,c}$, are relatively *insensitive* to m_t and to the scalar masses, M_{H^0}, M_{K^0}, and M_{H^+}. The prediction for the left-right asymmetry, A_{LR}, is shown in Fig. 1.

The partial width of Z decay to the fermion pair $f\overline{f}$ is

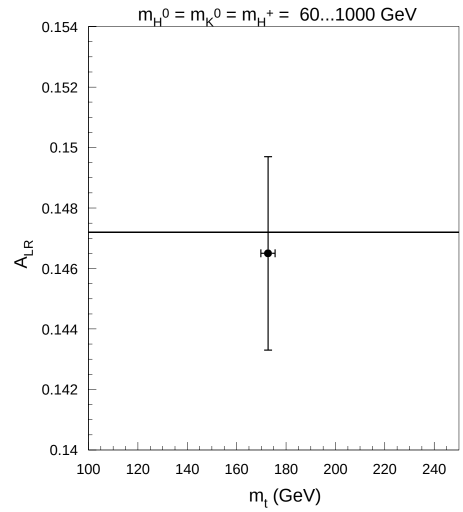

FIG. 1: Prediction for the left-right asymmetry, $A_{LR} = A_e$, as a function of m_t.The scalar masses, m_{H^0}, m_{K^0} and m_{H^+} are taken to be equal and are allowed to vary between 60 GeV to 1 TeV. The error bars on the data point represent the 1σ experimentally allowed region.

given by [2],

$$\Gamma_f = \Gamma_0 \left((g_V^f)^2 + (g_A^f)^2 \left(1 - \frac{6 m_f^2}{M_Z^2}\right)\right) \cdot \left(1 + Q_f^2 \frac{3\alpha}{4\pi}\right) + \Delta\Gamma_{QCD}^f , \tag{16}$$

where $\Gamma_0 = \frac{\sqrt{2} N_C^f G_\mu M_Z^3}{12\pi}$, $N_C^f = 1$ (3) for leptons (quarks), and $\Delta\Gamma_{QCD}^f$ summarizes the QCD corrections [2]. Note that g_V^f and g_A^f are the one-loop effective coupling constants, determined by Eq. 13 and 14. The factor $(1 + 3\alpha Q_f^2/4\pi)$ includes the corrections to the prefactor in the partial decay width. The total Z width is the sum of the fermion partial widths, $\Gamma_Z = \sum_f \Gamma_f$. Various ratios at the Z-pole are included in the fit and are defined as, $R_Z = \Gamma_{\text{had}}/\Gamma_e$, $R_c = \Gamma_c/\Gamma_{\text{had}}$, and $R_b = \Gamma_b/\Gamma_{\text{had}}$.

A numerical fit to just the W mass showed that there were large cancellations between the various contributions in the Higgs triplet model and the lightest neutral Higgs boson could be heavy, as shown in Fig. 2. This has been observed previously in generic models where $\rho \neq 1$ at tree level [2, 6]. Furthermore, the prediction for M_W^2 exhibits a logarithmic dependence, rather than a quadratic one, on m_t. In other words, the m_t^2 dependence in the case with a triplet Higgs has been absorbed into the definition of $\sin^2\theta_{\text{eff}}$ (or equivalently, the definition of ρ.)

It is interesting to note the pivotal role of Γ_Z in the fit. In the global fit without including the constraint from the experimental value of Γ_Z, the allowed parameter space for M_{H^0} is rather broad, ranging from 100 GeV to 1 TeV. This is consistent with the fit obtained

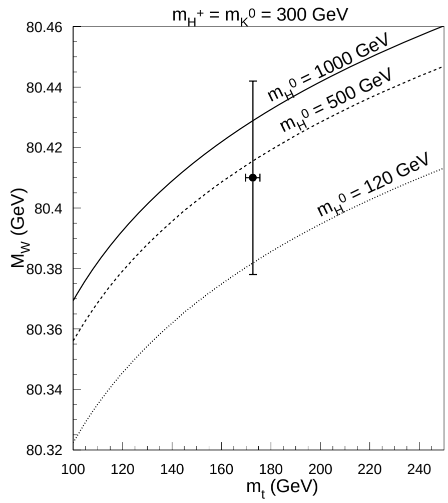

FIG. 2: Prediction for M_W as a function of m_t for M_{H^0} = 120, 500 and 1000 GeV[6]. The masses for M_{H^+} and M_{K^0} are taken to be 300 GeV. The error bars on the data point represent the 1σ experimentally allowed region.

to the W mass alone. However, if we include the constraint from the Γ_Z measurement, the best fit to the data then occurs for a light Standard Model like Higgs boson with 100 GeV $< M_{H^0} <$ 200 GeV and degenerate $M_{H^\pm} = M_{K^0}$, as illustrated in Fig. 3. Note that the fit is not sensitive to the mass of the degenerate Higgs bosons. This is consistent with the reults of Ref. [4]. We

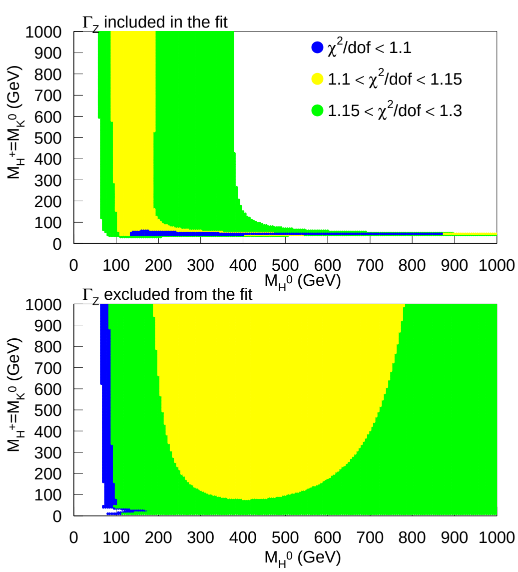

FIG. 3: The allowed parameter space in the M_{H^0} and M_{H^+} plane for various χ^2 values with and without the constraint from Γ_Z. The mass M_{K^0} is taken to be equal to M_{H^0}. The top quark mass is taken to be $m_t = 172.7$ GeV.

have also investigated the case where a mass splitting between M_{H^+} and M_{K^0} is present, as shown in Fig. 4. In this case, there are large contributions proportional to differences in the scalar masses. When the mass splitting is large, the contributions from the heavy scalars can be significant. It is found that the case in which the charged Higgs is heavier than the additional neutral Higgs, *i.e.* $M_{H^+} \gg M_{K^0}$, is disfavored, while the cases of $M_{H^+} \sim M_{K^0}$ and $M_{H^+} \ll M_{K^0}$ are allowed.

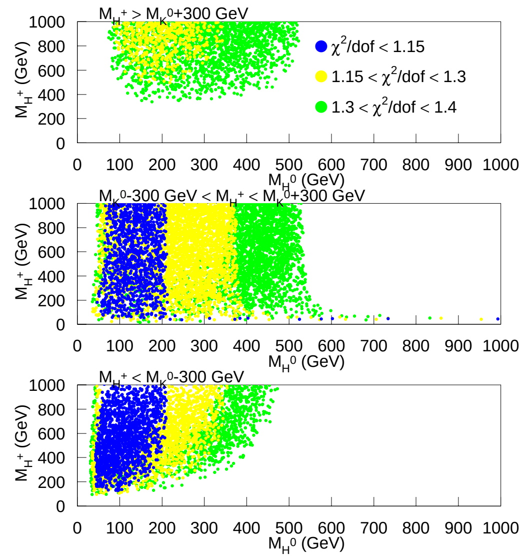

FIG. 4: The allowed parameter space in the M_{H^0} and M_{H^+} plane for various χ^2 values. Here we consider mass splitting between M_{K^0} and M_{H^+}. Three possible cases are considered: (*i*) $M_{H^+} - M_{K^0} > 300$ GeV; (*ii*) $\left|M_{H^+} - M_{K^0}\right| < 300$ GeV; (*iii*) $M_{K^0} - M_{H^+} > 300$ GeV. The top quark mass is taken to be $m_t = 172.7$ GeV.

The reason why Γ_Z plays such an important role in the global fit in the triplet model can be understood in the following way. The asymmetries in the triplet case do not receive corrections up to $\mathcal{O}((\frac{1}{16\pi^2})^2)$. Thus the only observables in the triplet model that are sensitive to m_t and M_{H^0} are M_W and Γ_Z. As a result, Γ_Z plays an important role in the χ^2 fit and can significantly constrain the allowed parameter space for M_{H^0}. On the other hand, all the observables considered in the global fit in the SM are sensitive to m_t and M_{H^0}. The constraint on Γ_Z alone therefore does not have such a large effect. This also has the implication that the χ^2/dof value for the global fit in the SM is not as good as that in the triplet model.

We comment that even though in the triplet model many observables exhibit only very mild logarithmic or no dependence on m_t up to $\mathcal{O}(\frac{1}{16\pi^2})$, the Z-width Γ_Z still depends on m_t quadratically, as can be seen in Fig. 5. Furthermore, in the triplet model, Γ_Z decreases as m_t increases, while in the SM case, Γ_Z increases as m_t. Due to this strong dependence on m_t through Γ_Z, it is still

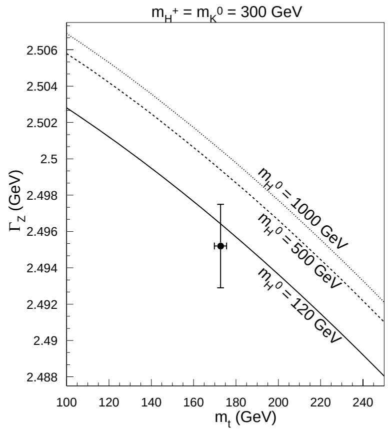

FIG. 5: Prediction for Γ_Z as a function of m_t for $M_{H^0} =$ 120, 500 and 1000 GeV. The masses for M_{H^+} and M_{K^0} are taken to be 300 GeV. The error bars on the data point represent the 1σ allowed region.

possible to place limits on m_t using the precision data in the Higgs triplet model.

It is interesting to note that in the littlest Higgs model with T-parity, a SM-like Higgs boson as heavy as 800 GeV is allowed by the global fit [14] (in this case, as tree level custodial symmetry is preserved, a three-parameter global fit is appropriate). This is because the new heavy top quark which exists in this model gives a positive contribution to the ρ parameter which cancels the large negative contribution from the heavy Higgs boson.

4. Conclusion: In models with a triplet Higgs boson, an extra input parameter is required for a consistent renormalization scheme, which significantly changes the predictions from those of the SM. Using the constraints on M_W alone, Ref. [6] found that a heavy Higgs boson with a mass as large as $\sim$ 1 TeV is allowed in a model with a triplet Higgs. This letter contains our results from a global fit to 11 electroweak measurements in the Higgs triplet model.

A large range for M_{H^0} is allowed in the triplet Higgs model by all precision data except for Γ_Z, which rules out many of the otherwise allowed values for M_{H^0}. As a result, a mass range of $100 - 200$ GeV is favored for the lightest neutral Higgs boson mass. This is in contrast with the SM where the Higgs mass allowed by the Γ_Z measurement alone is heavier than the Higgs mass favored by the global fit [11]. A global fit for models with a triplet Higgs has been performed before [4], in which the allowed range for the Higgs mass is very close to the range we found. A major difference between the previous analysis presented in [4] and our work is that the one-loop contributions from the heavy scalar fields were not included in the former case. The one-loop contributions from the heavy scalar particles could be substential in some parameter space, because many observables depend on scalar masses quadratically and the triplet Higgs does not decouple. (This non-decoupling behavior has also been observed in [15]. See Note Added).

An important conclusion that should be drawn from our study is the importance of the one-loop contributions as well as the crutial role of a global fit in deriving conclusions about the allowed masses. Our result demonstrates that the SM is not the only model which can fit the electroweak data.

Note added: After we submitted our paper, a paper [15] came out in which the electroweak radiative corrections are analyzed with a different renormalization scheme. The non-decoupling behavior of the triplet Higgs is also observed in this paper.

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[16] Note that a factor of $2s_\theta c_\theta$ is missing in the first term of Eq. (3.10) in Ref. [2].