# SUPPRESSION OF TRANSVERSE INSTABILITY BY A DIGITAL DAMPER

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*Abstract*

With cooling, beam phase space density increases, which makes the beam motion intrinsically unstable. To suppress instabilities, dampers are required. With a progress of digital technology, digital dampers are getting to be more and more preferable. Conversion of an analog signal into digital one is described by a linear operator with explicit time dependence. Thus, the analog-digital converter (ADC) does not preserve a signal frequency. Instead, a monochromatic input signal is transformed into a mixture of all possible frequencies, combining the input one with multiples of the sampling frequency. Stability analysis has to include a cross-talk between all these combined frequencies. In this paper, we are analyzing a problem of stability for beam transverse microwave oscillations in a presence of digital damper; the impedance and the space charge are taken into account. The developed formalism is applied for antiproton beam in the Recycler Ring (RR) at Fermilab.

## INSTABILITY BY ITSELF

Beam particles interact through the vacuum chamber. By itself, it always leads to an instability (image charges is a single exception). The interaction is described by the wake fields and their Fourier images – impedances $Z(\omega)$. This leads to a complex frequency shift of the coherent transverse motion (coasting beam):

$$\Delta\omega_Z(n) = -iNr_0\bar{\beta}Z(\omega_b + n\omega_0)/(2\gamma T_0^2)\,, \qquad (1)$$

where $N$ is the number of particles, $r_0$ is the classical radius, $\bar{\beta}$ is an average beta-function, and $T_0$ is the revolution time.

Frequency spread in the beam acts against the instability, but not always. The beam coherent motion cannot be transferred to an incoherent motion of an arbitrary particle. This energy transfer is efficient only for particles, which individual frequencies are equal to the beam coherent frequency, as they see it. This stabilization mechanism is Landau damping. Beam space charge separates coherent and incoherent frequencies by

$$\Delta\omega_{SC} = Nr_0/(2\gamma^2\varepsilon_\perp T_0^2)\,,$$

reducing density of the resonant particles. To be resonant, a particle has to compensate the space charge tune shift by its chromatic offset $\propto\xi$ and the Doppler shift $\propto \eta n$:

$$\Delta\omega_b(n) = \omega_0\,|\,n\eta - \xi\,|\,\delta p\,/\,p\ .$$

If the beam is bunched, the ratio between the space charge shift and the chromatic shift is not changed. This ratio is determined by a specific phase space density $D$:

$$x_n \equiv \Delta\omega_{sc}/\Delta\omega_b(n) \propto D \equiv N/(6\varepsilon_\perp\cdot 4\varepsilon_\parallel)\,,$$

where $6\varepsilon_\perp$ and $4\varepsilon_\parallel$ are 95% normalized transverse and longitudinal emittances. The Landau damping rate is determined by the phase space density of the resonant particles. For a Gaussian distribution this rate is

$$\Lambda_L(n) = \sqrt{\pi/2}\;\Delta\omega_b(n)x_n^2\exp(-x_n^2/2)\,. \qquad (2)$$

The stability condition $\Lambda_L(n) \geq \mathrm{Im}(\Delta\omega_Z)$ can be approximately presented as $x \geq x_{\mathrm{th}}$ with $x_{\mathrm{th}}$~3–5 logarithmically dependent on the impedance, assuming $\Delta\omega_{SC} >> |\Delta\omega_Z|$. Recycler impedance dominates by its resistive wall, vertical one is

$$\mathrm{Re}\,Z(\omega) = 20\sqrt{\omega_0/\omega}\ \ \mathrm{M\Omega/m} \quad\Rightarrow\quad \Delta\omega_{sc}/\Delta\omega_Z \cong 10-20\,.$$

The Landau damping is an extremely steep function of the phase space density, which is not changed with bunching. Thus, for the threshold phase space density $x_{\mathrm{th}}$, the beam bunching can be neglected and the coasting beam model be used with sufficiently good accuracy [1]. Stability analysis for RR with the finite bunch length taken into account was presented in Ref. [2].

For RR, a Gaussian beam of 1.8·10¹² pbars within 50 eV·s and 7 mm·mrad (both 95%) is calculated to be at the instability threshold for the chromaticity ξ = -6. This corresponds to the effective phase space density $D = 0.5$ (in units of $10^{10}$/(mm mrad eV s)). In the reality, the threshold density is about 1.6 times higher. The discrepancy is mainly related to the non-Gaussian tails of the beam energy distribution.

## ANALOG-DIGITAL CONVERTER

A damper essentially consists of a pickup, pre-amplifier, delay line, analog-digital converter (ADC), notch filter, low-pass filter (LPF), and kicker, see Ref. [3].

An amplitude and phase of the high-order LPF are presented at Fig. 1.

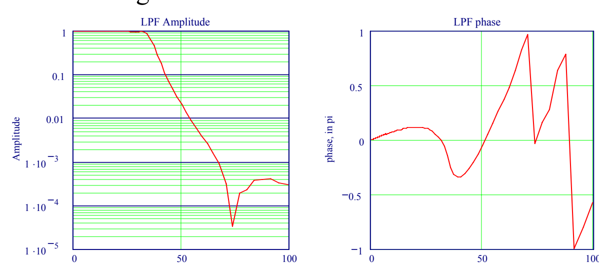


Fig. 1: Amplitude and phase characteristics of the LPF.

In principle, ADC is a linear operator with explicit periodic time dependence. In a simplest case, the ADC output is proportional to the beam offset, taken at the nearest time of "sampling". For more complicated schemes, the output is a linear combination of several nearest samplings.

For RR, originally the sample frequency was 53 MHz, being exactly 588 harmonic of the revolution (to filter out all the revolution harmonics). The input signal was detected at 4 times higher frequency, and then an average of these 4 numbers went as an output. An example of the ADC transformation is presented at Fig. 2 for the input frequency 10.6 MHz. ADC transforms any input frequency $\omega$ into a sequence of all the composite frequencies $\omega + k\omega_s$, shifted from the input one by multiples of the sample frequency $\omega_s$.

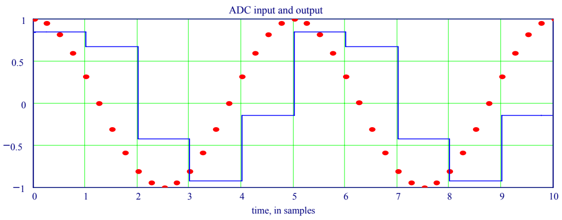


Fig. 2: An example of ADC transformation. The input is shown by red dots (10.6 MHz), the output is a blue line.

In this equidistant sequence of frequencies, there is a single one, $\omega_*$, lying inside an interval $[0, \omega_s]$, and this value can be taken as a parameter of the entire set of the cross-talking frequencies. The continuous parameter $\omega_*$ is referred below as a *marking frequency*. The analog-digital conversion $\hat{T}$ can be described in terms of the matrix $T_{pq}$, transforming incoming frequency $\omega_p = \omega_* + p\omega_s$ into a set of the outgoing frequencies $\omega_q = \omega_* + q\omega_s$, with $p,\ q = 0,\ \pm 1,\ \pm 2,\ \ldots$ . In other words,

$$\hat{T}\exp(-i\omega_p t) = \sum_{q=-\infty}^{\infty} T_{pq}\exp(-i\omega_q t)\,;$$
$$T_{pq} = \frac{2}{N_a}\exp\left[i\omega_p\tau_s\left(1-\frac{1}{2N_a}\right)\right]\frac{\sin^2(\omega_p\tau_s/2)}{\omega_q\tau_s\sin(\omega_p\tau_s/(2N_a))}\,. \tag{3}$$

Here $N_a = 4$ is the number of averaging points, $\tau_s = 1/f_s = 2\pi/\omega_s \approx 20$ ns is the output sampling time. In the following calculations, the linear phase factor in $T_{pq}$ is assumed to be compensated by a proper delay and mixing, so it is just omitted and the matrix $T_{pq}$ gets to be real.

## MODE DYNAMICS

With the ADC, the frequency $\omega$ (representing actually the wave length of the beam perturbation) is no longer a good parameter for the beam modes, each consisting of all the composite harmonics. Instead, the beam modes constitute infinite sets, each set with its own marking frequency $\omega_*$. High enough composite harmonics are strongly damped by Landau damping; thus, they can be neglected and the infinite set of the composite amplitudes being reasonably cut.

Let $A_p$ be an amplitude of the harmonic $\omega_p = \omega_* + p\omega_s$. Were the digital damper the only way for the beam to interact with itself, the beam dynamics would be described as

$$dA_p/dt = -\Lambda_0 \sum\nolimits_q T_{pq} A_q\,,$$

with $\Lambda_0$ as a low-frequency rate, determined by the pre-amplifier. A solution of this set of linear equations is expressed in terms of eigen-vectors, which eigen-values are the damping rates for the set of the beam modes. Impedance and Landau damping just add their terms to the matrix diagonal elements. Thus, the set of the dynamic equations follows:

$$dA_p/dt = -\Lambda_0 \sum\nolimits_q T_{pq} A_q - \Lambda_L(p)A_p - i\Delta\omega_Z(p)A_p\,. \tag{4}$$

The ADC matrix T is strongly degenerated: all its eigenvalues, but one, are exact zeroes. With impedance, half of these zeroes are getting unstable. With sufficient density of the resonant particles, stability can be provided by the Landau damping.

Without damper, for 2.8·10[12] antiprotons inside 35eV·s and 7mm·mrad (95%), the impedance-driven rate and the Landau damping rate are presented in Fig. 3 below. For

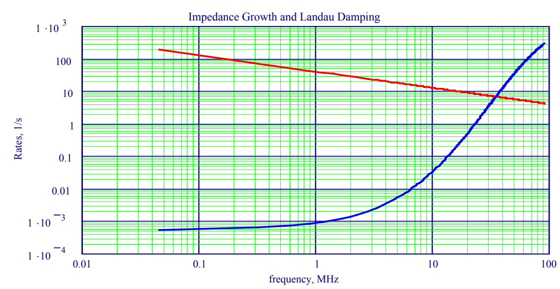


Fig. 3: Impedance-driven rate (red) and Landau damping rate (blue), 1/s, versus frequency, MHz. The modes below 35 MHz are unstable.

these parameters, the modes below 35 MHz are unstable.

Results for the stability analysis for the same beam and the damper are presented in Fig. 4. On the left, the damping rate for the less stable mode is shown as a function of the marking frequency for the damper's low-frequency rate of 600 turns. Clearly, it is the instability threshold (the density D=1.1). On the right, the average frequency of this less stable mode is calculated, where all the composite frequencies are weighted with amplitudes of their perturbations squared. The curve minimum is close to the Nyquist frequency 26.5 MHz.

The calculated threshold is the same for the LPF off. The reason is that if the filter is wider then the Landau damping boundary, it makes nothing. Otherwise, it makes the damper inefficient in a frequency range between the filter cut-off and the Landau damping start. The filter is useful though for another purpose – suppression of the amplifier noise to prevent emittance growth.

Offset of the sampling frequency from a revolution multiple would stop the harmonics cross-talk through the beam. As a result, the calculated threshold increases on ~ 20% for the mentioned parameters.

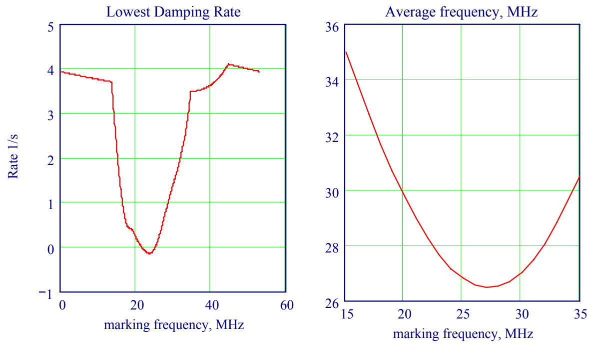


Fig. 4: Damping rate, 1/s, of the least stable mode as a function of the marking frequency (left), and the average frequency of this mode vs. marking frequency (right). All the frequencies are in MHz.

## OBSERVATIONS AND MODERNIZATION

Without damper, the instability blow-up was observed at the density $D$ = 0.8 versus $D$ = 0.5 calculated for the Gaussian distribution. This looks about consistent with the real distribution which tails are higher than Gaussian.

With the damper, the beam was cooled up to $D$ = 1.5, (with 53 MHz of the ADC output). A benefit from the damper is close to the predicted factor of 2 (1.1/0.5 ≈1.5/0.8).

For doubled sample rate, 106 MHz, the calculated threshold is 30% higher. The result is not sensitive to the averaging parameter $N_a$, averaging is still useful though for the noise suppression.

A goal with $6{\cdot}10^{12}$ antiprotons in 40 eVs and 5 mm mrad corresponds to the effective density $D$ = 3, is about 2 times higher than the threshold with the existing damper. This requires broader band of the kicker.

With a 4 times shorter kicker (already installed) and the sampling rate of 212 MHz, the calculated threshold goes up to $D$ = 2.5 (e. g. $6{\cdot}10^{12}$ antiprotons in 50 eVs and 5 mm mrad ). Remembering that the measured threshold of the non-Gaussian beam was ~ 1.5 times higher than one calculated for the Gaussian model, this means that the new kicker and already effective 212 MHz of the sampling should be sufficient for the Run II goal.

## SUMMARY

Threshold of the resistive wall transverse instability is calculated without damper; the space charge is taken into account.

A specific of digital dampers as linear operators with explicit periodic time dependence modifies structure of the eigen-modes. The stability problem with the digital damper is solved.

Calculations are in a reasonable agreement with measurement data.

Quadrupling of the sample frequency and 4 times shorter kicker should make the entire damper sufficient for the Run II goals.

## REFERENCES

[1] A. Burov, FERMILAB-TM-2336-AD, 2005.
[2] V. Balbekov, FERMILAB-PUB-06-070-AD, 2006.
[3] J. Crisp, M. Hu, V. Tupikov, FERMILAB-CONF-05-167-AD, 2005.