FERMILAB-CONF-06-248-T

Determining the Neutrino Mass Hierarchy

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ABSTRACT

In this proceedings I review the physics that future experiments will use to determine the neutrino mass hierarchy.

1. Introduction

Our current knowledge of neutrino masses leaves the possibility that the solar neutrino doublet ($\nu_1,\ \nu_2$) has a mean mass smaller than or greater than the remaining atmospheric neutrino (ν_3). If the solar doublet has the larger mass then this is called the "inverted" hierarchy, otherwise the hierarchy is called "normal". Determining this hierarchy is important for model building regardless of whether the neutrinos are quasi-degenerate or highly hierarchial. In Fig.1[1)], the two possible hierarchies are shown plus an example of the flavor content of the three neutrino mass eigenstates consistent with all existing data. Here, the thickness of the lines has been used to show the variation of these flavor fractions with the unknown parameter the CP violating phase. Note that the solar doublet is ν_e rich whereas the atmospheric singlet, ν_3, is ν_e poor. In fact, the upper current limit from CHOOZ[2)] on fraction of ν_e in ν_3 is 4%. Knowledge of the mass ordering of the solar doublet as well as the fraction of ν_e in ν_2 comes from SNO's measurement[3)] of the ^{8}Boron solar neutrinos which arrive at the earth as a nearly pure ν_2 mass eigenstate[4)] because of matter effects[5)].

There are various approaches to determining whether the neutrino mass hierarchy is normal or inverted which will be briefly reviewed in the following sections: first, the direct mass measurements followed by using matter effects in $\nu_\mu \leftrightarrow \nu_e$ transitions.

2. Direct Mass Measurements

Fig.2[6)] shows the various masses measured in double β-decay, in cosmology and Tritium β-decay verses the mass of the lightest neutrino for both the normal and inverted mass spectrums. Clearly, below some value for these masses only the normal hierarchy survives. Therefore if one can place a upper bound below this value one has determined the mass hierarchy to be normal. In a double β-decay experiment, a signal below this limit must be seen to exclude the possibility that the neutrinos are

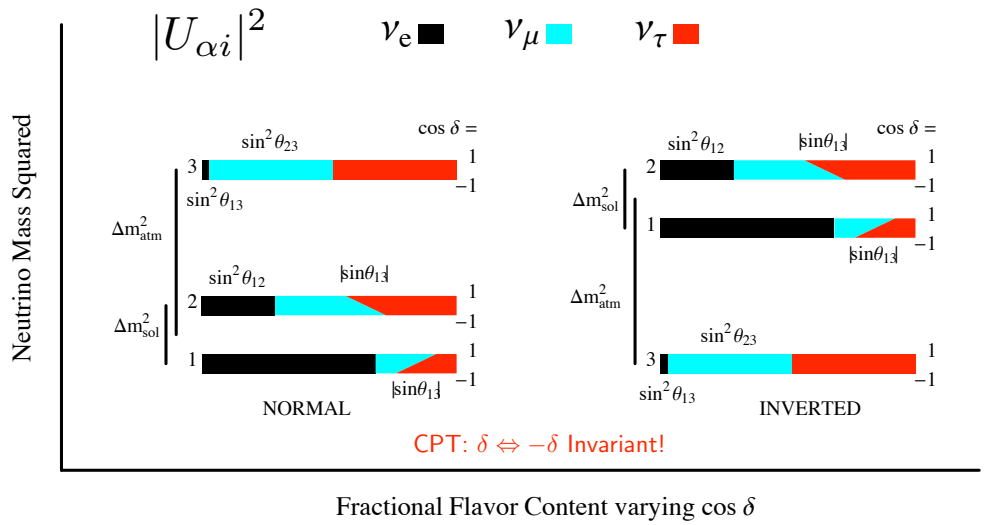

Figure 1: The fractional flavor content of the three neutrino mass eigenstates showing the two possible hierarchies. Here the thickness of the line has been used to show the variation with respect to cosine of the CP violating phase. Since this figure must be the same for neutrinos and anti-neutrinos, if CPT is conserved, this figure cannot be sensitive to whether sine of the CP violating phase is positive or negative.

Dirac particles.

3. Long Baseline: $\nu_\mu \longrightarrow \nu_e$

The transition probability $\nu_\mu \to \nu_e$ can be simple written as the square of a sum of three amplitudes, one associated with each neutrino mass eigenstate, as follows[7)]

$$\begin{aligned} P(\nu_\mu \to \nu_e) &= |\, U^*_{\mu 1} e^{-i m_1^2 L/2E} U_{e1} + U^*_{\mu 2} e^{-i m_2^2 L/2E} U_{e2} + U^*_{\mu 3} e^{-i m_3^2 L/2E} U_{e3} \,|^2 \\ &= |2 U^*_{\mu 3} U_{e3} \sin\Delta_{31} e^{-i\Delta_{32}} + 2 U^*_{\mu 2} U_{e2} \sin\Delta_{21}|^2 \\ &\approx |\sqrt{P_{atm}} e^{-i(\Delta_{32}+\delta)} + \sqrt{P_{sol}}|^2. \end{aligned} \tag{1}$$

where the unitarity of MNS-matrix has been used to eliminate the $U^*_{\mu 1} U_{e1}$ term and Δ_{jk} is used as a shorthand for the the kinematic phase, $\delta m^2_{jk} L/4E$. As the notation suggests the amplitude $\sqrt{P_{atm}}$ only depends on δm^2_{31} and $\sqrt{P_{sol}}$ only depends on δm^2_{21}. For propagation in the vacuum, these amplitudes are simple given by

$$\begin{aligned} \sqrt{P_{atm}} &= \sin\theta_{23} \sin 2\theta_{13} \sin\Delta_{31} \\ \sqrt{P_{sol}} &= \cos\theta_{23} \sin 2\theta_{12} \sin\Delta_{21}. \end{aligned} \tag{2}$$

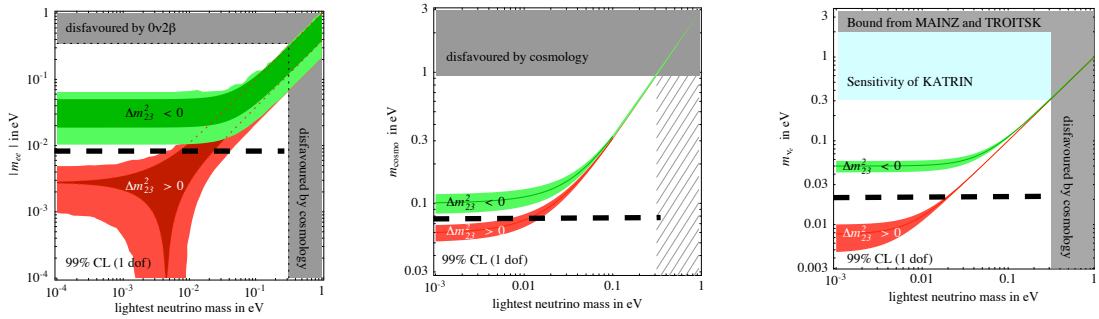

Figure 2: The "mass" measured in double β-decay, in cosmology and Tritium β-decay verses the mass of the lightest neutrino. Below the dashed lines, only the normal hierarchy is allowed. This figure was adapted from hep-ph/0503246[6].

Whereas in matter, the approximate invariance of $\{\delta m^2 \sin 2\theta\}_{jk}$ for (jk)=(31) and (21) implies that

$$\begin{aligned} \sqrt{P_{atm}} &= \sin\theta_{23}\ \sin 2\theta_{13}\ \frac{\sin(\Delta_{31} - aL)}{(\Delta_{31} - aL)}\ \Delta_{31} \\ \sqrt{P_{sol}} &= \cos\theta_{23}\ \sin 2\theta_{12}\ \frac{\sin(aL)}{(aL)}\ \Delta_{21}. \end{aligned} \tag{3}$$

The matter potential is given by $a = G_F N_e/\sqrt{2} \approx (4000\ km)^{-1}$ and the sign of Δ_{31} (and Δ_{32}) determines the hierarchy; normal $\Delta_{31} > 0$ whereas inverted $\Delta_{31} < 0$.

For anti-neutrinos $a \to -a$ and $\delta \to -\delta$. Thus the phase between $\sqrt{P_{atm}}$ and $\sqrt{P_{sol}}$ changes from $(\Delta_{32} + \delta)$ to $(\Delta_{32} - \delta)$. This changes the interference term from

$$2\sqrt{P_{atm}}\sqrt{P_{sol}}\cos(\Delta_{32} + \delta) \quad \Rightarrow \quad 2\sqrt{P_{atm}}\sqrt{P_{sol}}\cos(\Delta_{32} - \delta). \tag{4}$$

Expanding $\cos(\Delta_{32} \pm \delta)$, one has a CP conserving part $2\sqrt{P_{atm}}\sqrt{P_{sol}}\cos\Delta_{32}\cos\delta$ and the CP violating part $\mp 2\sqrt{P_{atm}}\sqrt{P_{sol}}\sin\Delta_{32}\sin\delta$. Notice that for this term to be non-zero the kinematical phase Δ_{32} cannot be $n\pi$. This is the neutrino counter part to the non-zero strong phase requirement for CP violation in the quark sector.

The transition probability given in Eq.[1] has a many interesting and non-obvious properties. For example, in Fig. 3 the asymmetry between neutrinos and anti-neutrinos is given as a function of $\sin^2 2\theta_{13}$ at the first maximum of the oscillation, $\Delta_{32} = \pi/2$. The peak occurs when the amplitudes associated with the atmospheric term and the solar term are equal in magnitude near $\sin^2 2\theta_{13} \approx 0.002$. Also the location of the zero mimicking solutions[10] are given at the first maximum of the oscillation. Along these lines the transition probability is just given by the solar term even-though $\sin^2 2\theta_{13} \neq 0$.

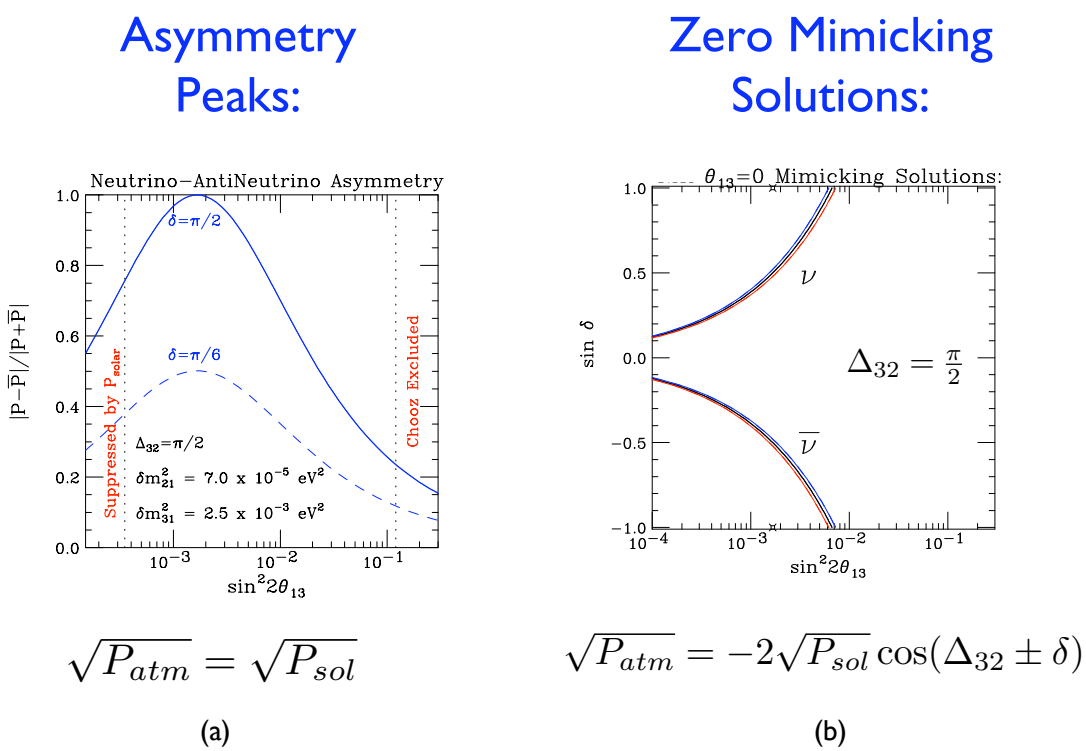

Figure 3: The left panel (a) shows the asymmetry between neutrino and anti-neutrinos as a function of $\sin^2 2\theta_{13}$. The peak occurs when $\sqrt{P_{atm}} = \sqrt{P_{sol}}$ at $\sin^2 2\theta_{13} \approx 0.002$. The right panel (b) shows the location of the zero mimicking solutions at the first oscillation maximum.

3.1. Off Axis Counting Experiments: T2K and NOVA

The experiments NOνA and T2K are introduced in the talks by Bernstein[8)] and Nishikawa[9)] respectively. The allowed regions in the bi-probability space for the NOνA experiment are shown in Fig. 4(a). The upper (red) region is for the inverted hierarchy and the lower (blue) region for the normal hierarchy. The maximum value of θ_{13} for which there is overlap in these two regions has been named the critical value[11)], θ_{crit}, and is given by the following expression,

$$\theta_{crit} \approx \frac{\pi^2}{8} \frac{\sin 2\theta_{12}}{\tan\theta_{23}} \frac{\delta m^2_{21}}{\delta m^2_{31}} \left(\frac{4\Delta^2_{31}/\pi^2}{1 - \Delta_{31}\cot\Delta_{31}} \right) /(aL) \tag{5}$$

For this experiment the numerical value of $\sin^2 2\theta_{crit}$ is approximately 0.10, i.e. it is close to the CHOOZ bound. Whereas Fig. 4(b) shows the four allowed solutions, two for the normal hierarchy and two for the inverted hierarchy, in the $\sin\delta$ v $\sin^2 2\theta_{13}$ plane assuming that the measured values of the transitions probability for neutrinos and anti-neutrinos is given by the cross near (2,2) % in Fig.4(a). Notice that the

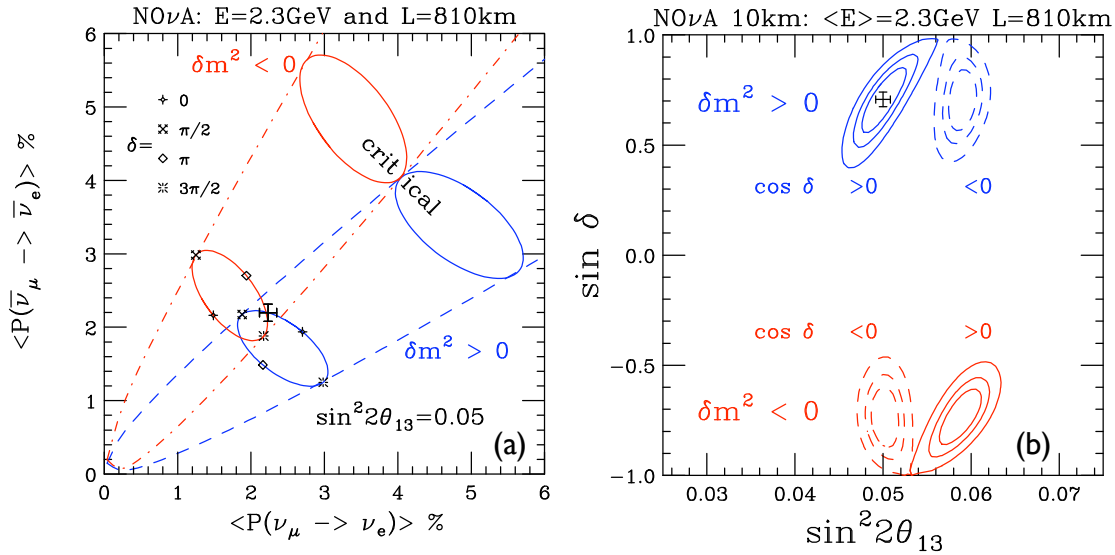

Figure 4: The left panel (a) is the bi-probability plot, $\nu_\mu \to \nu_e$ verses $\bar{\nu}_\mu \to \bar{\nu}_e$ for the NOνA experiment showing the critical value of θ_{13} as well as two (out of four) of the ellipses which pass through the data point (large cross) with $\sin^2 2\theta_{13} = 0.05$. The right panel (b) shows the four solutions for this data point in the $\sin\delta$ v $\sin^2 2\theta_{13}$ plane. Notice that the two solution for the normal (or inverted) hierarchy have values of $\sin\delta$ which are numerical very close but differ in the sign of $\cos\delta$ and that the separation between the solutions the normal hierarchy and those with the inverted hierarchy is 1.4 in $\sin\delta$.

two solutions for the normal or inverted hierarchies have very similar values for $\sin\delta$ whereas the mean value of $\sin\delta$ for the normal hierarchy solutions differs substantially from the mean value for the inverted hierarchy. Both of these characteristics of the solutions are general features. Also one can relate the mean value of $\sin\delta$ for the normal hierarchy solutions, $\langle\sin\delta\rangle_+$, to that of the mean value of the inverted hierarchy solutions, $\langle\sin\delta\rangle_-$, using θ_{crit} as follows[11)]

$$\boldsymbol{\langle\sin\delta\rangle_+ - \langle\sin\delta\rangle_- \;=\; 2\langle\theta_{13}\rangle/\theta_{crit}.} \tag{6}$$

At the largest value of the transition probabilities in the overlap region, the difference between the two mean values of $\sin\delta$ is 2 and the value of θ_{13} is θ_{crit}, thus Eq.[6] is trivially satisfied. At smaller values of θ_{13} the right hand side just simply scales linearly with θ_{13}. Using the value of $\sin^2 2\theta_{crit}$ calculated from Eq.[5], the above expression is more usefully written as

$$\langle\sin\delta\rangle_+ - \langle\sin\delta\rangle_- \;=\; 1.4\sqrt{\frac{\sin^2 2\theta_{13}}{0.05}} \qquad \text{for NO}\nu\text{A.} \tag{7}$$

This expression has many uses: the boundary of the inverted (normal) hierarchy in

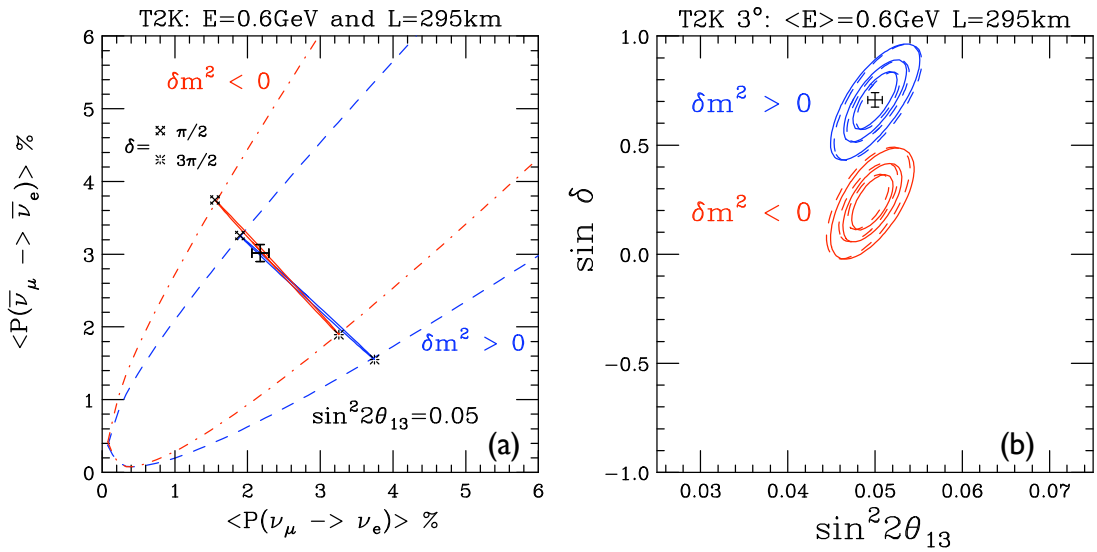

Figure 5: The left panel (a) is the bi-probability plot, $\nu_\mu \to \nu_e$ verses $\bar{\nu}_\mu \to \bar{\nu}_e$ for the T2K experiment. The critical value of θ_{13} is out of range of this figure. Two of the ellipses which pass through the data point (large cross) with $\sin^2 2\theta_{13} = 0.05$ are also shown. The right panel (b) shows the four solutions for this data point in the $\sin\delta$ verses $\sin^2 2\theta_{13}$ plane. Notice that the two solution for the normal (or inverted) hierarchy essentially overlap since the E/L for this experiment is close to vacuum oscillation maximum. The separation between the solutions the normal hierarchy and those with the inverted hierarchy is 0.47 in $\sin\delta$.

the normal (inverted) hierarchy allowed region is given for NOνA by

$$\mathrm{sign}(\delta m^2_{31})\langle \sin\delta\rangle \;=\; -1 + 1.4\sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}. \tag{8}$$

Similarly the relationship between $\langle \sin\delta\rangle$ for the normal and inverted hierarchies is

$$\langle \sin\delta\rangle_+ - \langle \sin\delta\rangle_- \;=\; 0.47\sqrt{\frac{\sin^2 2\theta_{13}}{0.05}} \qquad \text{for T2K.} \tag{9}$$

The difference in this expression for T2K and NOνA is just a factor of 3 which comes primarily from the fact that matter density times path length for NOνA is three times that for T2K. Fig.5(a) and (b) are the equivalent figures for T2K as Fig.4 is for NOνA. Clearly the region of overlap between the hierarchies is much larger for T2K as the matter effect is 3 times smaller. Again, the boundary of the inverted (normal) hierarchy in the normal (inverted) hierarchy allowed region is given for T2K by

$$\mathrm{sign}(\delta m^2_{31})\langle \sin\delta\rangle \;=\; -1 + 0.47\sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}. \tag{10}$$

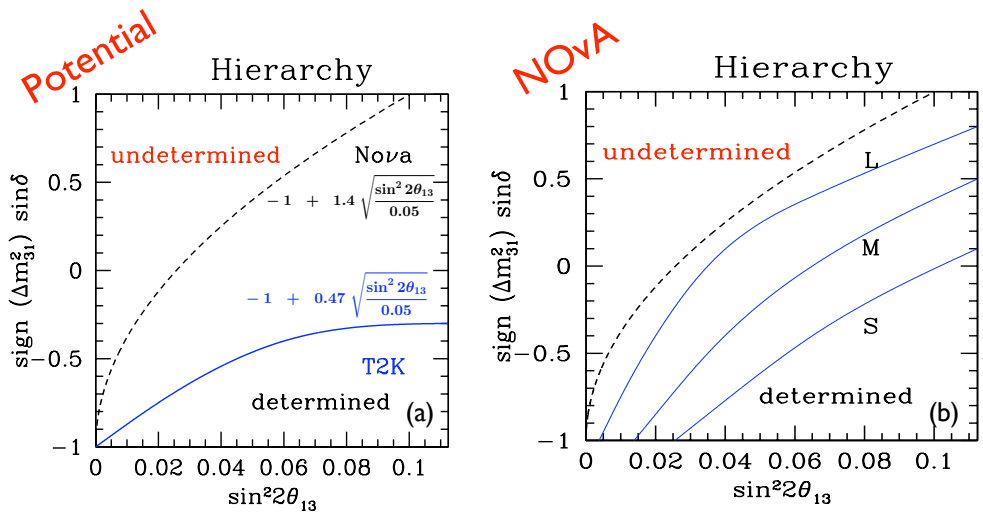

Figure 6: Left panel (a): the regions in the $\sin\delta$ v $\sin^2 2\theta_{13}$ plane for which the mass hierarchy could, in principal, be determined for the NOνA and T2K experiments with both neutrinos and anti-neutrino running. NOνA's region is much large due to the much large matter effect for the NOνA baseline. Right panel (b): NOνA reach once statistics and backgrounds are included, for details see hep-ph/0505202[10].

Fig.6(a) shows the regions in the $\sin\delta$ v $\sin^2 2\theta_{13}$ plane for which the mass hierarchy can be determined, in principal, for each experiment. Whereas, Fig.6(b) shows the reach for NOνA for three different setups labeled S, M and L (The ratios of the statistical power are S:M:L = 1:5:25).

With neutrino and anti-neutrino running in both experiments, we expect the true solutions to coincide within uncertainties

$$| \langle \sin\delta \rangle^{T2K}_{true} - \langle \sin\delta \rangle^{NO\nu A}_{true} | \approx 0, \tag{11}$$

whereas Eq.[6] implies that the fake solutions differ by

$$| \langle \sin\delta \rangle^{T2K}_{fake} - \langle \sin\delta \rangle^{NO\nu A}_{fake} | = 0.94\sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}. \tag{12}$$

Thus, even when $\sin^2 2\theta_{13} = 0.01$ the difference in the values of $\sin\delta$ for the fake solutions in the two experiments is greater than 0.4. Therefore , if $\sin\delta$ can be determined with an uncertainty of approximately 0.2 in both experiments then the hierarchy can be determined down to $\sin^2 2\theta_{13} = 0.01$. However, these are challenging levels of precision for $\sin\delta$ at such small values of $\sin^2 2\theta_{13}$.

Another alternative way to determine the hierarchy using NOνA and T2K, is to compare neutrino running in both experiments at the same E/L[12]. This would require an adjustment to the proposed location of the NOνA experiment or a change in the off-axis angle for T2K[13]. The Super-NOνA[14] idea exploits this idea by building a second detector in the NuMI beamline near Madison, WI.

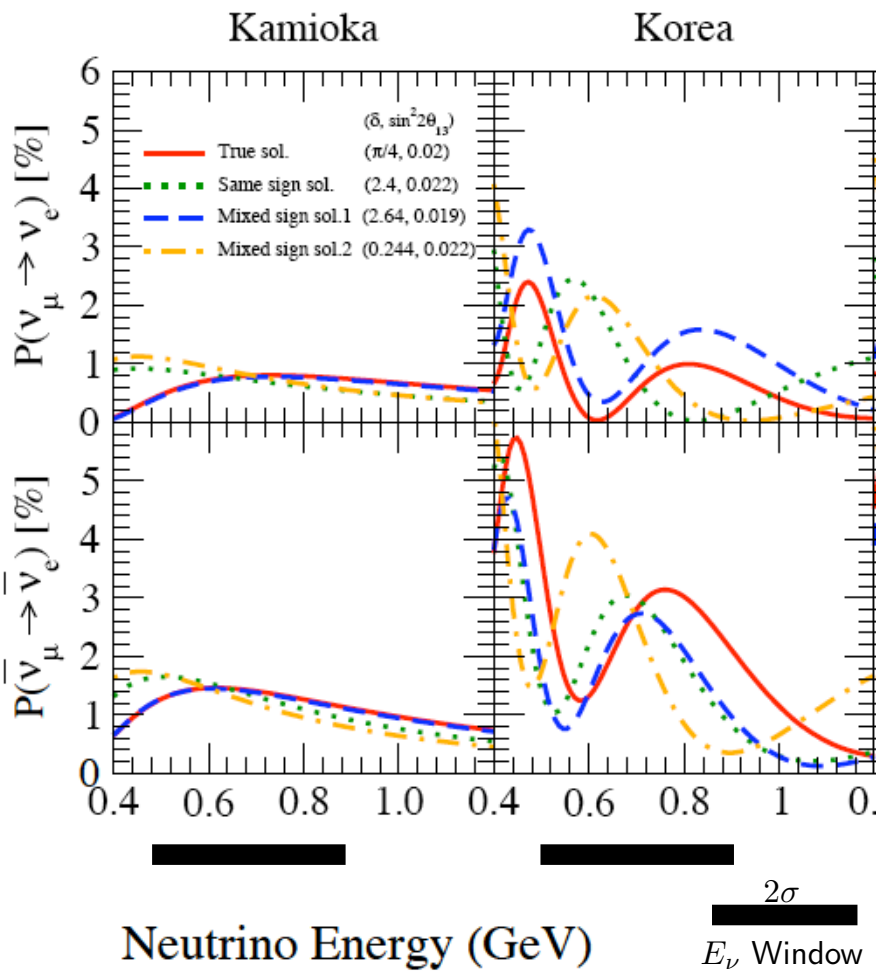

Figure 7: The T2KK spectra for the four degenerate solutions at Kamiokande (left panels) and at a detector placed in Korea (right panels), from hep-ph/0504026[15].

3.2. Spectrum Measurements

An alternative way to breaking the degeneracy between solutions with the normal and inverted hierarchies is to look at spectrum measurements at the second oscillation maximum where the changes in the oscillation probability are more dynamic. The T2KK[15] proposal exploits this idea. Fig. 7 shows the spectrum for the degenerate solutions at Kamiokande compared to what one would see at the second oscillation maximum with a detector in Korea. Clearly the four solutions have similar spectra at Kamiokande but have sizable variation at the second oscillation maximum over the energy window of the off axis beam. See Kajita's talk[16].

Another possibility is to place a detector in a wide band beam and measure the spectra over two or more oscillation peaks like the BNL proposal[17]. Fig.8 shows the

variation in the spectra coming from the choice of hierarchy and the value of the CP violating phase. Excellent π^0 rejection is a very important practical consideration to make this proposal a reality.

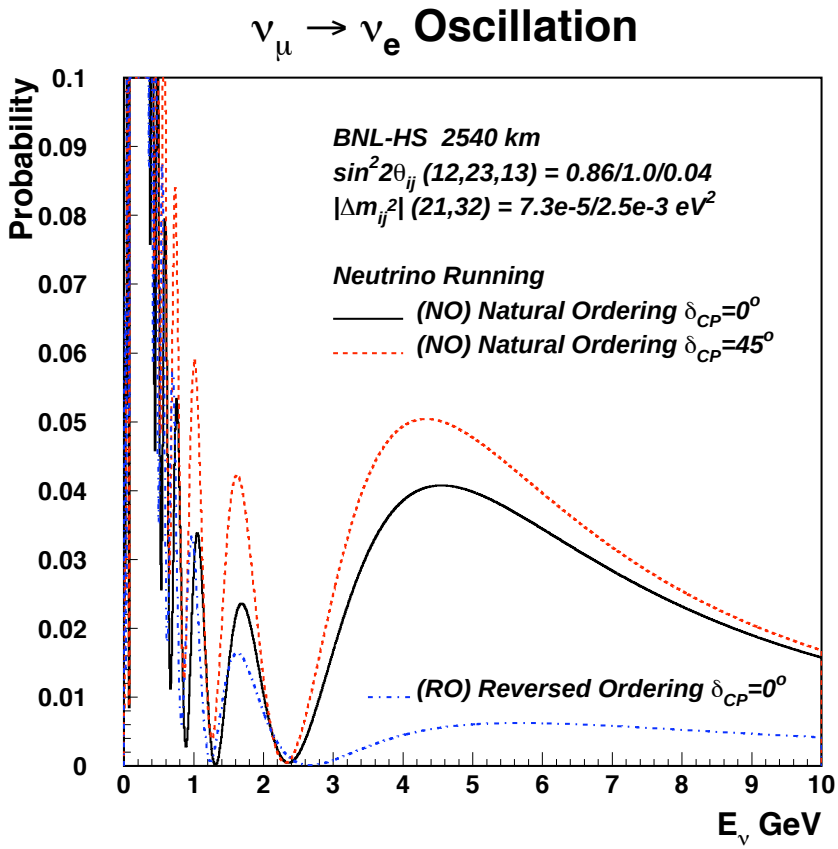

Figure 8: The transition probability for a wide band beam at very distance detector. Matter effects are important at the first peak whereas CP violating effects are more important at the second and following peaks, from hep-ph/0303081[17].

3.3. New Neutrino Beamlines

A neutrino beam produced from a Neutrino Factory is the ultimate machine for determining the neutrino hierarchy because of the intense sources possible and the very large distance possible between source and detector. The large matter effect available in this setup not only amplifies the effects of small θ_{13} but also produces a large asymmetry between neutrino and anti-neutrino running the sign of which depends on the hierarchy. Fig. 9 shows the bi-probability plot[18] as well as the ratio of wrong sign events[19] for both neutrino and anti-neutrino running. More discussion on determining the hierarchy with a Neutrino Factory can be found in Winter's talk[20].

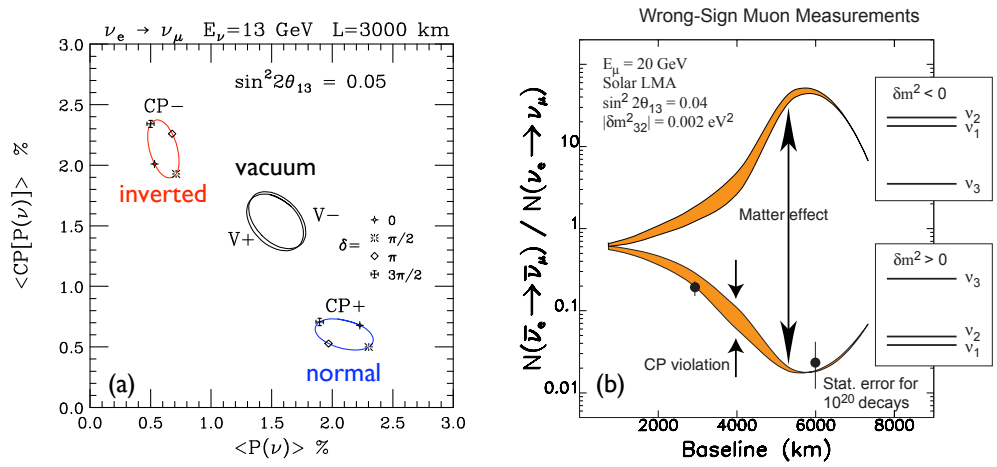

Figure 9: Left panel (a) is the bi-probability plot for the Neutrino Factory setup, from hep-ph/0204171[18]. The right panel (b) is the ratio of wrong sign events for both neutrino and anti-neutrino running at the Neutrino factory, from hep-ex/0008064[19].

4. Summary and Conclusions

It is possible that one of the various mass measurements, from double beta decay, cosmology or tritium beta decay, will determine the neutrino mass hierarchy if the hierarchy is normal. This possibility is independent of the value of $\sin^2 2\theta_{13}$ however it does require the neutrino spectrum to be strongly hierarchial i.e. the mass of lightest neutrino must be smaller than ∼0.01 eV. The most likely way the hierarchy will be determined comes from studying matter effects in $\nu_\mu \leftrightarrow \nu_e$ provided that $\sin^2 2\theta_{13} > 0.0001$. For values of $\sin^2 2\theta_{13}$ not far from the CHOOZ bound, the counting experiments T2K and NOνA could determine the hierarchy with sufficient statistics. For smaller values of $\sin^2 2\theta_{13}$ spectrum measurements near the second peak are probably required, eg T2KK or a long baseline wide band beam. For very small values of $\sin^2 2\theta_{13}$ a neutrino factory will be required.

5. Acknowledgements

I thank Prof. Baldo Ceolin for organizing this wonderfully productive workshop and for my invitation to speak. I am deeply grateful to my collaborators Hisakazu Minakata, Hiroshi Nunokawa and Olga Mena for many discussions on all aspects of this topic.

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