

**$B_s$  Mixing at the Tevatron**

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Abstract

The Tevatron collider at Fermilab provides a very rich environment for the study of  $B_s$  mesons.  $B_s$  Mixing is the most important analysis within the  $B$  Physics program of both experiments. In this paper we summarize the most recent results on this topic from both DØ and CDF experiments. There were very important updates in both experiments after I gave my talk, hence the organizers warmly recommended me to include the latest available results on  $B_s$  mixing, instead of what I presented there.

## 1 Introduction

The Tevatron collider at Fermilab, operating at  $\sqrt{s} = 1.96 \text{ TeV}$ , has a huge  $b$  production rate which is 3 orders of magnitude higher than the production rate at  $e^+e^-$  colliders running on the  $\Upsilon(4S)$  resonance. Among the produced  $B$  particles there are as well heavy and excited states which are currently uniquely accessible at the Tevatron, such as for example  $B_s$ ,  $B_c$ ,  $\Lambda_b$ ,  $\theta_b$ ,  $B^{**}$  or  $B_s^{**}$ . Dedicated triggers are able to pick 1  $B$  event out of 1000 QCD events by selecting leptons and/or events with displaced vertices already on hardware level.

The aim of the  $B$  Physics program of the Tevatron experiments DØ and CDF is to provide constraint to the CKM matrix which takes advantage of the unique features of a hadron collider.  $B_s$  Mixing is the flagship analysis within the  $B$  Physics program of both experiments.

Both the DØ and CDF detectors are symmetric multi-purpose detectors having both silicon vertex detectors, high resolution tracking in a magnetic field and lepton identification <sup>1, 2)</sup>. CDF is for the first time in an hadronic environment able to trigger on hardware level on large track impact parameters which indicates displaced vertices. Thus it is very powerful in fully hadronic  $B$  modes. Analyses are based on an integrated luminosity of about  $1 \text{ fb}^{-1}$ . Details on both analyses are given in <sup>3, 4)</sup>.

## 2 $B_s^0$ Mixing in the Standard Model

In the Standard Model, the  $B_s^0$  meson exists in two  $CP$ -conjugate states,  $|B_s^0\rangle = |\bar{b}s\rangle$  and  $|\bar{B}_s^0\rangle = |b\bar{s}\rangle$ . The two mass eigenstates of the  $B_s^0$  meson,  $B_s^H$  and  $B_s^L$  ( $H$  = ‘heavy’ and  $L$  = ‘light’), are not  $CP$ -eigenstates, but are mixtures of the two  $CP$ -conjugate quark states:

$$|B_s^H\rangle = p|B_s^0\rangle - q|\bar{B}_s^0\rangle \text{ and } |B_s^L\rangle = p|B_s^0\rangle + q|\bar{B}_s^0\rangle, \quad \text{with } p^2 + q^2 = 1. \quad (1)$$

The mass and lifetime differences between the  $B_s^H$  and  $B_s^L$  can be defined as

$$\Delta m \equiv m_H - m_L, \quad \Delta\Gamma \equiv \Gamma_L - \Gamma_H \quad \text{and} \quad \Gamma = \frac{\Gamma_H + \Gamma_L}{2}, \quad (2)$$

where  $m_{H,L}$  and  $\Gamma_{H,L}$  denote the mass and decay width of  $B_s^H$  and  $B_s^L$ . The probability  $\mathcal{P}$  for a  $B_s^0$  meson produced at time  $t = 0$  to decay as  $\bar{B}_s^0$  at proper time  $t > 0$  is given by

$$\mathcal{P}_{B_s^0}^{\text{mix}} = \mathcal{P}(B_s^0 \rightarrow \bar{B}_s^0) = \frac{1}{2} \Gamma e^{-\Gamma t} [1 - \cos(\Delta m_s t)], \quad (3)$$

neglecting effects from  $CP$  violation as well as a possible lifetime difference between the heavy and light mass eigenstates of the  $B_s^0$ . A measurement of the

oscillation frequency  $\Delta m_s$  gives a direct measurement of the mass difference between the two physical  $B_s^0$  meson states.

Particle-antiparticle oscillations have been observed and well established in the  $B_d$  system. The mass difference  $\Delta m_d$  is measured to be  $\Delta m_d = (0.505 \pm 0.005) \text{ ps}^{-1}$  [5]. However, observing the oscillation signal in the  $B_s^0$  system has been challenging so far. The 95% C.L. limit for the mass difference is  $\Delta m_s > 14.4 \text{ ps}^{-1}$  [5].

The canonical  $B$  mixing analysis, in which oscillations are observed and the mixing frequency,  $\Delta m$ , is measured, proceeds as follows. The  $B$  meson flavor at the time of its decay is determined by exclusive reconstruction of the final state. The proper time,  $t = m_B L/pc$ , at which the decay occurred is determined by measuring the decay length,  $L$ , and the  $B$  momentum,  $p$ . Finally the production flavor must be tagged in order to classify the decay as being mixed or unmixed at the time of its decay.

Oscillation manifests itself in a time dependence of the mixed asymmetry:

$$\mathcal{A}_{unmix}(t) = \frac{N_{unmixed}(t) - N_{mixed}(t)}{N_{unmixed}(t) + N_{mixed}(t)} = \cos \Delta m t \quad (4)$$

In practice, the production flavor will be correctly tagged with a probability  $P_{tag}$ , which is significantly smaller than one, but larger than one half (which corresponds to a random tag). The measured mixing asymmetry in terms of dilution,  $\mathcal{D}$ , is

$$\mathcal{A}_{unmix}^{meas}(t) = \mathcal{D} \mathcal{A}_{unmix} = \mathcal{D} \cos \Delta m t \quad (5)$$

where  $\mathcal{D} = 2P_{tag} - 1$ .

To measure time-dependent oscillations three ingredients are needed:

- Large  $B_s^0$  samples with good signal-to-background ratio, where the  $b$  flavor at decay time is known. Sufficient statistic is needed to be sensitive to high mixing frequencies.
- Proper decay time with good resolution, which is specially important in order to resolve high  $\Delta m_s$  mixing frequency.
- $b$  flavor at production time, where the production flavor additional information from the event has to be evaluated in order to tag as either unmixed or mixed.

### 3 Reconstructed $B_s$ Decays

DØ exploits the high statistics muon trigger to study semileptonic  $B_s^0$  decays. Several thousands candidates have been reconstructed in the  $B_s^0 \rightarrow$

$\mu^+ D_s^- X, D_s^- \rightarrow \phi \pi^-$  mode. Additionally DØ is also working on reconstructing  $B_s^0 \rightarrow \mu^+ D_s^- X, D_s^- \rightarrow K^{*0} K^-$  candidates. Throughout this document references to a specific charge state imply the charge-conjugate state as well.

CDF performs the  $B_s$  mixing analysis using both fully reconstructed  $B_s^0$  decays ( $B_s^0 \rightarrow D_s^- (\pi^+ \pi^-) \pi^+$ ) and semileptonic decays ( $B_s^0 \rightarrow \ell^+ D_s^- X$ ). In both cases the  $D_s$  is reconstructed in the  $D_s^- \rightarrow \phi \pi^-$ ,  $D_s^- \rightarrow K^{*0} K^-$  and  $D_s^- \rightarrow \pi^+ \pi^- \pi^-$  modes.

Fig. 1 shows the reconstructed semileptonic  $B_s^0 \rightarrow \ell^+ D_s^- X$  candidates from both DØ and CDF.

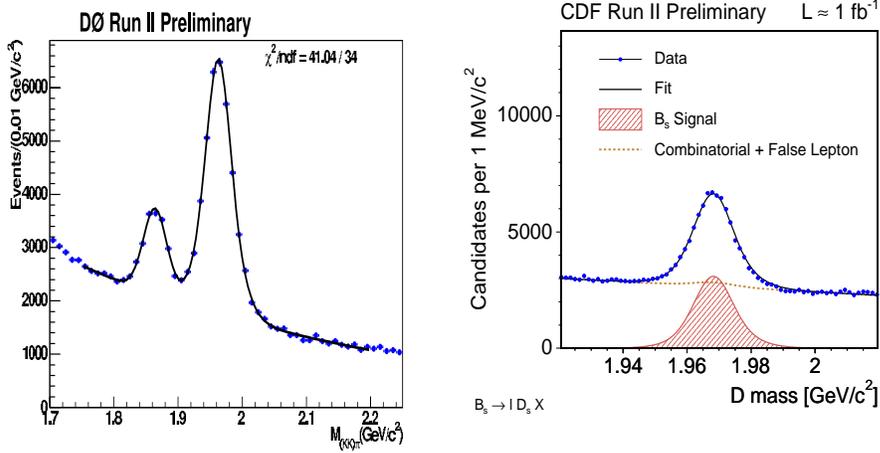


Figure 1:  $D_s$  meson Mass distributions for  $\mu D_s, D_s \rightarrow \phi \pi$  at DØ (left) and for all combined  $\ell D_s$  decay modes at CDF (right) for  $\sim 1 \text{ fb}^{-1}$  data sample.

#### 4 Decay Length Reconstruction and Resolution

The transverse decay length  $L_{xy}(B)$  is defined as the displacement  $\vec{X}$  in the transverse plane from the primary event vertex to the reconstructed  $B$  decay point, projected onto the  $B$  transverse momentum

$$L_{xy}^B = \frac{\vec{X} \cdot \vec{B}(B)}{|\vec{p}_T^B(B)|}. \quad (6)$$

The  $B$  meson decay time is then given by

$$ct(B) = L_{xy}^B \frac{m(B)}{p_T(B)}, \quad (7)$$

where  $m(B)$  is the  $B$  mass <sup>5)</sup>. For the semileptonic  $B$  decays we must substitute the  $B$  decay by the  $\ell D$  system. Since the  $B$  meson is not fully reconstructed, the pseudo proper decay time  $t^*$  of the reconstructed  $B$  meson is computed from the measured decay length  $L_{xy}^B$  as

$$ct^* = L_{xy}^B \frac{m(B)}{p_T(\ell D)} \quad (8)$$

and introduce a correction factor

$$K = \frac{p_T(\ell D)}{p_T(B)}. \quad (9)$$

This  $k$ -factor corrects between the reconstructed  $p_T(\ell D)$  and the unknown  $p_T(B)$  in the data. The  $k$ -factor distribution  $\mathcal{F}(K)$  is obtained from a MC simulation of the signal semileptonic decays taking into account the sample composition. One improvement made by both DØ and CDF is the use different  $k$ -factor distributions as a function of the lepton- $D$  mass. As shown in Fig. 2, that distribution changed a lot along the lepton- $D$  mass, making events with large lepton- $D$  mass much more valuable than the others.

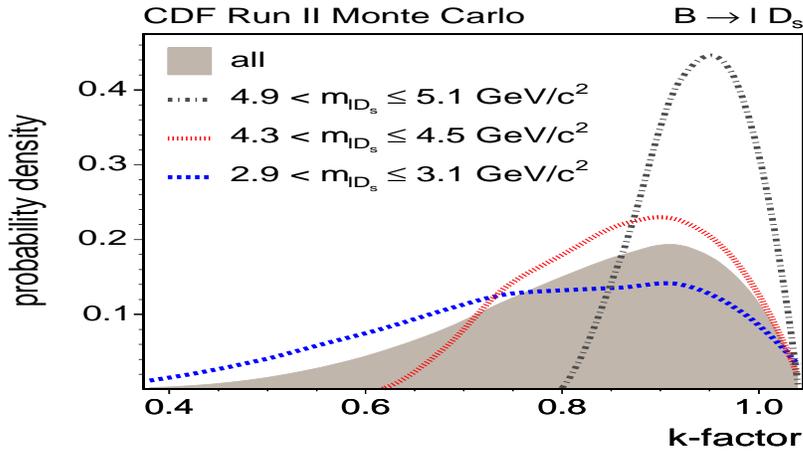


Figure 2:  $k$ -factor distribution for several  $m(\ell D)$  mass regions for  $B_s^0 \rightarrow \ell^+ D_s^-, D_s^- \rightarrow \phi \pi^-$  decays.

Due to some cuts on variables related to the reconstructed proper decay time distribution, the proper decay time does not follow a pure exponential

(modulo resolution and  $k$ -factor effects), but it is biased. This bias, expressed as an efficiency curve is obtained using Monte Carlo simulation.

The determination of proper time uncertainties is notoriously difficult, so experiments usually introduce a scale factor to adjust these uncertainties. CDF introduces an event-by-event correction, which depends on the topology and on several kinematical quantities.  $D\mathcal{O}$  applies an overall correction with a double Gaussian distribution: the narrow Gaussian has a width of  $0.998\sigma$  and comprises 72% of the total, and the second Gaussian has a width of  $2.777\sigma$ , being  $\sigma$  the default event-by-event error on the pseudo-proper decay time.

## 5 Flavor Tagging

One of the components of measuring neutral  $B$  mesons flavor oscillations is identifying whether the  $B$  meson was produced as a  $B$ , which contains  $\bar{b}$  anti-quark, or a  $\bar{B}$ , which contains  $b$  quark. We refer to this  $B$  hadron flavor identification as “ $b$  flavor tagging”. The methods of  $b$  flavor tagging may be classified into two categories: opposite-side and same-side  $b$  flavor taggers. Opposite-side taggers exploit the fact that  $b$  quarks in hadron colliders are mostly produced in  $b\bar{b}$  pairs. Same-side flavor tags are based on the charge of particles produced in association with the production of the  $B$  hadron. The performance of the  $b$  flavor tags may be quantified conveniently by their efficiency  $\epsilon$  and their dilution  $D$ . Efficiency is the fraction of  $B$  hadrons which the flavor tag can be applied, while dilution has already been defined above in the text.

### 5.1 Soft-Lepton-Tagging

In 20 % of cases the opposite semileptonic  $b$  decays either into an electron or a muon ( $b \rightarrow l^- X$ ). The charge of the lepton is correlated to the charge of the decaying  $B$  meson. Depending on the type of the  $B$  meson there is a certain probability of oscillation between production and decay (0 % for  $B^\pm$ , 17.5 % for  $B_d$  and 50 % for  $B_s$ ). Therefore this tagging algorithm already contains an intrinsic dilution. Another potential source of miss-tag is the transition of the  $b$  quark into a  $c$  quark, which then forms a  $D$  meson and subsequently decays semileptonically ( $\bar{b} \rightarrow \bar{c} \rightarrow l^- X$ ). Due to the different decay length and momentum distribution of  $B$  and  $D$  meson decays this source of miss-tag can mostly be eliminated.

### 5.2 Jet-Charge Tagging

The average charge of an opposite-side  $b$ -jet is weakly correlated to the charge of the opposite  $b$  quark and can thus be used to determine the opposite-side  $b$  flavor. The main challenge of this tagger is to select the  $b$ -jet. Information of a displaced vertex or displaced tracks in the jet helps to identify  $b$ -jets. This

tagging algorithm has high tagging efficiency, but the dilution is relatively low. By separating sets of tagged events of different qualities e.g. how  $b$  like the jet is, it is possible to increase the overall tagging performance.

### 5.3 Same-Side Tagging

During fragmentation and the formation of the  $B_{s/d}$  meson there is a left over  $\bar{s}/\bar{d}$  quark which is likely to form a  $K^+/\pi^+$ . Hence if there is a nearby charged particle, which is additionally identified as a kaon/pion, it is quite likely that it is the leading fragmentation track and its charge is then correlated to the flavor of the  $B_{s/d}$  meson. While the performance of the opposite-side tagger does not depend on the flavor of the  $B$  on the signal side, the same-side tagger performance depends on the signal fragmentation processes. Therefore the opposite-side performance can be measured in  $B_d$  mixing and can then be used for setting a limit on the  $B_s$  mixing frequency. But for using the same-side tagger for a limit on  $\Delta m_s$ , it is needed to rely on Monte Carlo simulation. CDF has performed extensive data and Monte Carlo comparisons on all quantities related to the tagging and used different tagging algorithms to probe different aspects of the fragmentation. The final algorithm used to determine the tagging candidate is to select the most likely kaon track, which used a combined particle identification likelihood with the information from the  $dE/dx$  and from the Time-of-Flight. A comparison between data and Pythia Monte Carlo for the average dilution obtained by using that variable is shown in Fig. 3. A very good agreement is found in the high statistics  $B^0$  and  $B^+$  modes. Finally the overall tagging performance as computed in Monte Carlo is:  $\epsilon D^2(B_s \rightarrow D_s(\phi\pi)\pi) = 4.0^{+0.8\%}_{-1.2\%}$ . When applying the same-side kaon tagger to different subsample, CDF accounts for the fact that the tagger performance varies with the average transverse momentum of the  $B_s^0$  mesons in each sample.

### 5.4 $\Delta m_d$ Measurement and Calibration of Taggers

For setting a limit on  $\Delta m_s$  the knowledge of the tagger performance is crucial. Therefore it has to be measured in kinematically similar  $B_d$  and  $B^+$  samples.

The  $\Delta m_s$  and  $\Delta m_d$  analysis are complex fits with many parameters which combine several  $B$  flavor and several decay modes, various different taggers and deals with complex templates for mass and lifetime fits for various sources of background. Therefore the measurement of  $\Delta m_s$  is beside the calibration of the opposite-side taggers a very important to test and trust the fitter framework, although the actual  $\Delta m_d$  result at the Tevatron is not competitive with the  $B$  factories.

Both CDF and DØ have demonstrated that the whole machinery is working, being  $\Delta m_d$  measurements compatible with the PDG average value. The combined tagging performance of the opposite-side taggers is about 1.5% for

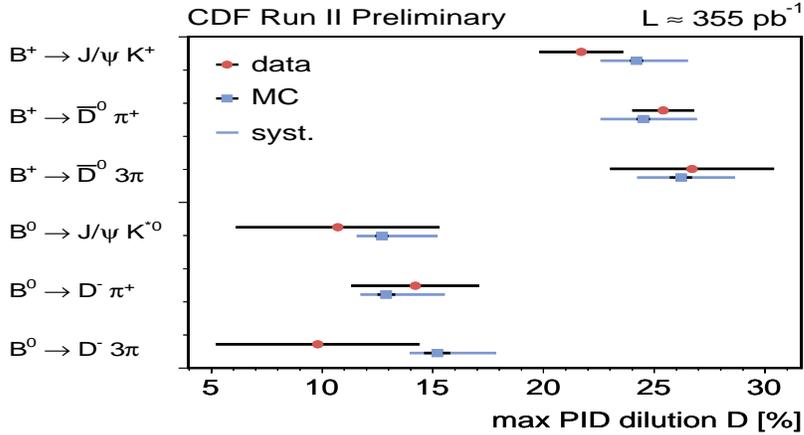


Figure 3: Comparison between CDF data and Pythia Monte Carlo for the average dilution obtained by selecting the most likely kaon track as tagging candidate.

CDF and about 2.5% for  $D\bar{0}$ . The main difference between both results comes from the better muon coverage at  $D\bar{0}$ . The tagger performance of the different opposite-side taggers is summarized in Tab. 1.

Tagger	$\epsilon D^2$ (%)	
	$D\bar{0}$	CDF
Muon	$1.48 \pm 0.17$	$0.55 \pm 0.05$
Electron	$0.21 \pm 0.07$	$0.30 \pm 0.03$
JQT	$0.50 \pm 0.11$	$0.70 \pm 0.06$
Combined	$2.48 \pm 0.22$	$1.55 \pm 0.08$

Table 1: *Tagging performance of the different opposite-side taggers.*

The fitted asymmetry using the combined opposite taggers on the semileptonic decay modes from  $D\bar{0}$  is displayed in Fig. 4.

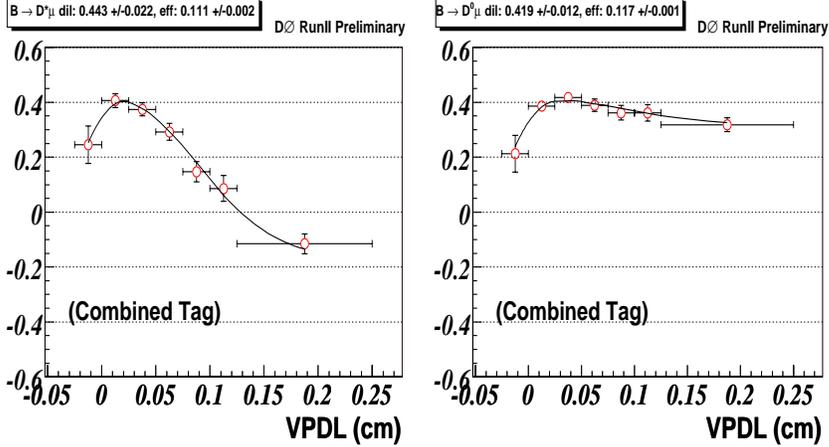


Figure 4: Asymmetry fit projection for  $\Delta m_d$  using a combined opposite-side tagger in semileptonic decays from  $D\phi$ .

## 6 Amplitude Scan

An alternative method for studying neutral  $B$  meson oscillations is the so called “amplitude scan”, which is explained in detail in Reference [6](#)). The likelihood term describing the tagged proper decay time of a neutral  $B$  meson is modified by including an additional parameter multiplying the cosine, the so-called amplitude  $A$ .

The signal oscillation term in the likelihood of the  $\Delta m$  thus becomes

$$\mathcal{L} \propto \frac{1 \pm AD \cos(\Delta mt)}{2} \quad (10)$$

The parameter  $A$  is left free in the fit while  $\mathcal{D}$  is supposed to be known and fixed in the scan. The method involves performing one such  $A$ -fit for each value of the parameter  $\Delta m$ , which is fixed at each step; in the case of infinite statistics, optimal resolution and perfect tagger parameterization and calibration, one would expect  $A$  to be unit for the true oscillation frequency and zero for the remaining of the probed spectrum. In practice, the output of the procedure is accordingly a list of fitted values  $(A, \sigma_A)$  for each  $\Delta m$  hypothesis. Such a  $\Delta m$  hypothesis is excluded to a 95% confidence level in case the following relation is observed,  $A + 1.645 \cdot \sigma_A < 1$ .

The sensitivity of a mixing measurement is defined as the lowest  $\Delta m$  value for which  $1.645 \cdot \sigma_A = 1$ .

The amplitude method will be employed in the ensuing  $B_s$  mixing analysis. One of its main advantages is the fact that it allows easy combination among different measurements and experiments.

The plot shown in Fig. 5 is obtained when the method is applied to the hadronic  $B_d$  samples of the CDF experiment, using the exclusively combined opposite-side tagging algorithms.

The expected compatibility of the measured amplitude with unit in the vicinity of the true frequency,  $\Delta m_d = 0.5 \text{ ps}^{-1}$ , is confirmed.

However, we observe the expected increase in the amplitude uncertainty for higher oscillation frequency hypotheses. This is equivalent to saying that the significance is reduced with increasing frequency.

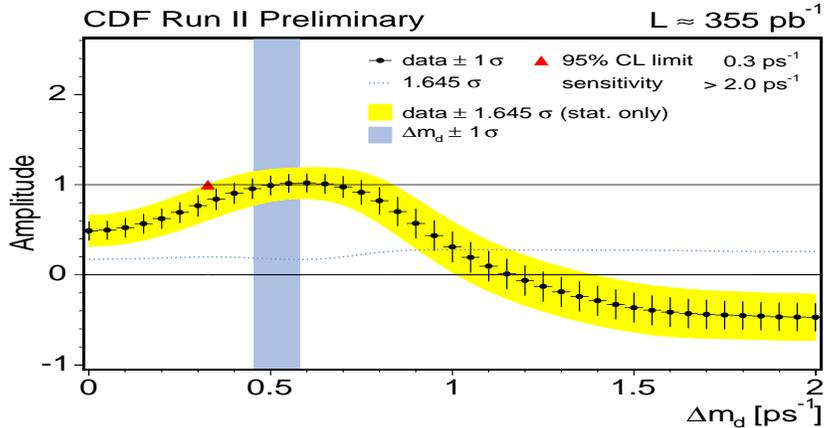


Figure 5: *Amplitude scan for  $\Delta m_d$  in hadronic decay modes (CDF). The scan is compatible with 1 around the result of the actual  $\Delta m_d$  fit.*

## 7 $B_s^0$ Mixing Results

### 7.1 $D\bar{0}$

The result of the  $D\bar{0}$  amplitude scan on  $\sim 1 \text{ fb}^{-1}$  is shown in Fig. 8. The sensitivity is  $14.1 \text{ ps}^{-1}$ , and the 95% C.L. limit is  $\Delta m_s > 14.8 \text{ ps}^{-1}$ . Fig. 7 shows the dependence of  $\mathcal{L}$  as a function of  $\Delta m_s$ , when the amplitude is fixed to  $\mathcal{A} = 1$ . The preferred value is  $\Delta m_s = 19 \text{ ps}^{-1}$ , with a 90% interval of  $17 < \Delta m_s < 21 \text{ ps}^{-1}$ . The probability that random tags background could fluctuate to mimic such a signature is about 5%.

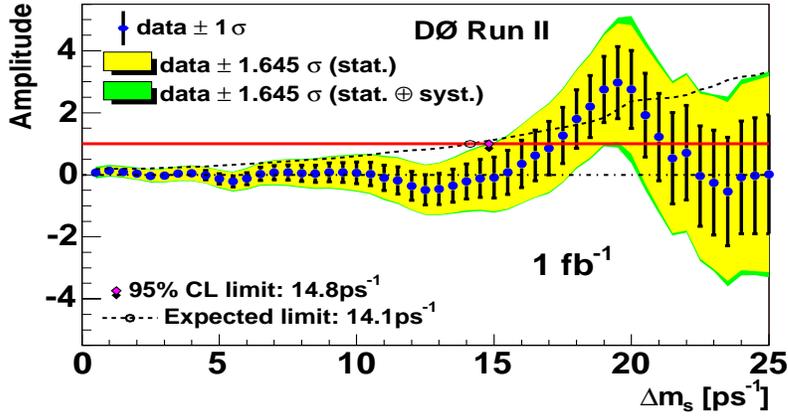


Figure 6:  $B_s^0$  oscillation amplitude as a function of the oscillation frequency,  $\Delta m_s$ , for the DØ analysis.

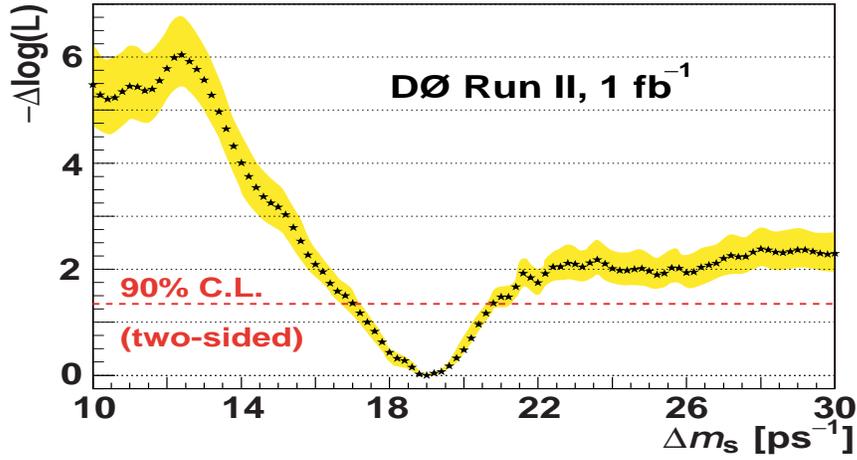


Figure 7: Value of  $-\Delta\mathcal{L}$  as a function of  $\Delta m_s$  for the DØ analysis.

## 7.2 CDF

The result of the CDF combined amplitude scan on  $\sim 1 \text{ fb}^{-1}$  is shown in Fig. 8. The sensitivity for the combination of all hadronic and semileptonic modes is

25.3 ps<sup>-1</sup>, and the 95% C.L. limit is  $\Delta m_s > 16.7$  ps<sup>-1</sup>.

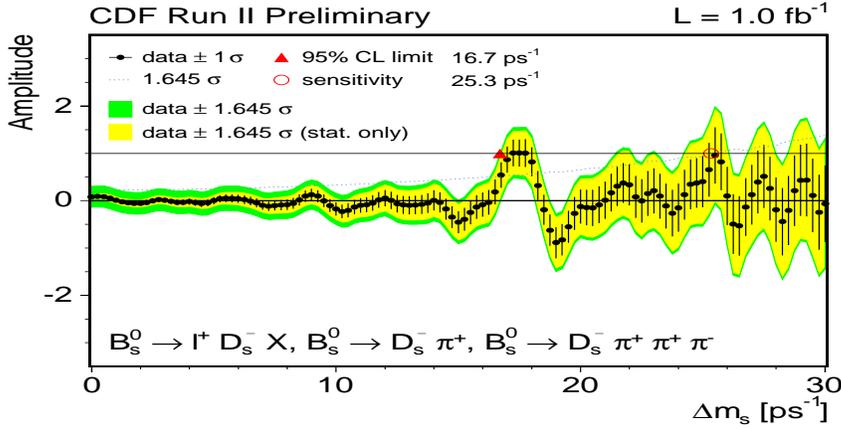


Figure 8:  $B_s^0$  oscillation amplitude as a function of the oscillation frequency,  $\Delta m_s$ , for the CDF analysis.

The 95% confidence level is significantly lower than the expected limit because the amplitude shows a value consistent with unity near  $\Delta m_s = 17.25$  ps<sup>-1</sup>. To assess the significance of this deviation, CDF looks at the ratio of the likelihood function at  $\mathcal{A}=0$  and  $\mathcal{A}=1$ , as shown in Fig. 9. The maximum likelihood ratio is at  $\Delta m_s = 17.33$  ps<sup>-1</sup> and has a value of 6.06. The probability that random tags background could fluctuate to mimic such a signature is 0.5%. Under the hypothesis that this is a signal for  $B_s^0 - \bar{B}_s^0$  oscillations, CDF measures  $\Delta m_s = 17.33_{-0.21}^{+0.42}(\text{stat.}) \pm 0.07(\text{syst.})$  ps<sup>-1</sup>. The systematic error of this measurement is completely dominated by the  $ct$  scale uncertainty, which is of the order of 0.4%.

## 8 Conclusions

The large amount of data collected by the CDF and DØ experiments are improving our knowledge about  $B_s$  mesons, in particular on  $B_s^0$  Mixing. Two very interesting results have recently appeared in the market. Both of them have used a total integrated luminosity of about 1 fb<sup>-1</sup>.

DØ has performed a study  $B_s^0 - \bar{B}_s^0$  oscillations using  $B_s^0 \rightarrow \mu^+ D_s^- X$  decays and an opposite-side flavor tagging algorithm. The expected limit at 95% C.L. is 14.1 ps<sup>-1</sup>. At  $\Delta m_s = 19$  ps<sup>-1</sup>, the amplitude method yields a

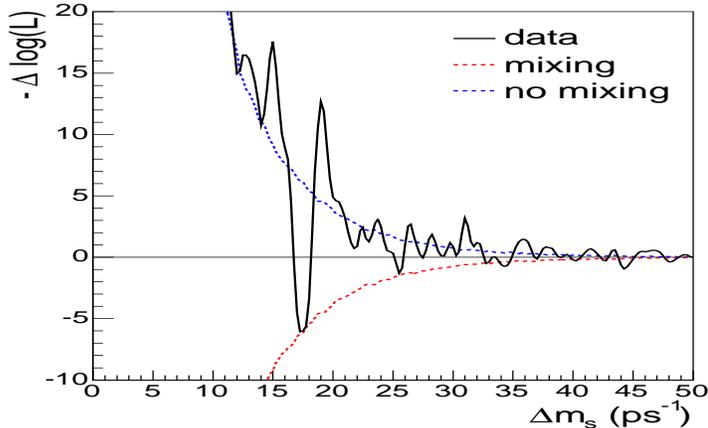


Figure 9: Combined likelihood ratio as a function of  $\Delta m_s$  for the CDF analysis.

result that deviates from the hypothesis  $\mathcal{A} = 0$  ( $\mathcal{A} = 1$ ) by 2.5 (1.6) standard deviations, corresponding to a two-sided C.L. of 1%(10%). Assuming Gaussian uncertainties a 90% C.L. interval of  $17 < \Delta m_s < 21 \text{ ps}^{-1}$  is set.

CDF has searched for  $B_s^0$  flavor oscillations using hadronic and semileptonic decays. Opposite-side and for first time same-side tags provide information about the  $B_s^0$  production flavor. Using an amplitude scan method, CDF obtains a 95% confidence level limit on the oscillation frequency  $\Delta m_s > 16.7 \text{ ps}^{-1}$  for a  $\Delta m_s$  sensitivity of  $25.3 \text{ ps}^{-1}$ . The observed limit is noticeable lower than the expected limit due to a statistical significant signature consistent with  $B_s^0 - \bar{B}_s^0$  oscillations, being the probability that random tags background could fluctuate to mimic such a signature 0.5%. Assuming this is a signal for  $B_s^0 - \bar{B}_s^0$  oscillations, we measure  $\Delta m_s = 17.33_{-0.21}^{+0.42}(\text{stat.}) \pm 0.07(\text{syst.}) \text{ ps}^{-1}$ .

## 9 Acknowledgments

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