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Neutrinos: In and Out of the Standard Model [1]

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Abstract. The particle physics Standard Model has been tremendously successful in predicting the outcome of a large number of experiments. In this model Neutrinos are massless. Yet recent evidence points to the fact that neutrinos are massive particles with tiny masses compared to the other particles in the Standard Model. These tiny masses allow the neutrinos to change flavor and oscillate. In this series of Lectures, I will review the properties of Neutrinos In the Standard Model and then discuss the physics of Neutrinos Beyond the Standard Model. Topics to be covered include Neutrino Flavor Transformations and Oscillations, Majorana versus Dirac Neutrino Masses, the Seesaw Mechanism and Leptogenesis.

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In the Standard Model the neutrinos, $(\nu_e,\ \nu_\mu,\ \nu_\tau)$, are massless and interact diagonally in flavor,

$$\begin{array}{llll} W^+ & \to e^+ + \nu_e & Z & \to \nu_e + \bar{\nu}_e \\ W^+ & \to \mu^+ + \nu_\mu & Z & \to \nu_\mu + \bar{\nu}_\mu \\ W^+ & \to \tau^+ + \nu_\tau & Z & \to \nu_\tau + \bar{\nu}_\tau . \end{array} \tag{1}$$

Since they travel at the speed of light, their character cannot change from production to detection. Therefore, in flavor terms, massless neutrinos are relatively uninteresting compared to quarks.

1. NEUTRINO OSCILLATIONS IN VACUUM:

If neutrinos have mass, then time passes for them and they can change character since they are not traveling at the speed of light. Typically, the neutrino states that interact with the W and Z bosons are not necessarily the states that propagate simply in time but they are related by a unitary matrix,

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{2}$$

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where $(\nu_\mu,\ \nu_\tau)$ are the flavor states, eg $W^+ \to \mu^+ + \nu_\mu$ and $(\nu_1,\ \nu_2)$ are the mass eigenstates. The angle θ is the mixing angle to be determined experimentally and eventually explained by the theory of fermion masses. The mass eigenstates propagate in time as $|\nu_j\rangle \to e^{-ip_j\cdot x}|\nu_j\rangle$ with $p_j^2 = m_j^2$. (The greek (latin) letters $\alpha, \beta \ldots (i, j \ldots)$ refer to flavor (mass) eigenstates.)

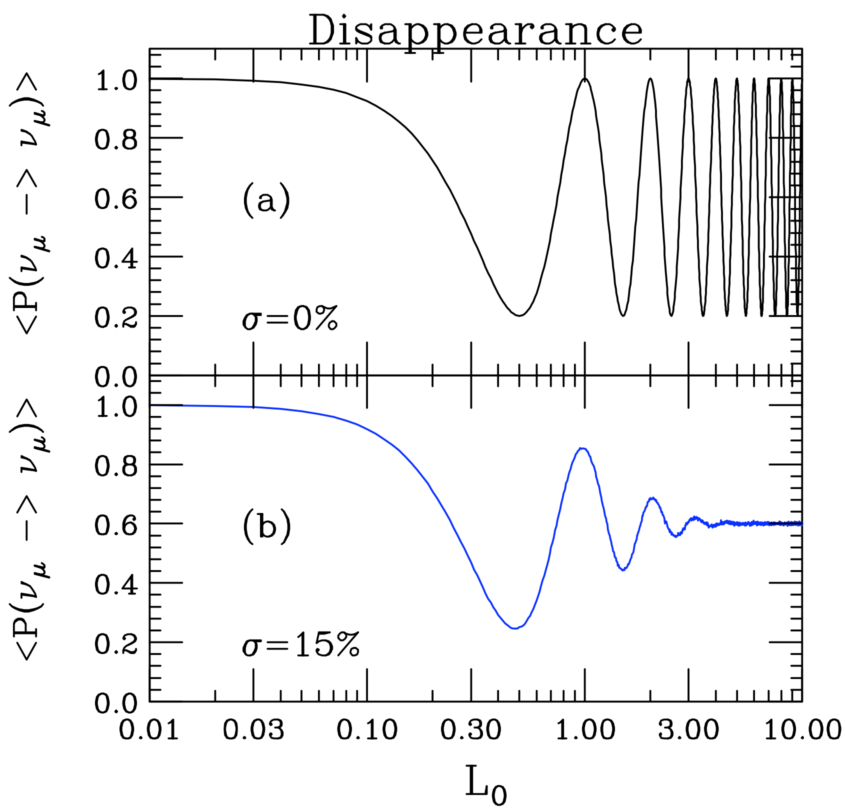

FIGURE 1. The survival probability for a muon neutrino versus distance traveled in units of the oscillation length, $4\pi E/\delta m^2$: (a) for fixed neutrino energy, (b) using a gaussian energy spread equal to 15% of the mean energy of the neutrino. Notice that even for this narrow band beam the oscillations have disappeared after three oscillations!

Thus, the life of a neutrino can be represented as follows (at the amplitude level):

$$\text{At Production:} \quad |\nu_\mu\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

$$\text{During Propagation:} \quad |\nu_1\rangle \to e^{-ip_1\cdot x}|\nu_1\rangle \text{ and } |\nu_2\rangle \to e^{-ip_2\cdot x}|\nu_2\rangle$$

$$\text{At Detection:} \quad \begin{cases} |\nu_1\rangle = \cos\theta|\nu_\mu\rangle - \sin\theta|\nu_\tau\rangle \\ |\nu_2\rangle = \sin\theta|\nu_\mu\rangle + \cos\theta|\nu_\tau\rangle \end{cases}$$

Thus, the transition probability for a neutrino to change flavor is

$$P(\nu_\mu \to \nu_\tau) = |\cos\theta(e^{-ip_1\cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2\cdot x})\cos\theta|^2. \tag{3}$$

Using the same E formulation, we have that $p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$ and therefore

$$P(\nu_\mu \to \nu_\tau) \;=\; \sin^2\theta\cos^2\theta |e^{-im_2^2 L/2E} - e^{-im_1^2 L/2E}|^2 = \sin^2 2\theta \sin^2 \Delta \qquad (4)$$

where $\Delta \equiv \delta m^2 L/4E$ is the kinematic phase, with $\delta m^2 = m_2^2 - m_1^2$. The disappearance probability is given by

$$P(\nu_\mu \to \nu_\mu) \;=\; 1 - P(\nu_\mu \to \nu_\tau) = 1 - \sin^2 2\theta \sin^2 \Delta. \qquad (5)$$

If we put the $\hbar$'s and c's into the appearance probability we find

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2 \left(\frac{\delta m^2 c^4 L}{4\hbar c E}\right). \qquad (6)$$

In the semi-classical limit, $\hbar \to 0$, the oscillation length goes to zero and the oscillations are averaged out. This is the same limit as letting δm^2 become large. This is precisely what happens in the quark sector. In Fig. 1 we have shown the oscillation probability for both fixed energy and a gaussian spread of 15% of the mean neutrino energy. Notice that oscillations are observable only for a limited range of distance. At small distance the simple flavor description is a good one. But at very large distance using the probability description with mass eigenstates works well since the oscillations are averaged out. The neutrino mass eigenstates are effectively incoherent. Thus, in terms of probabilities[2]

At Production:	the fraction of $\lvert\nu_\mu\rangle$ that is $\lvert\nu_1\rangle$ is $\cos^2\theta$
	the fraction of $\lvert\nu_\mu\rangle$ that is $\lvert\nu_2\rangle$ is $\sin^2\theta$
During Propagation:	flavor fractions in $\lvert\nu_1\rangle$ and $\lvert\nu_2\rangle$ remain unchanged
At Detection:	the fraction of $\lvert\nu_1\rangle$ that is $\lvert\nu_\mu\rangle$ is $\cos^2\theta$
	the fraction of $\lvert\nu_1\rangle$ that is $\lvert\nu_\tau\rangle$ is $\sin^2\theta$
	the fraction of $\lvert\nu_2\rangle$ that is $\lvert\nu_\mu\rangle$ is $\sin^2\theta$
	the fraction of $\lvert\nu_2\rangle$ that is $\lvert\nu_\tau\rangle$ is $\cos^2\theta$

Thus, in the ν_μ beam, the fraction of ν_1 is $f_1 = \cos^2\theta$ and ν_2 is $f_2 = \sin^2\theta$, independently of the neutrino energy, and the survival probability is

$$\begin{aligned} P(\nu_\mu \to \nu_\mu) &= f_1 \cos^2\theta + f_2 \sin^2\theta \qquad (7) \\ &= \cos^4\theta + \sin^4\theta = 1 - \sin^2 2\theta \left\langle \sin^2\Delta \right\rangle, \end{aligned}$$

since $\left\langle \sin^2\Delta \right\rangle = 1/2$. Notice that the full treatment given earlier is really only useful for distances around (1/5 to 5 times, say) the oscillation length, $L_0 = 4\pi E/\delta m^2$. At small distance, the oscillations haven't built up enough to be significant, whereas as at the large distance the oscillations are average out.

[2] ν_μ is the neutrino produced in association with μ^+.

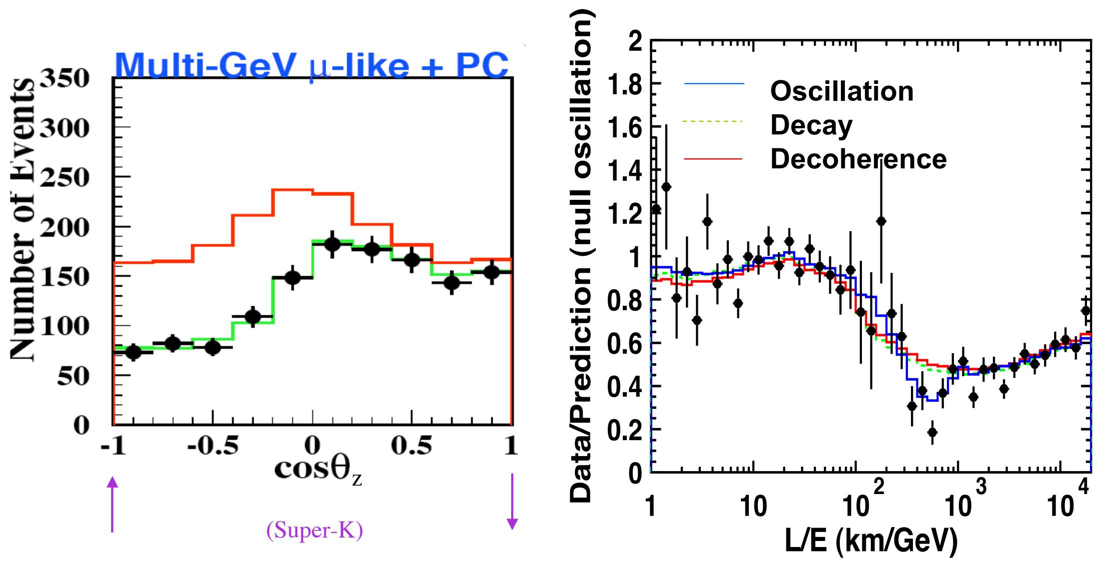

FIGURE 2. SuperKamiokande's evidence for neutrino oscillations both in the zenith angle and L/E plots.

2. EVIDENCE FOR NEUTRINO OSCILLATIONS:

2.1. Atmospheric and Accelerator Neutrinos

SuperKamiokande(SK) has very compelling evidence for ν_μ disappearance in their atmospheric neutrino studies, see [1]. In Fig. 2 the zenith angle dependence of the multi-GeV ν_μ sample is shown together with their L/E plot. This data fits very well the simple two component neutrino hypothesis with

$$\delta m^2_{atm} = 2-3\times 10^{-3} eV^2 \qquad \text{and} \qquad \sin^2\theta_{atm} = 0.50 \pm 0.15 \tag{8}$$

This corresponds to a L/E for oscillations of 500 km /GeV and nearly maximal mixing. No evidence for the involvement of the ν_e is observed so the assumption is that $\nu_\mu \to \nu_\tau$.

Two beams of ν_μ neutrinos have been sent to two detectors located at large distance: K2K experiment, [2], is from KEK to SK with a baseline of 250 km and the MINOS experiment, [3], from Fermilab to the Soudan mine with a baseline of 735 km. Both experiments see evidence for ν_μ disappearance which is summarized in Fig. 3

2.2. Reactor and Solar Neutrinos:

The KamLAND reactor experiment, [5], sees evidence for neutrino oscillations and not only at a different L/E than the atmospheric and accelerator experiments but also this oscillation involves the ν_e. These flavor transitions have also been seen in solar neutrino

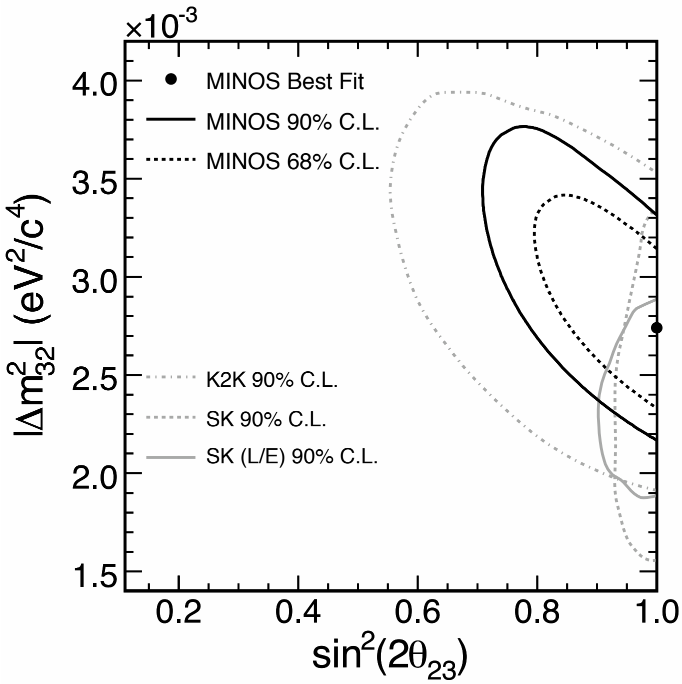

FIGURE 3. The allowed regions in the δm^2_{atm} v $\sin^2\theta_{atm}$ plane for MINOS data as well as for K2K data and two of the SK analyses. MINOS's best fit point is at $\sin^2\theta_{atm} = 1$ and $\delta m^2_{atm} = 2.7\times 10^{-3}$ eV^2.

experiments. The best fit values for $\delta m^2_{\odot}$ and $\sin^2\theta_{\odot}$ are

$$\delta m^2_{\odot} = 8.0 \pm 0.4 \times 10^{-5} eV^2 \qquad \text{and} \quad \sin^2\theta_{\odot} = 0.31 \pm 0.03. \tag{9}$$

Thus, the L/E for this oscillation is 15 km/MeV which is 30 times larger than the atmospheric scale and the mixing angle, though large, is not maximal.

Fig. 4 shows the disappearance probability for the $\bar{\nu}_e$ from many reactor experiments as well as the flavor content of the ^{8}Boron solar neutrino flux measured by SNO, [6], and SK, [7]. The reactor result can be understood in terms of vacuum neutrino oscillations and the fit to the disappearance probability, Eq. [4], suitably averaged over E and L, provides a good fit.

Solar neutrinos are somewhat more complicated because of the matter effects that the neutrinos experience from the production region until they exit the sun, at least for the ^{8}Boron neutrinos. The pp and ^{7}Be neutrinos are little effected by the matter and undergo quasi-vacuum oscillations whereas the ^{8}Boron neutrinos exit the sun mainly as a ν_2 mass eigenstate because of matter effects and therefore do not undergo oscillations. This difference is primarily due to the difference in the energy of the neutrinos: pp (^{7}Be) have a mean energy of 0.2 MeV (0.9 MeV) whereas ^{8}B have a mean energy of 10 MeV and the matter effect is proportional to energy of the neutrino.

The kinematic phase for solar neutrinos is

$$\Delta_{\odot} = \frac{\delta m^2_{\odot} L}{4E} = 10^{7\pm1}. \tag{10}$$

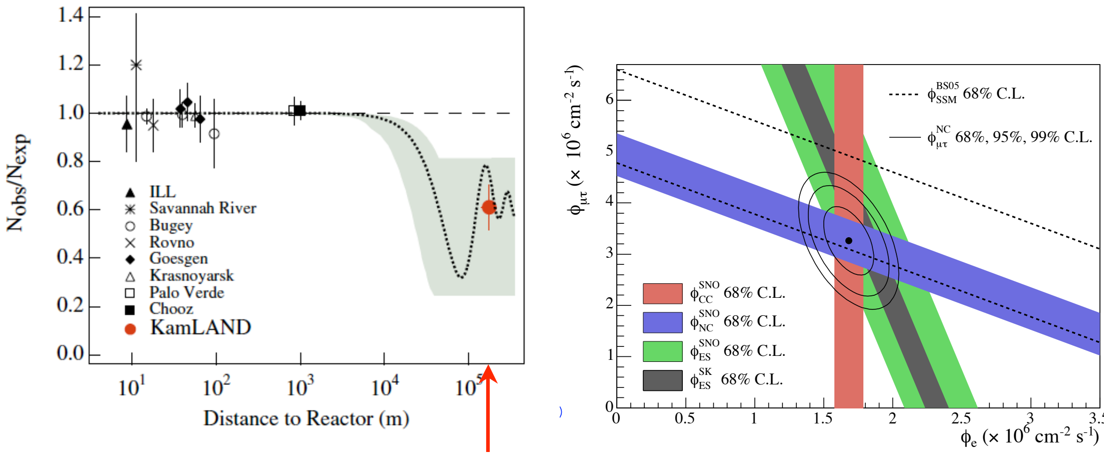

FIGURE 4. The disappearance of $\overline{\nu}_e$ observed by reactor experiments as a function of distance from the reactor. The flavor content of the ^{8}Boron solar neutrinos for the various reactions for SNO and SK. CC: $\nu_e + d \to e^- + p + p$, NC: $\nu_x + d \to \nu_x + p + n$ and ES: $\nu_\alpha + e^- \to \nu_\alpha + e^-$.

Therefore, the solar neutrinos are "effectively incoherent" when they reach the earth. Hence the ν_e survival probability is given by[3]

$$\begin{aligned} \langle P_{ee} \rangle &= f_1 \cos^2\theta_\odot + f_2 \sin^2\theta_\odot \qquad (11) \\ \text{where} \quad f_1 + f_2 &= 1 \quad \text{and} \quad \cos^2\theta_\odot + \sin^2\theta_\odot = 1. \end{aligned}$$

Now the pp and ^{7}Be solar neutrinos behave essentially as in vacuum and therefore $f_1 \approx \cos^2\theta_\odot = 0.69$ and $f_2 \approx \sin^2\theta_\odot = 0.31$ whereas the mass eigenstate fraction for the ^{8}B are substantially different, see Fig. 5.

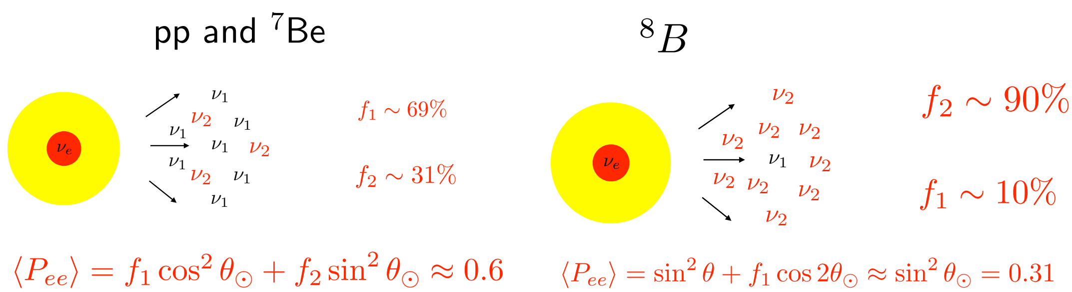

FIGURE 5. The sun produces ν_e in the core but once they exit the sun thinking about them in the mass eigenstate basis is useful. The fraction of ν_1 and ν_2 is energy dependent above ~ 1 MeV and has a dramatic effect on the ^{8}Boron solar neutrinos, as first observed by Davis.

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[3] Given the relationship between the quantities in this expression there are many equivalent ways to write the same expression.

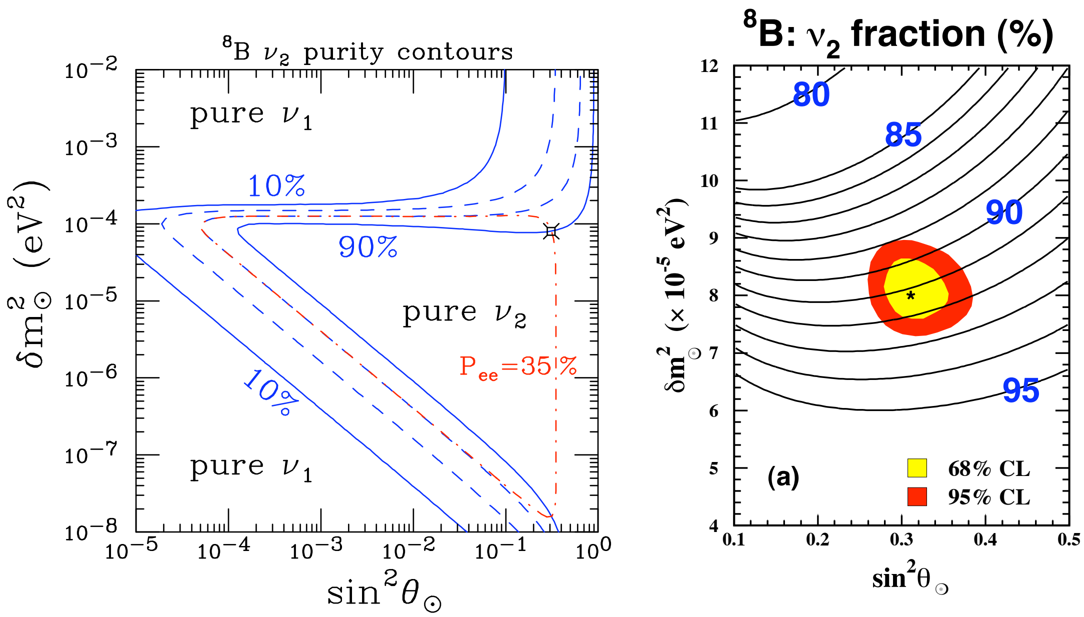

FIGURE 6. The ν_2 fraction (%) in the $\delta m^2_\odot$ versus $\sin^2\theta_\odot$ plane. (a) The solid and dashed (blue) lines are the 90, 65, 35 and 10% iso-contours of the fraction of the solar ^{8}B neutrinos that are ν_2's. The current best fit value, indicated by the open circle with the cross, is close to the 90% contour. The iso-contour for an electron neutrino survival probability, $\langle P_{ee}\rangle$, equal to 35% is the dot-dashed (red) "triangle" formed by the 65% ν_2 purity contour for small $\sin^2\theta_\odot$ and a vertical line in the pure ν_2 region at $\sin^2\theta_\odot = 0.35$. Except at the top and bottom right hand corners of this triangle the ν_2 purity is either 65% or 100%. (b) Focuses in on the current allowed region. The 68 and 95% CL are shown by the shaded areas with the best fit values indicated by the star using the combined fit of KamLAND and solar neutrino data given in [6] .

In a two neutrino analysis, the *day-time* CC/NC of SNO, which is equivalent to the day-time average ν_e survival probability, $\langle P_{ee}\rangle$, is given by

$$\left.\frac{\mathrm{CC}}{\mathrm{NC}}\right|_{\mathrm{day}} = \langle P_{ee}\rangle = f_1\cos^2\theta_\odot + f_2\sin^2\theta_\odot, \tag{12}$$

where f_1 and $f_2 = 1 - f_1$ are understood to be the ν_1 and ν_2 fractions, respectively, averaged over the ^{8}B neutrino energy spectrum weighted with the charged current cross section. Therefore, the ν_1 fraction (or how much f_2 differs from 100%) is given by

$$f_1 = \frac{\left(\left.\frac{\mathrm{CC}}{\mathrm{NC}}\right|_{\mathrm{day}} - \sin^2\theta_\odot\right)}{\cos 2\theta_\odot} = \frac{(0.347 - 0.311)}{0.378} \approx 10 \pm ??\,\%, \tag{13}$$

where the central values of the recent SNO analysis, [6], have been used. Due to the correlations in the uncertainties between the CC/NC ratio and $\sin^2\theta_\odot$ we are unable to estimate the uncertainty on f_1 from their analysis. Note, that if the fraction of ν_2 were 100%, then $\frac{\mathrm{CC}}{\mathrm{NC}} = \sin^2\theta_\odot$.

Using the analytical analysis of the Mikheyev-Smirnov-Wolfenstein (MSW) effect given in Ref. [8], the mass eigenstate fractions are given by

$$f_2 = 1 - f_1 = \langle \sin^2\theta^N_\odot + P_x \cos 2\theta^N_\odot\rangle_{^8\mathrm{B}}, \tag{14}$$

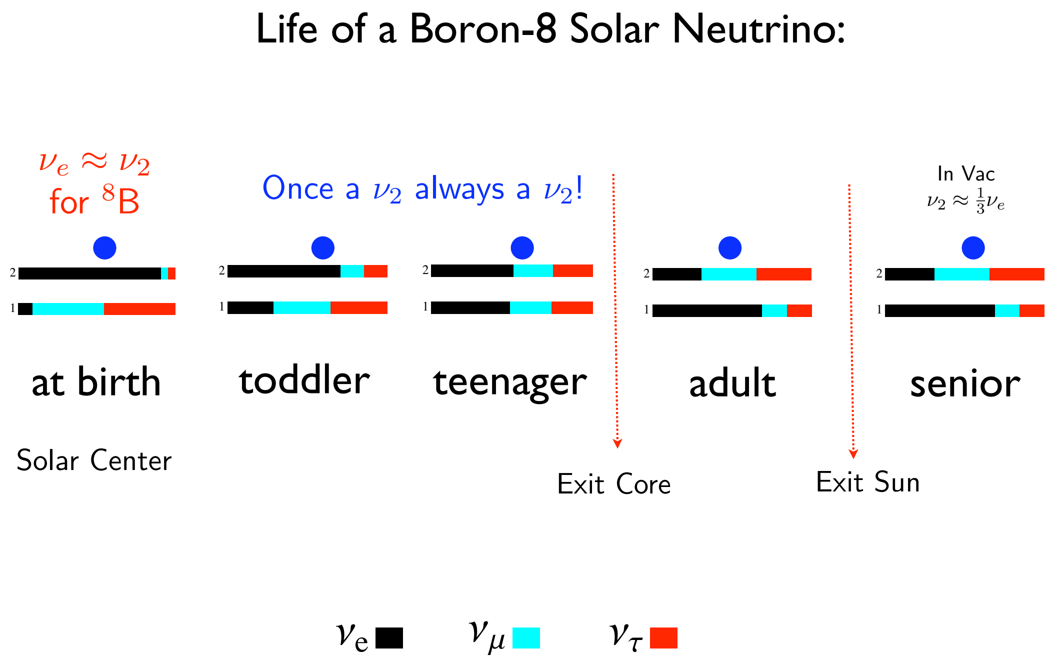

FIGURE 7. Life of a ^{8}Boron solar neutrino from its birth at the center of the sun to its "death" in a detector at the earth. Notice how the flavor content of the the ν_2 mass eigenstate evolves as the neutrino travels through the solar core.

where $\theta_\odot^N$ is the mixing angle defined at the ν_e production point and P_x is the probability of the neutrino to jump from one mass eigenstate to the other during the Mikheyev-Smirnov resonance crossing. The average $\langle \cdots \rangle_{^8\mathrm{B}}$ is over the electron density of the ^{8}B ν_e production region in the center of the Sun predicted by the Standard Solar Model and the energy spectrum of ^{8}B neutrinos weighted with SNO's charged current cross section. Fig. 6 shows the iso-contours of this averaged ν_2 fraction using a threshold of 5.5 MeV on the kinetic energy of the recoil electrons, this figure is taken from Ref. [9]. Thus, the ^{8}B energy weighted average fraction of ν_2's observed by SNO is

$$f_2 = 91 \pm 2\% \quad \text{at the 95\% CL.} \tag{15}$$

Hence, the ^{8}B solar neutrinos are the purest mass eigenstate neutrino beam known so far and SK famous picture of the sun taken with neutrinos is more than 80% ν_2!!!

3. NU STANDARD MODEL:

The Neutrino Standard Model has emerged as follows[4]:

- 3 light (m_i <1 eV) Majorana Neutrinos: $\Rightarrow$ only 2 δm^2

[4] If MiniBooNE confirms the LSND result then this section will require major revision.

$|\delta m^2_{atm}| \sim 2.5 \times 10^{-3}$ eV^2 and $\delta m^2_{solar} \sim +8.0 \times 10^{-5}$ eV^2

- Only three Active flavors (no steriles): $e,\ \mu,\ \tau$
- Unitary Mixing Matrix: 3 angles ($\theta_{12},\ \theta_{23},\ \theta_{13}$), 1 Dirac phase ($\delta$), 2 Majorana phases ($\alpha, \beta$)

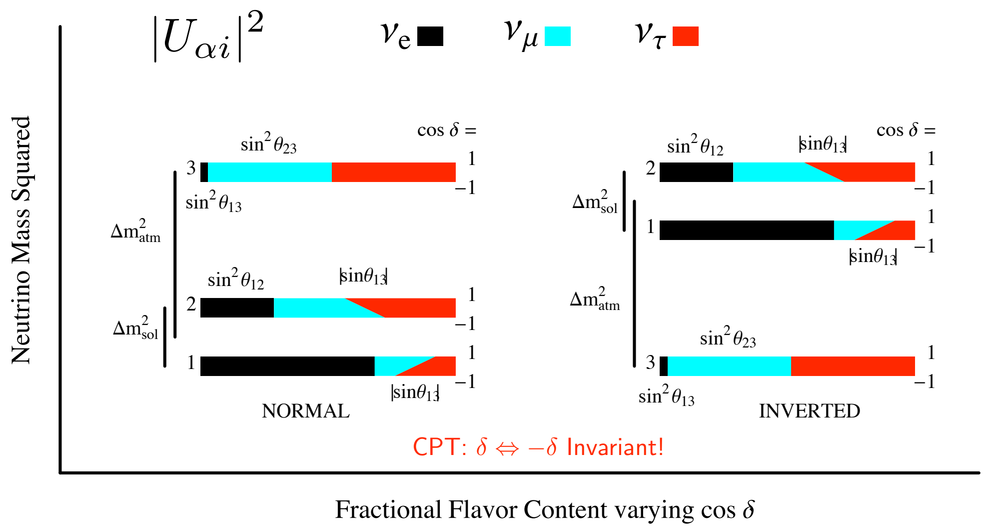

FIGURE 8. Flavor content of the three neutrino mass eigenstates showing the dependence on the cosine of the CP violating phase, δ. If CPT is conserved, the flavor content must be the same for neutrinos and anti-neutrinos. This figure was adapted from Ref. [10].

where the MNS mixing matrix relating flavor to mass eigenstates, $|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle$ is given by

$$U_{\alpha i} =$$
$$\begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix} \quad (16)$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. The (23) sector is identified with the atmospheric δm^2_{atm} and the (12) sector is identified with the solar $\delta m^2_{\odot}$. The (13) sector is responsible for the ν_e flavor transitions at the atmospheric scale so far unobserved, see [11]. Therefore,

$$\begin{aligned} \sin^2\theta_{12} &= 0.31 \pm 0.03 \\ \sin^2\theta_{23} &= 0.50 \pm 0.15 \\ \sin^2\theta_{13} &< 0.04 \end{aligned}$$

and the mass splittings[5] are

$$|\delta m^2_{32}| = 2.7 \pm 0.4 \times 10^{-3}\mathrm{eV}^2 \qquad \text{and} \qquad \delta m^2_{21} = +8.0 \pm 0.4 \times 10^{-5}\mathrm{eV}^2.$$

The mass of the lightest neutrino is unknown but the heaviest one must be lighter than about 1 eV. These mixing angles and mass splittings are summarized in Fig. 8 which also shows the dependence of the flavor fractions on the CP violating Dirac phase, δ. The Majorana phases are unobservable in oscillations since oscillations depend on $U^*_{\alpha i}U_{\beta i}$ but they have observable CP conserving effects in neutrinoless double beta decay.

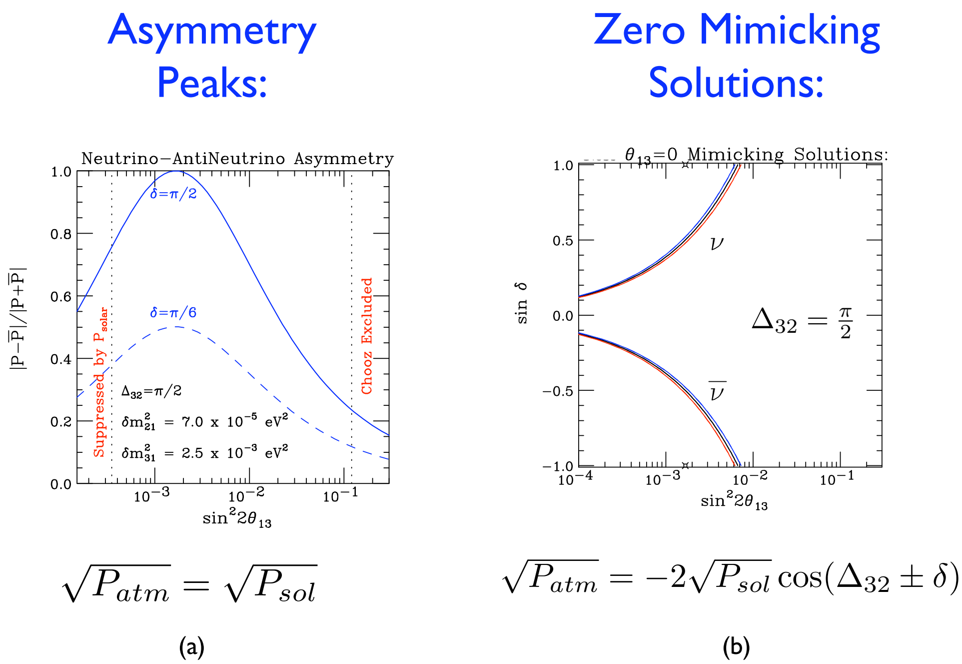

FIGURE 9. (a) The neutrino-antineutrino asymmetry as function of $\sin^2 2\theta_{13}$ at the first vacuum oscillation maximum. The asymmetry peaks when $\sin^2 2\theta_{13} = 0.002$. (b) The zero mimicking solutions at the first vacuum oscillation maximum. Along these lines there is no evidence of non-zero θ_{13}.

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[5] The δm^2 MINOS actually measures is

$$\frac{(|U_{\mu 2}|^2|\delta m^2_{32}| + |U_{\mu 1}|^2|\delta m^2_{31}|)}{(|U_{\mu 2}|^2 + |U_{\mu 1}|^2)}.$$

3.1. Genuine Three Flavor Effects: $\nu_\mu \rightarrow \nu_{\mathbf{e}}$

The most likely genuine three flavor effects to be first observed are $\nu_\mu \rightarrow \nu_e$ and/or its CP and T conjugate processes. That is, in one of following transitions

$$\begin{array}{ccccc} & & \text{CP} & & \\ & \nu_\mu \rightarrow \nu_e & \Longleftrightarrow & \bar{\nu}_\mu \rightarrow \bar{\nu}_e & \\ \text{T} & \Updownarrow & & \Updownarrow & \text{T} \\ & \nu_e \rightarrow \nu_\mu & \Longleftrightarrow & \bar{\nu}_e \rightarrow \bar{\nu}_\mu & \\ & & \text{CP} & & \end{array}$$

Processes across the diagonal are related by CPT. The first row will be explored in very powerful conventional beams, Superbeams, whereas the second row could be explored in Nu-Factories or Beta Beams.

In vacuum, the probability for $\nu_\mu \rightarrow \nu_e$ is derived like so, [12],

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) &= |\, U^*_{\mu 1} e^{-im_1^2 L/2E} U_{e1} + U^*_{\mu 2} e^{-im_2^2 L/2E} U_{e2} + U^*_{\mu 3} e^{-im_3^2 L/2E} U_{e3}\, |^2 \\ &= |2 U^*_{\mu 3} U_{e3} \sin\Delta_{31} e^{-i\Delta_{32}} + 2 U^*_{\mu 2} U_{e2} \sin\Delta_{21}|^2 \\ &\approx |\sqrt{P_{atm}} e^{-i(\Delta_{32}+\delta)} + \sqrt{P_{sol}}|^2 \end{aligned} \qquad (17)$$

where $\sqrt{P_{atm}} = \sin\theta_{23} \sin 2\theta_{13} \sin\Delta_{31}$ and $\sqrt{P_{sol}} \approx \cos\theta_{23} \sin 2\theta_{12} \sin\Delta_{21}$. For anti-neutrinos δ must be replaced with $-\delta$ and the interference term changes

$$2\sqrt{P_{atm}}\sqrt{P_{sol}} \cos(\Delta_{32}+\delta) \quad \Rightarrow \quad 2\sqrt{P_{atm}}\sqrt{P_{sol}} \cos(\Delta_{32}-\delta).$$

This allows for the possibility that CP violation maybe able to be observed in the neutrino sector since it allows for $P(\nu_\mu \rightarrow \nu_e) \neq P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$.

In matter, $\sqrt{P_{atm}}$ and $\sqrt{P_{sol}}$ are modified as follows

$$\begin{aligned} \sqrt{P_{atm}} &\Rightarrow \sin\theta_{23} \sin 2\theta_{13} \frac{\sin(\Delta_{31} \mp aL)}{(\Delta_{31} \mp aL)} \Delta_{31} \\ \sqrt{P_{sol}} &\Rightarrow \cos\theta_{23} \sin 2\theta_{12} \frac{\sin(aL)}{(aL)} \Delta_{21} \end{aligned} \qquad (18)$$

where $a = \pm G_F N_e/\sqrt{2} \approx (4000\, km)^{-1}$ and the sign is positive for neutrinos and negative for anti-neutrinos. This change follows since in both the (31) and (21) sectors the product $\{\delta m^2 \sin 2\theta\}$ is approximately independent of matter effects. In Fig. 10 the bi-probability plots are shown for both T2K, [13], and NOνA, [14] . It is possible that these two experiments will determine the mass ordering (normal or inverted hierarchy, see Fig. reffig: pmns-sq), and observe CP violation in the neutrino sector.

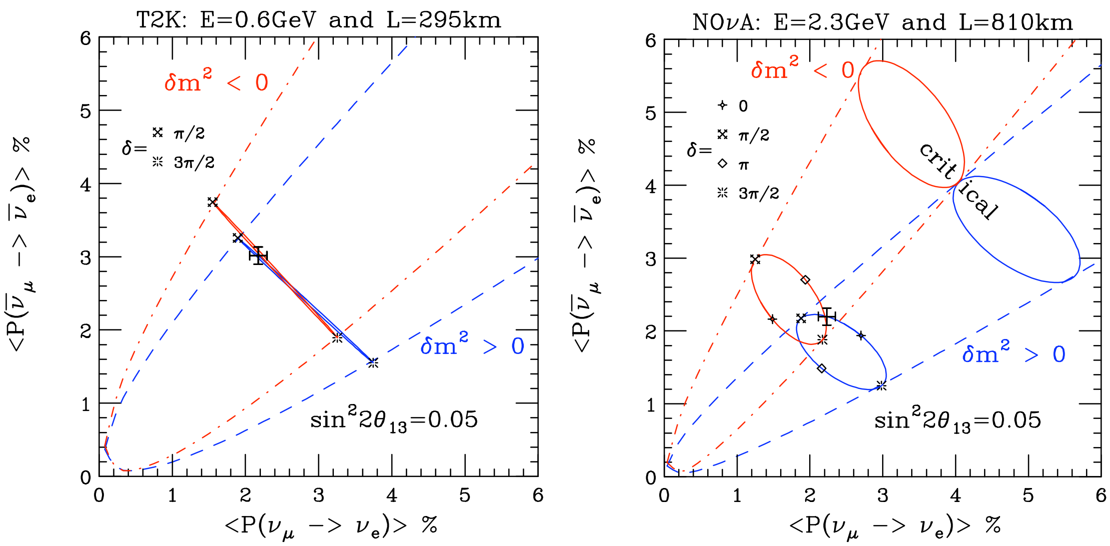


FIGURE 10. The bi-probability plots for both T2K and NOνA. The matter effects and hence the separation between the hierarchies is 3 times large for T2K than NOνA primarily due to the fact NOνA has three times the baseline as T2K. See [15] to understand how to use these plots to untangle CP violation and the mass hierarchy.

4. NEUTRINO MASS

4.1. Absolute Neutrino Mass

Tritium beta decay, neutrinoless double beta decay and cosmology all have the potential to provide us information on the absolute scale of neutrino mass. The Katrin tritium beta decay experiment, [16], has sensitivity down to 200 meV for the "mass" of ν_e defined as

$$m_{\nu_e} \quad = \quad |U_{e1}|^2 m_1 + |U_{e2}|^2 m_2 + |U_{e3}|^2 m_3. \tag{19}$$

Neutrinoless double beta decay, see [17] for review, measures the following combination of neutrino mass,

$$m_{\beta\beta} \quad = \quad |\sum m_i U_{ei}^2| = |m_1 c_{13}^2 c_{12}^2 + m_2 c_{13}^2 s_{12}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta}|, \tag{20}$$

assuming the neutrinos are Majorana. It maybe possible to eventually reach below 10meV for $m_{\beta\beta}$ in double beta decay.

Cosmology measures the sum of the neutrino mases,

$$m_{cosmo} = \sum_i m_i. \tag{21}$$

If $\sum m_i \approx 50\ eV$ the universe's critical density would be saturated. The current limit, [18], is a few % of this number, $\sim$1eV. Given the systematic uncertainties inherent in cosmology, a convincing limit of less than 100 meV seems difficult.

Fig. 11 shows the allowed values for these masses for both the normal and inverted hierarchy.

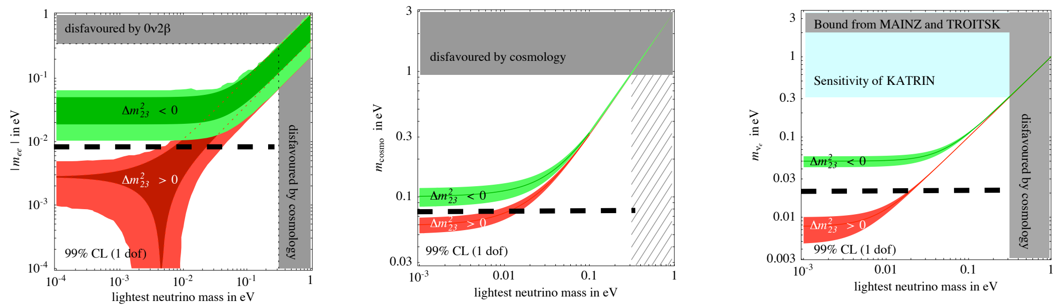
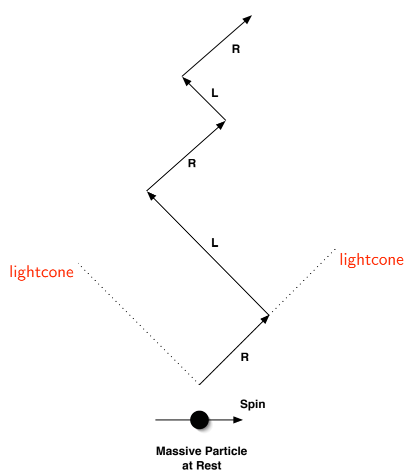


FIGURE 11. The "mass" measured in double β-decay, in cosmology and Tritium β-decay versus the mass of the lightest neutrino. Below the dashed lines, only the normal hierarchy is allowed. This figure was adapted from hep-ph/0503246 [19].

4.2. Majorana v Dirac

Fermion mass is a coupling of left handed to right handed states. Consider a massive fermion at rest, then one can consider this state as a linear combination of a massless right handed particle and a massless left handed particle as shown in Fig. 12. For a particle with an electric charge, like an electron, the left handed particle must have the same charge as the right handed particle. This is a Dirac mass. For a neutral particle, like a sterile neutrino, there is another possibility, the left handed particle could be coupled to the right handed anti-particle, this is the Majorana mass.



$$P^2 = M^2, \quad P \cdot S = 0 \quad and \quad S^2 = -1$$

$$u(P,S) = \underbrace{\frac{(1+\gamma_5)}{2} u(\frac{P+MS}{2})}_{\text{right massless}} + e^{i\phi} \underbrace{\frac{(1-\gamma_5)}{2} u(\frac{P-MS}{2})}_{\text{left massless}}$$

FIGURE 12. The left diagram shows how a massive particle at rest can be considered as a linear combination of two massless particles, one right handed and one left handed. The equation at the right shows the decomposition of a massive Dirac spinor into two massless spinors with different momenta, one right handed and the other left handed (from the Appendix of [20]).

Therefore for a neutral particle there is the possibility of having both Dirac and Majorana masses, as

$$
\begin{array}{lccccl}
\text{Left Chiral} & \nu_L & \Longleftrightarrow & \bar{\nu}_R & \\
 & \Updownarrow & & \Updownarrow & \text{Dirac Mass} \\
\text{Right Chiral} & \nu_R & \Longleftrightarrow & \bar{\nu}_L & \\
 & & \begin{array}{c}\text{Majorana}\\ \text{Mass}\end{array} & &
\end{array}
$$

For the neutrino, the left chiral field couples to $SU(2) \times U(1)$ therefore a Majorana mass term is forbidden by gauge symmetry. However, the right chiral field carries no quantum numbers. Therefore, the Majorana mass term is unprotected by any symmetry and it is expected to be very large. The Dirac mass terms are expected to be of the order of the charge lepton or quark masses. Thus, the mass matrix for the neutrinos is as in Fig. 13.

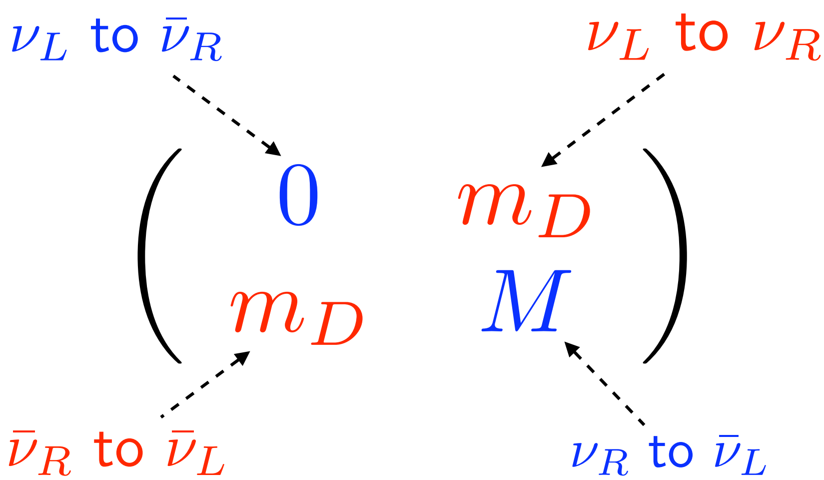


FIGURE 13. The neutrino mass matrix with the various right to left couplings. m_D is the Dirac mass terms while 0 and M are Majorana masses for the charged and uncharged (under $SU(2) \times U(1)$) chiral components.

After diagonalizing the neutrino mass matrix, one is left with two Majorana neutrinos, one heavy Majorana neutrino with mass ~M and one light Majorana neutrino with mass m_D^2/M. This is the famous seesaw mechanism, [21]. The light neutrino is the one observed in current experiments whereas the heavy neutrino is responsible for leptogenesis at very high energy scales since its decays are CP violating and depend on the Majorana phases in the MNS matrix, Eq. 16.

Majorana neutrinos not only allow for neutrinoless double beta decay but also for the possibility that the a muon neutrino, say, produces a positive charged muon, violating lepton number. However, this process would be suppressed by $(m_\nu/E)^2$ which is tiny, 10^{-20}, and, therefore is unobservable.

5. SUMMARY

Neutrino Mass $\Leftrightarrow$ Flavor Change

Open questions:

- Majorana v Dirac
- Light Steriles ???
- Mass Hierarchy $m_3 > m_2 > m_1$ OR $m_2 > m_1 > m_3$
  (labeling such that $|U_{e3}|^2 < |U_{e2}|^2 < |U_{e1}|^2$)
- fraction of ν_e in ν_3 ($< 4\%$) (value of $\sin^2\theta_{13}$)
- Is CP violated ? ($\sin\delta \neq 0$)
- Mass of Heaviest Neutrino
- Mass of Lightest Neutrino
- New Interactions, Surprises !!!

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REFERENCES

1. Y. Ashie *et al.* [Super-Kamiokande Collaboration], Phys. Rev. D **71**, 112005 (2005).
2. E. Aliu *et al.* [K2K Collaboration], Phys. Rev. Lett. **94**, 081802 (2005);
   M. H. Ahn *et al.* [K2K Collaboration], Phys. Rev. Lett. **90**, 041801 (2003).
3. E. Ables *et al.* [MINOS Collaboration], FERMILAB-PROPOSAL-0875.
4. [MINOS Collaboration], arXiv:hep-ex/0607088.
5. K. Eguchi *et al.* [KamLAND Collaboration], Phys. Rev. Lett. **90**, 021802 (2003). T. Araki *et al.* [KamLAND Collaboration], Phys. Rev. Lett. **94**, 081801 (2005).
6. B. Aharmim *et al.* [SNO Collaboration], arXiv:nucl-ex/0502021.
7. S. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Lett. B **539**, 179 (2002).
8. S. J. Parke, Phys. Rev. Lett. **57**, 1275 (1986); S. J. Parke and T. P. Walker, Phys. Rev. Lett. **57**, 2322 (1986); S. J. Parke, “RESONANT NEUTRINO OSCILLATIONS,” FERMILAB-CONF-86-131-T, *Proceedings of theTopical Mtg. of 14th SLAC Summer Inst. on Particle Physics, Stanford, CA, Aug 5-8, 1986*
9. H. Nunokawa, S. Parke and R. Z. Funchal, “What fraction of B-8 Solar neutrinos arrive at the Earth as a ν_2 mass eigenstate", Fermilab-Pub-05-049, arXiv:hep-ph/0601198.
10. O. Mena and S. J. Parke, Phys. Rev. D **69**, 117301 (2004) [arXiv:hep-ph/0312131].
11. M. Apollonio *et al.* [CHOOZ Collaboration], Phys. Lett. B **466**, 415 (1999).
12. A. Cervera *et al.*, Nucl. Phys. B **579**, 17 (2000) [Erratum-*ibid.* B **593**, 731 (2001)].
13. Y. Hayato *et al.*, Letter of Intent, available at `http://neutrino.kek.jp/jhfnu/`
14. D. S. Ayres *et al.* [NOvA Collaboration], hep-ex/0503053. FERMILAB-PROPOSAL-0929, March 21, 2005. Revised NOνA Proposal available at `http://www-nova.fnal.gov/NOvA_Proposal/Revised_NOvA_Proposal.html`

15. O. Mena and S. J. Parke, “Untangling CP violation and the mass hierarchy in long baseline experiments,” Phys. Rev. D **70**, 093011 (2004) [arXiv:hep-ph/0408070].
16. A. Osipowicz *et al.* [KATRIN Collaboration], “KATRIN: A next generation tritium beta decay experiment with sub-eV sensitivity for the electron neutrino mass,” arXiv:hep-ex/0109033.
17. S. R. Elliott and P. Vogel, Ann. Rev. Nucl. Part. Sci. **52**, 115 (2002) [arXiv:hep-ph/0202264].
18. D. N. Spergel *et al.*, “Wilkinson Microwave Anisotropy Probe (WMAP) three year results: Implications for cosmology,” arXiv:astro-ph/0603449.
19. A. Strumia and F. Vissani, Nucl. Phys. B **726**, 294 (2005) [arXiv:hep-ph/0503246].
20. G. Mahlon and S. J. Parke, “Angular Correlations in Top Quark Pair Production and Decay at Hadron Phys. Rev. D **53**, 4886 (1996) [arXiv:hep-ph/9512264].
21. M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman, (North-Holland, 1979), p. 315;
T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, 1979), p. 95;
R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).