The Standard Model: Alchemy and Astrology

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An brief unconventional review of Standard Model physics, containing no plots.

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1. Introduction

In the days of Copernicus, the most prestigious activities at the Krakow Academy were studies of alchemy and astrology. Since that time a number of scientific revolutions, Copernican and otherwise, have advanced us to a more sophisticated view of the universe based on particle physics and astrophysics. A Standard Model (SM) has emerged with a precise comprehensive description of the constituents of matter and their interactions. This model is, by far, the most predictive and best tested scientific framework yet developed.

There is a temptation to regard the Standard Model as a fixed edifice, completed and proofed. In the same spirit, the advent of the Large Hadron Collider (LHC) is seen as the groundbreaking for a new framework of physics beyond the Standard Model, with this new physics being the dominant concern of the LHC program.

This way of thinking is incorrect. Our understanding of the Standard Model has evolved greatly in the decades since the first elements of the theory were put into place. This evolution will continue until a number of profound mysteries are resolved. Discoveries of physics beyond the SM may provide key insights, but the mysteries themselves involve the structure and dynamics of the Standard Model proper.

The LHC will indeed provide us with the first direct access to a new framework of fundamental physics. The LHC will also probe the Standard

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Model in a new high energy, high luminosity environment. Both sorts of activity will provide discoveries. They will also be tightly coupled, since a better understanding of SM processes will be required to extract the new framework, while the new framework will elucidate mysteries of the Standard Model. Far from marking the end of the Standard Model era, LHC turn-on will induce a rash of technical and conceptual developments, culminating in a much more sophisticated view of the same constituents and interactions that comprise our current picture.

2. The theoretical inputs of the Standard Model

In order to understand the Standard Model, it is enlightening to list the minimal set of theoretical inputs that define it. Having done this, will we see in the next section that the SM possesses a number of interesting derived properties. This separation of inputs from derived properties is ahistorical, i.e., I will employ modern insights that were not available or not sufficiently appreciated at the time that the SM was invented.

• All interactions are local.

• Quantum mechanics is correct, at least up to energy scales around a TeV.

• Special relativity, or more precisely four-dimensional Poincaré invariance, is respected by interactions and kinematics on these same scales.

This set of assumptions implies that particles physics up to some high energy scale can be completely described by an effective relativistic quantum field theory.

The next set of assumptions is about interactions:

• There are gauge forces, mediated (at least at large momentum transfers) by exchanges of gauge bosons.
• The local gauge symmetry is $SU(3)_c \times SU(2)_L \times U(1)_Y$.

• Gravity exists but is ignored.

The next set of assumptions is about constituents:

• The fundamental matter constituents are two-component complex Weyl fermions. They come in five varieties, as shown in Table 1.

• There are three copies (generations) of this matter content.

The next set of assumptions involve the Higgs:

• The local gauge symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$ is spontaneously broken to $SU(3)_c \times U(1)_{em}$ via the Higgs mechanism.

• This same Higgs scalar also has direct Yukawa couplings to pairs of fermions. These pairs have the same color charges but different hypercharges and weak charges:

$$\lambda_{ij}^L H \bar{L}_i E_j + \lambda_{ij}^U H^c \bar{Q}_i U_j + \lambda_{ij}^D H \bar{Q}_i D_j + \text{hermitian conjugates}. \quad (1)$$

• The matrices of Yukawa couplings are neither real nor diagonal.

The final assumption looks rather obscure:

• Only include operators up to dimension four.

Historically, this assumption was included to make the theory renormalizable. From a modern point of view, this input is poorly motivated.

3. The derived properties of the Standard Model

3.1. mass and energy scales

With one exception particle masses in the Standard Model are derived from dynamics and dimensionless couplings. For example, pure $SU(3)_c$ gauge theory is classically scale invariant, but picks up a logarithmic scale dependence at one-loop from the running of the gauge coupling. If I run this coupling down from some arbitrary ultraviolet cutoff to the infrared confining regime, I can trade the dimensionless gauge coupling and the cutoff for a dimensionful dynamically determined energy scale $\Lambda_{QCD}$.

The only dimensionful input parameter of the Standard Model is the negative Higgs mass-squared parameter $-m_H^2$. Together with the dimensionless Higgs quartic self-coupling $\lambda$, these input parameters determine the Higgs vacuum expectation value $v/\sqrt{2}$, $v = 246.2$ GeV, as well as the Higgs
mass. The derived value of $v$, combined with the dimensionless SM Yukawa couplings, then determine the masses of the quarks and leptons.

At finite temperature and vanishing chemical potential the SM has two derived critical temperatures. One is the quark deconfinement temperature of about 175 MeV, where we observe a crossover transition from hadrons to a quark–gluon plasma. The other is a temperature of about 100 GeV where we expect a weakly first order phase transition restoring the full $SU(2)_L \times U(1)_Y$ symmetry.

In the Standard Model neutrinos do not have mass, in contradiction with experiment. I will return to this problem below.

3.2. sowing the seeds of its own destruction

Because we have forbidden higher dimension operators by hand, the Standard Model has no explicit cutoff dependence. However, if the Higgs self-coupling is too large – corresponding to a physical Higgs boson mass greater than about 180 GeV – then the SM generates its own ultraviolet cutoff $\Lambda_{LP}$. This is because $\lambda$ runs logarithmically with energy scale, and if $\lambda$ is large enough at the electroweak scale the sign of the effect is to increase $\lambda$ at higher energies. At some energy scale $\Lambda_{LP}$ the coupling hits a Landau pole and the electroweak sector of the Standard Model breaks down.

If the Higgs self-coupling at the electroweak scale is too small – corresponding to a physical Higgs boson mass less than about 130 GeV – then the running goes the other way, and at some high energy scale the sign of this quartic coupling goes negative. At best, this destabilizes the vacuum; at worst, theories with this kind of disease are unphysical. One could attempt to compensate by invoking dimension 6 Higgs self–couplings, but this would violate one of our defining theoretical inputs.

3.3. flavor

The Standard Model has large accidental global flavor symmetries. I call these accidental because if we had introduced generic higher dimension operators ab initio, these symmetries would be violated. In any event most of them are not exact.

Baryon number $B$ and lepton number $L$ are accidental global symmetries. They are exact at the perturbative level, but the combination $B + L$ is broken by nonperturbative effects.

The SM has chiral symmetries for quarks and leptons, due to the fact that gauge invariance forbids direct mass terms. The chiral symmetries are broken nonperturbatively by a QCD condensate, also breaking electroweak symmetry dynamically. This dynamical effect, scaled up to much higher energies, is the basis of technicolor models.
In the absence of Yukawa couplings, the Standard Model has a huge $[U(3)]^5$ global flavor symmetry. These symmetries are explicitly broken by the Yukawas, but since the Yukawas are mostly quite small numerically, these symmetries are still important.

Because the Yukawa matrices are not diagonal, the SM has flavor–changing charged currents at tree level, as encoded in the CKM matrix. The SM has no flavor–changing neutral currents (FCNCs) at tree level, and at loop level FCNC’s have an extra suppression (besides the loop factor) coming from the GIM mechanism.

There are two sources of $CP$ violation in the Standard Model. One is a single physical phase in the CKM matrix, coming from the fact that the quark Yukawa matrices are complex. This phase is rather large. The other source of $CP$ violation is instantons, a nonperturbative effect in QCD. This effect is parametrized by an angle $\theta_{QCD}$. For unknown reasons this angle is either zero or very small, $\theta_{QCD} < 10^{-9}$.

Last but not least, the SM has an accidental global $SU(2)$ symmetry known as “custodial $SU(2)$”. This is because the Higgs sector of the SM has an $O(4) \sim SU(2)_L \times SU(2)_R$ global symmetry that acts on the four real components of the complex Higgs doublet; custodial $SU(2)$ is the diagonal remnant left unbroken after the Higgs gets a vev. This symmetry is broken by the hypercharge gauge coupling and by the fermion doublet mass splittings.

4. Disturbing features

The Standard Model has many disturbing features. Some of these have been nagging particle physicists for decades, while others have only become apparent in recent years [1].

4.1. the hierarchy problem

The SM ignores gravity, which (modulo the possibility of extra spatial dimensions) is an extremely good approximation for tree-level processes in particle experiments. But the existence of gravity, combined with naive (four-dimensional) scaling, implies the existence of a Planckian regime $M_{\text{Planck}} \sim 10^{19} \text{ GeV}$ where gravity becomes strong. There are presumably new Planckian degrees of freedom associated with this threshold. In the absence of supersymmetry, the SM Higgs should interact with these states via loops. So why isn’t $|m_H| \sim M_{\text{Planck}}$?

Suppose that this problem is somehow solved. Then we observe that the SM has a number of potential gauge anomalies. These anomalies want to induce a one-loop breaking of the local $SU(2)_L \times U(1)_Y$ gauge invariance. The only reason that this does not happen is that the fermion content of
each generation exactly matches the 15 SM nonsinglet members of the 16 of $SO(10)$ (the sixteenth state can be added to provide neutrino masses). Furthermore, if we extrapolate the SM gauge couplings to higher energies, we find that they roughly unify at a scale $M_{\text{GUT}} \sim 10^{14}$ GeV (the unification is more precise, though still not perfect, if we add the assumption of a supersymmetry threshold at $\sim 1$ TeV \cite{2}). Thus we have two strong hints that the SM has an underlying grand unified structure. So why isn’t $|m_H| \sim M_{\text{GUT}}$?

Suppose that this problem is somehow solved. Then we go back to our previous observation that the SM sows the seeds of its own destruction, through the running of the Higgs self-coupling $\lambda$. Over 95% of the allowed mass range for the Higgs, this implies a mass scale $M_{LP}$ or $M_{V_I}$ at which the SM breaks down due to a Landau pole or a vacuum instability. So why isn’t $|m_H| \sim M_{LP}$?

4.2. flavor

In the Standard Model the quark and lepton Yukawa couplings are inputs. Since the couplings run, their precise values are dependent on both scale and renormalization scheme; further subtleties arise in extracting the Yukawas of the light quarks. But roughly speaking, if these couplings were true input parameters, determined e.g. by initial conditions of the early universe, we would expect them to be of order one.

Instead the SM Yukawas are hierarchical, with values as low as $\sim 3 \times 10^{-6}$, and intrafamily mass ratios as large as 40. The Yukawa mixings also have a hierarchical structure, as evidenced by the famous Wolfenstein parametrization of the CKM matrix:

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4), \quad (2)$$

where $\lambda$ is a small parameter ($\lambda \approx 0.2$).

There is only one diagonal Yukawa coupling that is of order one, and that is the top quark Yukawa. But even this case is mysterious. The top Yukawa is not really of order one: it is equal to one! For example, using the 2005 combined Tevatron value for the pole mass of the top quark, the corresponding Yukawa coupling is $\lambda_t = 0.99 \pm 0.01$. The entire particle physics community has chosen (so far) to regard this fact as a 1% coincidence. I should point out that similar percent level equalities e.g. supersymmetric gauge coupling unification or the ratio of the total mass-energy density of the universe to the critical density, have spawned huge theoretical frameworks bolstered by thousands of papers.
Obviously the flavor structure of the Standard Model is not random, and is begging for explanation. It is still possible for some of this hierarchical structure to arise from initial conditions in the early universe, if we invoke anthropic arguments and allow ourselves the decadent luxury of positing $10^{500}$ different vacuum bubbles out beyond the Hubble horizon. But the more straightforward and economical explanations are (i) the Standard Model is formulated with too few degrees of freedom: the Yukawa couplings should be promoted to fields, whose vacuum expectation values are determined by a combination of dynamics and symmetries, or (ii) the Standard Model is formulated with too many degrees of freedom: the quarks and leptons are not fundamental, and the flavor structure is a feature of the dynamics that maps the true fundamental constituents (strings, preons, etc) to the light SM states that we observe.

4.3. higher dimension operators

From the discussion above of the hierarchy problem it seems impossible that the Standard Model lagrangian provides an accurate description of nature at arbitrarily high energies. Thus we should regard the SM as an effective field theory. We then expect that we have probably neglected higher dimension operators constructed out of SM fields and suppressed by powers of an ultraviolet cutoff $\Lambda > v$. Of course there may be several different cutoffs involved, representing a variety of physical thresholds. In the early days of the SM it made sense to neglect such operators, since our experiments for the most part probed energy scales less than $v$.

In more recent times many experiments have probed SM processes at energy scales much larger than $v$, either directly (at the Tevatron), through processes sensitive to loops (at LEP, the B factories, etc) or through processes sensitive to new small violations of the accidental symmetries of the Standard Model. Dozens of experiments have had the opportunity to observe the effects of dimension 5 and dimension 6 operators constructed out of SM fields, including both operators that violate accidental symmetries of the SM and operators that preserve those symmetries.

With one important exception, none of these experiments have observed clear evidence for any such higher dimension operators. This surprising result is worth reviewing in some detail. To organize our thinking, I will divide the SM higher dimension operators into three classes: those that violate $B$ and/or $L$, those that violate the approximate flavor symmetries of the SM, introducing new sources of flavor violation besides those already provided by the SM Yukawa matrices, and those that respect all SM accidental symmetries, i.e. respect $B$ and $L$ and are Minimal Flavor Violating (MFV).
4.3.1. $B$ and/or $L$ violating operators

Experimental bounds on proton decay and on charged lepton flavor violating processes ($\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, $\mu \rightarrow e$ conversion) tell us that generic dimension 6 operators that violate $B$ and $L$ either do not exist or are suppressed by a superheavy mass scale. This is an important result, indicating that conservation of $B$ and $L$ is not so accidental after all.

The discovery of neutrino masses means that the SM requires some kind of extension. By adding right-handed neutrinos (SM singlet fermions) as new degrees of freedom, we can preserve the coupling rules for the SM. An alternative that does not require new degrees of freedom is to introduce the unique (gauge invariant) dimension 5 operator that can be constructed out of SM fields:

$$O_5 = \frac{f}{\Lambda} \left( L^T C i\tau_2 \bar{T} \right) \left( H^T i\tau_2 \bar{T} H \right),$$

where the $\tau_i$ are $SU(2)_L$ matrices, and $f$ is a dimensionless coupling with generation indices suppressed. This operator violates lepton number by 2 units, and gives Majorana masses to neutrinos $m_\nu \sim f v^2/\Lambda$. If we require $f$ of order one for the heaviest neutrino and take the heaviest neutrino to saturate the current experimental upper bound of a few tenths of an eV, then the cutoff scale $\Lambda$ is a few times $10^{14}$ GeV. The Majorana nature of the neutrino mass may be confirmed in the near future by the observation of neutrinoless double beta decay.

Thus it appears that neutrino data favors augmenting the SM by its unique dimension 5 operator, but this operator is suppressed by a superheavy mass scale. It is not at all clear what this implies about the likelihood of observing any of the large number of dimension 6 operators.

4.3.2. flavor violating operators

Many dimension 6 operators would provide new sources of quark flavor violation beyond that induced by the CKM matrix. A large number of experiments have been performed in the $b$, charm and kaon sectors looking for such effects. So far no clear signals have been observed anywhere, and impressive limits have been set. Some FCNC operators are only compatible with experiment if they are suppressed by a cutoff exceeding 1000 TeV.

This situation may change with the next round of experiments, but currently the simplest interpretation of this data is that the CKM matrix is the only source of quark flavor violation, up to scales of a few TeV or higher.
4.3.3. symmetry preserving operators

An important class of dimension 6 operators are the “oblique” operators. These operators are purely electroweak, flavor diagonal, and at leading order they only affect the $W$ and $Z$ vacuum polarization. They violate no accidental symmetries except for custodial $SU(2)$. If present, these operators would shift the values of the oblique parameters $S$ and $T$ [3]. The latest global fits to electroweak precision data show no clear evidence for any such effects [4]. Higgs naturalness implies that such operators are likely to exist, suppressed by a cutoff that is no larger than a couple of TeV, i.e. $|m_H|$ divided by the square root of a loop factor. The current experimental lower bounds exceed this estimate.

There is also a large class of dimension 6 four-fermion operators that are flavor diagonal. There is no argument based on SM symmetries that would forbid such operators. Nevertheless they are not seen in data. The cutoff scale for such operators is constrained, e.g. to be larger than a 10 TeV for $eeee$ couplings and 26 TeV for $eedd$ couplings [5].

5. Discovering the Standard Model

By the 1980s the basic elements of the Standard Model were clearly defined, and many key predictions had been spectacularly verified by experiment. However two particles predicted by the Standard Model – the top quark and the Higgs boson – had not been observed. Not only did the SM predict the existence of these particles, it also predicted the values of all of their quantum numbers, except their masses.

Standard Model radiative corrections contain diagrams with virtual top quarks and Higgs bosons. This leads to electroweak observables whose predicted values depend logarithmically on the ratios $m_t^2/m_Z^2$ and $m_h^2/m_Z^2$, where $m_h$ is the mass of the physical Higgs particle. Note we could replace $m_Z$ with $m_W$ in these ratios, since the difference is higher order. There are also leading order corrections that are directly proportional to $m_t^2$, rather than to a logarithm. These arise from the Yukawa couplings of the top to the Goldstone bosons that were eaten by the $W$ and $Z$ gauge bosons, and indicate the fact that top radiative corrections do not decouple in the limit $m_t \to \infty$ with $m_b$ fixed. With the advent of the LEP experiments and SLD, it was possible to observe the quadratic $m_t^2$ effects of virtual tops in precision electroweak data. This was a big discovery.

5.1. the Tevatron top

In 1995, the Tevatron experiments discovered a new strongly pair-produced state that decays promptly to a $W$ boson and a $b$-jet. This was a big discov-
ery. Of course such a state is compatible with the top quark predicted by the SM, and its mass was compatible with the less precise mass determinations from electroweak precision data.

During this past year, data from Run II of the Tevatron has allowed us for the first time to probe many properties of this new heavy particle, comparing this Tevatron top with the theoretical particle of the Standard Model. Here is a quick summary of what has been discovered:

- The Tevatron top has charge 2/3. A measurement of the jet charges of the $b$-jets produced from $t\bar{t}$ pairs now eliminates the possibility of a charge 4/3 particle at nearly 95% confidence [6].

- The Tevatron top has spin 1/2, a coupling to the $W$ that is more like $V^-A$ than $V^+A$, and a coupling to longitudinal $W$’s consistent with the SM Higgs mechanism to within an experimental uncertainty of about 20% . These results come from measuring the angles between the charged $e$ or $\mu$ and the $b$-jet in top decays. The distribution of these angles allows a fit to $f_0$ and $f_+$, the fraction of decays that produce a longitudinally polarized $W$ and a right-handed $W$, respectively. If the Tevatron top had spin 3/2, $f_+$ would be close to 1; if it had spin 1/2 but a $V^+A$ coupling, $f_+$ would be close to 0.3. For a SM top, producing a right-handed $W$ requires a $b$ quark helicity flip, suppressing this decay by the ratio $m_b^2/m_t^2$. Results from CDF and DZero [7, 8] show $f_+ < 0.09$ at 95% confidence, and $f_0$ within about 20% of its SM value.

These top results are just an example of many recent discoveries in Standard Model physics. One effect of these discoveries has been to rule out or place tight constraints on scenarios beyond the Standard Model, but the broader significance is that we are observing for the first time what Nature is really doing in these fundamental phenomena of particle physics.

5.2. the virtual virtual Higgs

With the LHC (and perhaps even the Tevatron) we expect a similar story to unfold regarding a new particle (or particles) that I will generically call the Higgs. It is often said, invoking the famous “blue band” plot, that the electroweak precision data already reveals the radiative effects of a light Higgs, much as the effects of virtual top were observed before the Tevatron discovery.

Strictly speaking, this claim is false, as can be seen by have a closer look at the global electroweak fits. For example, let me use the analysis of Appendix E of a recent combined analysis [9]. The data is used to fit
the purely electroweak quasi-observables \[ \epsilon_1, \epsilon_2, \epsilon_3 \] and \( \epsilon_b \). The effects of virtual top are clearly seen in these fits; e.g. using the leading order relation

\[ m_t^2 = -4\pi \epsilon_b \frac{\sqrt{2}}{G_F} \]  

(4)

I obtain the (unsophisticated) estimate \( m_t = 155 \pm 25 \text{ GeV} \). Virtual SM Higgs effects can arise to leading order from two different combinations:

\[ 3\epsilon_3 - \epsilon_2 \propto \ln \frac{m_h}{m_Z} = -0.002 \pm 0.004 , \]  

(5)

\[ 2\epsilon_1 + 3\epsilon_b \propto \ln \frac{m_h}{m_Z} = -0.004 \pm 0.005 , \]  

(6)

where my error bars do not take into account correlations.

Thus we have two way of detecting a virtual Higgs in existing data, and in both cases we have obtained a null result, despite part per mil accuracy. The prevailing interpretation of these results (with which I agree) is that the Higgs is light, with a mass not much above \( m_Z \) in logarithmic units. The reason why this interpretation is reasonable is because if there were no Higgs we would have expected to see some other radiative effects in the electroweak data.

Thus the statement that the precision data favors a light Higgs, as opposed to no Higgs at all, relies upon some theoretical baggage. This baggage originates from the observation that SM diagrams for longitudinal WW and WZ scattering give amplitudes that grow like \((\text{energy})^2\) and \((\text{energy})^4\), violating unitarity at energies a little above a TeV \( [11] \). Adding the SM Higgs restores a weakly coupled theory. Other alternatives have been explored, in which Kaluza-Klein gauge bosons or new strong interactions do the job of Higgs in restoring unitarity. Generically such alternatives do produce fairly large radiative effects, but no one claims to know that this is necessarily true in all cases.

5.3. QCD

We have ample evidence that QCD is the correct theory of strong interactions, and QCD has an unambiguous nonperturbative definition via Wilsonian ideas applied on the lattice. Nevertheless many fundamental questions about QCD are still unanswered \( [12] \). One way to gauge our ignorance is to ask for a detailed picture of the interior of a proton. We know that the answer to this question depends on the nature of the process used as a probe, in particular on the squared momentum transfer \( Q^2 \) and the parton momentum fraction \( x \). We know that there are at least three qualitatively different regimes: large \( x \) + large \( Q^2 \), large \( x \) + small \( Q^2 \), and small
The first is the standard perturbative regime, and is the only one in which we have a detailed picture of proton constituents. Yet even here we are mystified by basic issues such as how the spin of the proton is distributed among the partons. For the second regime we have a vague picture of valence quarks confined by gluonic flux tubes, but no detailed understanding. In the third regime, which is quite relevant for LHC, we are just beginning to grapple with the dynamics of parton saturation.

5.4. Prediction

During the LHC era, we will discover as many important new insights about the Standard Model as we discover about physics beyond the Standard Model. A decade from now, we will look back on our current understanding of the Standard Model and be amused at its lack of sophistication.

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