FERMILAB-CONF-06-036-AD

Ionization Cooling in a Low-Energy Proton Storage Ring

David Neuffer

Fermilab
PO Box 500, Batavia IL 60510

Abstract. At the FFAG05 meeting, Mori and Okabe presented a scenario in which the lifetime of protons in a low-energy storage ring (~10 MeV) is extended by energy-loss in a wedge foil, and this enables greater neutron production from the foil. The lifetime extension is due to the cooling effect of this energy loss. We have previously analyzed ionization cooling for muons at optimal cooling energies. The same equations, with appropriate adaptations, can be used to analyze the dynamic situation for proton-material interactions at low energies. In this note we discuss this extension and calculate cooling and heating effects at these very different parameters. The ring could provide a practical application of ionization cooling methods.

INTRODUCTION

Previously we have developed equations for the ionization cooling of muons.[1-6] As noted previously the cooling effects could apply to any charged particle. Ionization cooling would not be very practical for protons, since the protons would undergo nuclear reactions before they could be cooled by very much (~1 cooling time). For the ERIT application, where ERIT is an acronym for energy recovering internal target, the goal of the stored protons is to obtain a nuclear interaction (and secondary neutron production), and the ionization cooling is simply used to keep the proton in the ring until the nuclear reaction occurs. [7, 8] And passage through a cooling foil (that is also the neutron-producing interaction source) is necessary in any case. Thus use of the dynamics of ionization cooling to maximize the total number of beam-foil traversals could become practical in this application. In this note we will review the ionization cooling equations, apply them to low-energy proton beam storage, and evaluate effects in the ERIT case. Beam lifetime (and neutron flux) could be doubled at ERIT parameters, and increased even more by some variations. Much of this evaluation has previously been obtained by Okabe and Mori and their collaborations, but we confirm and extend the calculations, and note some potential variations. An experimental test of the method can be obtained in the KURRI 20 MeV FFAG ring. Significance of the method and the possible experiments are discussed.

Cooling equations at ERIT parameters

In this section we review the baseline ionization cooling equations. The detailed discussion is presented in order to identify the energy and mass dependent terms, and to assist in identifying possible mistakes that may occur in extrapolating ionization cooling effects from the quasirelativistic cooling of muons to the nonrelativistic motion of 10 MeV protons. The differential equation for rms transverse cooling of muons is [1-6]:

$$\frac{d\varepsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\varepsilon_N + \frac{\beta\gamma\ \beta_\perp}{2}\frac{d\left\langle \theta_{rms}{}^2 \right\rangle}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\varepsilon_N + \frac{\beta_\perp E_s^2}{2\beta^3 m_\mu c^2 L_R E} \qquad (1)$$

where the first term is the energy-loss cooling effect and the second is the multiple-scattering heating term. Here ε_N is the normalized emittance, E is the beam (total) energy, $\beta = v/c$ and γ are the usual kinematic factors, dE/ds is the energy loss rate, θ_{rms} is the rms multiple scattering angle, L_R is the material radiation length, $\beta_\perp$ is the betatron function at the absorber, and E_s is the characteristic scattering energy (~13.6 MeV). (The normalized emittance is related to the geometric emittance $\varepsilon_\perp$ by $\varepsilon_\perp = \varepsilon_N/(\beta\gamma)$, and the beam size is given by $\sigma_x = (\varepsilon_\perp\beta_\perp)^{1/2}$.) While these equations are expected to be generally valid, they were developed in the context of medium-energy muon cooling, and the extrapolation to very low energies may not be completely accurate. Some very low energy effects may not be included, and proton nuclear interactions are not included.

With protons we use m_p rather than m_μ, and at low energies the momentum is a more natural variable than energy. The cooling equation for low-energy protons can be written as:

$$\frac{d\varepsilon_N}{ds} = -\frac{g_t}{P_p}\frac{dP_p}{ds}\varepsilon_N + \frac{\beta_\perp E_s^2}{2\beta^2 m_p c^3 L_R P_p}, \qquad (2)$$

where we have inserted the partition number g_t to include the changes in transverse cooling rates that can occur with coupling to longitudinal motion. (g_t =1 without coupling.)

The energy loss can be estimated by the Bethe-Bloch equation:

$$\frac{dE}{ds} = 4\pi N_A \rho\, r_e{}^2 m_e c^2\,\frac{Z}{A}\left[\frac{1}{\beta^2}\ln\left(\frac{2m_e c^2\gamma^2\beta^2}{I(Z)}\right) - 1 - \frac{\delta}{2\beta^2}\right], \qquad (3)$$

where N_A is Avogadro's number, ρ, A and Z are the density, atomic weight and number of the absorbing material, m_e and r_e are the mass and classical radius of the electron, ($4\pi N_A r_e^2 m_e c^2$ = 0.3071 MeV cm^2/gm). The ionization constant $I(Z) \cong 16\ Z^{0.9}$ eV, and δ is the density effect factor. ($\delta \cong 0$.) Note that dE/ds is proportional to $1/\beta^2$ for low-energy particles. The formula is accurate down to ~5MeV, and becomes very inaccurate for E_p < 1MeV, where frictional energy loss dominates.

From equation 1 and 2, an expression for equilibrium emittance can be obtained:

$$\varepsilon_{N,eq} = \frac{\beta_\perp E_s^2}{2 g_t \beta\, m_p c^2 L_R \frac{dE}{ds}} \qquad (4)$$

At E_{kin}=10 MeV (P_p = 137.4 MeV/c), and with Be absorbers (L_R = 35.28cm), $\varepsilon_{N,eq}$= 0.000267 $\beta_\perp/g_t$. Because dE/ds varies as $1/\beta^2$, $\varepsilon_{N,eq}$ decreases with low energy, approximately proportional to $\beta\times\beta_\perp/g_t$. This dependence is complicated by the variation of $\beta_\perp$ and g_t, however. (We assume L_R is energy-independent, in the present discussion.)

The equation for longitudinal cooling with energy loss is:

$$\frac{d\sigma_E^2}{ds} = -2\frac{\partial \frac{dE}{ds}}{\partial E}\sigma_E^2 + \frac{d\left\langle \Delta E_{rms}{}^2 \right\rangle}{dt}, \tag{5}$$

in which the first term is the cooling term and the second is the heating term caused by random fluctuations in the particle energy loss. In the long-pathlength Gaussian-distribution limit, the second term in Eq. 2 is given by:

$$\frac{d\left\langle \Delta E_{rms}{}^2 \right\rangle}{ds} = 4\pi\left(r_e m_e c^2\right)^2 n_e \gamma^2 \left(1 - \frac{\beta^2}{2}\right), \tag{6}$$

where n_e is the electron density in the material.

Longitudinal beam cooling can occur if $\partial(dE/ds)/\partial E > 0$. The derivative is negative (or naturally heating) for low energy protons. However, the cooling term can be enhanced by placing the absorbers where transverse position depends upon energy (nonzero dispersion) and where the absorber density or thickness also depends upon energy, such as in a wedge absorber. This makes the beam particle path length through absorber material dependent on energy. (see figure 2) In that case the cooling derivative can be rewritten as:

$$\frac{\partial \frac{dE}{ds}}{\partial E} \Rightarrow \left.\frac{\partial \frac{dE}{ds}}{\partial E}\right|_0 + \frac{dE}{ds}\frac{\eta \rho'}{\beta c p \rho_0}, \tag{7}$$

where ρ'/ρ_0 is the change in density with respect to transverse position, ρ_0 is the reference density associated with dE/ds, and η is the dispersion $(\eta = dx/d(\Delta p/p))$. Increasing the longitudinal cooling rate in this manner decreases the transverse cooling by the same amount. The transverse cooling term is changed to:

$$\frac{d\varepsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\left(1 - \frac{\eta \rho'}{\rho_0}\right)\varepsilon_N, \tag{8}$$

Note that the coupled transverse cooling (and heating) changes occur in the same direction (i.e. horizontal or vertical) as the dispersion and wedge. However the sum of the cooling rates (over *x*, *y*, and *z*) remains constant. This sum can be represented, as with radiation damping, as a sum of cooling partition numbers, where the partition number is defined as the ratio of the cooling rate to the fractional momentum loss rate. For *x* and *y* emittance cooling the partition numbers are both naturally 1, but the partition number for longitudinal cooling is given by

$$g_L = \frac{\frac{d\varepsilon_L/ds}{\varepsilon_L}}{\frac{dp/ds}{p}}, \tag{9}$$

and is a function of muon energy. With $\delta = 0$ in the energy loss formula, and no coupling:

$$g_L = -\frac{2}{\gamma^2} + \frac{2(1 - \frac{\beta^2}{\gamma^2})}{\left(\ln\left[\frac{2m_e c^2 \beta^2 \gamma^2}{I(Z)}\right] - \beta^2\right)}. \tag{10}$$

With horizontal wedge enhancement of longitudinal cooling, g_L increases by $\eta\rho'/\rho_0$, and g_x becomes $g_x = 1 - \eta\rho'/\rho_0$, leaving the sum of the partition numbers Σ_g constant. (This coupling also mixes the heating terms; in initial approximation we neglect this complication.)

The sum of partition numbers $\Sigma_g = (g_x + g_y + g_L)$ is:

$$\Sigma_g = 2\beta^2 + 2\frac{(1-\frac{\beta^2}{\gamma^2})}{\left(\ln\left[\frac{2m_e c^2\beta^2\gamma^2}{I(Z)}\right]-\beta^2\right)}. \tag{11}$$

For relativistic particles, Σ_g is ~2, but for low energy particles it is significantly less than 2. For 10 MeV protons, $\Sigma_g \cong 0.3675$; this indicates that the energy loss is strongly antidamping in the longitudinal direction. Σ_g is at a minimal value at 10 MeV, which makes the total cooling particularly difficult.

Longitudinal cooling can be obtained by a relatively large value of $\eta\rho'/\rho_0$. The uncoupled value of g_L is ~-1.63 at 10 MeV. In a typical lattice, $\eta \cong 0.5$m. For $\eta \cong 0.5$m, we want $G=\rho'/\rho_0$ to be ~4m^{-1}, to obtain g_L ~0.37. If the cooling is obtained by a triangular wedge with thickness δ_0 at the reference orbit, then the wedge opening angle should be $\theta = 2\tan^{-1}(G\delta_0/2)$ see fig. 2. (The wedge apex is at x = -1/G and 1/G could be called the wedge length (L_W).) If a horizontal (x) wedge is introduced that is strong enough to obtain longitudinal cooling, then the x-emittance will be antidamped, with $g_x < -0.63$. The initial ERIT papers had longitudinal cooling with x antidamping, and no coupling. Coupling with vertical motion, reducing the vertical damping rate, would be required to obtain cooling in x, y, and z. For equal damping at 10 MeV, we would have $g_x = g_y = g_z \cong 0.12$, requiring x-y coupling and $\eta\rho'/\rho_o = 1.75$, or $G=1.75/\eta$.

The longitudinal cooling equation (eq. 5) only tracks energy spread. It can be transformed into a longitudinal emittance cooling equation, by adding longitudinal rf focusing that places the beam within a bunch[7]:

$$\frac{d\varepsilon_L}{ds} = -\frac{g_L}{\beta^2 E}\frac{dE}{ds}\varepsilon_L + \frac{\beta_\phi}{2}\frac{d\left\langle \Delta E_{rms}{}^2\right\rangle}{ds} \tag{12}$$

Here β_ϕ is a focusing function, defined by:

$$\beta_\phi{}^2 = \frac{\left\langle \phi^2\right\rangle}{\left\langle \Delta E^2\right\rangle} = \frac{1}{\beta^3\gamma\, eV'\cos\phi_s}\frac{2\pi}{\lambda_0}\frac{\alpha_p}{mc^2} \quad , \tag{13}$$

where λ_0 is the focusing rf wavelength, $eV'\sin\phi_s$ is the mean focusing rf gradient, and α_p is the momentum compaction. ($\alpha_p = 1/\gamma^2 - 1/\gamma_t^2$ in a synchrotron and $\alpha_p = 1/\gamma^2$ in a linac.) This example expresses the longitudinal bunch emittance in δE-$\delta\phi$ units

Example: 10 MeV Proton Storage Ring with Be Foils

Parameters of the ERIT example

Figure 1 shows an overview of the ERIT storage ring, where ERIT is an acronym for energy recovering internal target, and the name indicates that the ring has an rf system that recovers the energy lost in the target. The baseline parameters are displayed in table 1.

The purpose of the FFAG-ERIT ring is to maximize the production of neutrons from protons hitting the internal target, through the reaction p + Be → n + B, which has a cross-section of σ = 500 millibarn or $\sigma=0.5\times10^{-24}$ cm^2 for protons in the energy rage from ~5 to 20MeV. The ring enables multiturn passages of the beam through a Be foil, with a density of $\rho_A \cong 1.226\times10^{23}$ atoms/cm^3. If the foil thickness δ_0 is 5μ, then neutrons are produced at a rate of 3.065×10^{-5} neutrons/proton/turn. (If the neutron production cross-section were the dominant interaction the beam lifetime would be ~30000 turns.) With a circulating current of 40mA, and a revolution frequency of 3.85 MHz, we find 6.5×10^{10} protons in the ring. If we assume that the ring is filled at 1000 Hz and that the mean proton beam storage time is 1000 turns, we would obtain 2.0×10^{12} neutrons/s (with no neutron losses in material or transport).

With Be foils and E_p = 10MeV, the reference cooling absorber length is (p/(dp/ds))=0.276cm, which corresponds to ~550 turns with 5μ foils. This gives the cooling time for uncoupled transverse motion, and sets the scale for beam parameter changes.

Okabe presented a possible lattice design for a 10 MeV FFAG ERIT ring. His initially preferred design is a spiral-sector lattice with 8 magnets (k value = 2, F-angle=13.5º, spiral angle =26º). The mean orbit radius is 1.8m (C=11.3m). Parameters for that lattice are included in table 1. Key parameters are the betatron functions at the foil (~1m) and the dispersion at the foil (~0.6m).

The ring requires rf sufficient to recover energy loss in the foil and to maintain the beam in a stable rf bucket. In the initial example, an h=5 (19.3MHz), 200kV rf cavity was chosen. The mean energy loss per turn is 36 keV, placing the stable accelerating phase at 10.4º. With $\gamma_t \cong$ 1.7, the parameter β_ϕ is ~2.1 radians/MeV.

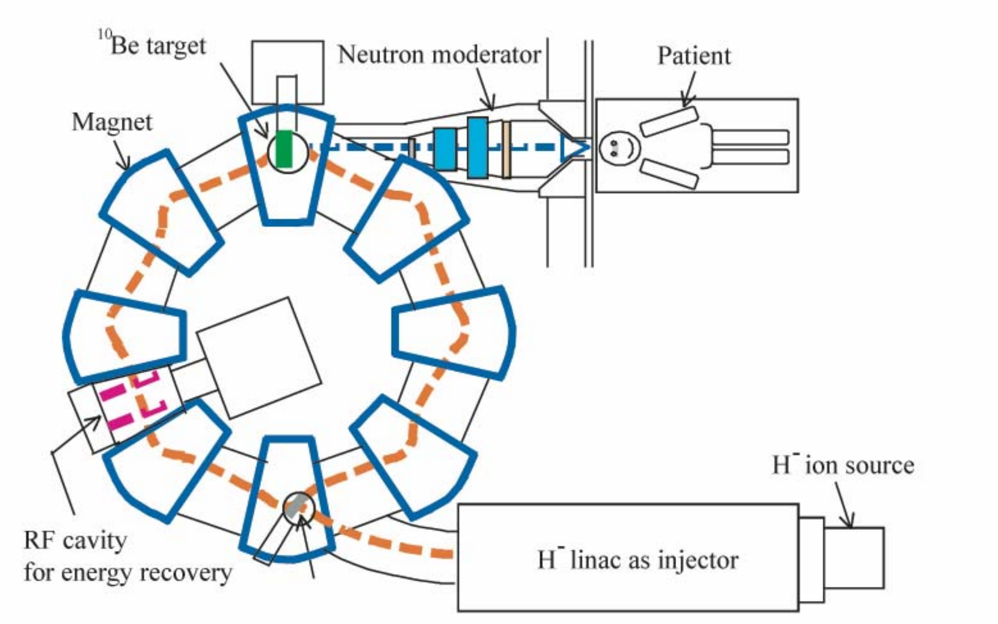


FIGURE 1. Schematic view of the components of the ERIT system, showing an injector 10 MeV H$^-$ linac, a storage ring for ~10 MeV protons (P_p=137 MeV/c)., and a Be foil target for multiturn production of neutrons, with an extraction line with moderator for medical use of the neutrons. (from ref. 7)

As described by Mori and Okabe,[8, 9] the increase in energy spread from energy loss in the foil causes beam loss. The proposal is to reduce that energy spread increase by using a "wedge" absorber which is thicker for higher-energy orbits and thinner for lower-energy orbits. That can place the energy spread in a damping situation, increasing the beam lifetime. The transverse energy loss (with multiple scattering heating) could also place the beam at transverse damping (ionization cooling) parameters. At their parameters, wedge damping of energy spread leads to horizontal antidamping, and the design choice would be to minimize energy spread increase while avoiding transverse beam loss, with the optimum chosen to maximize beam lifetime.

The beam dynamics in the ring would explore parameters relevant to ionization cooling, containing damping and antidamping, as well as multiple scattering and energy straggling terms. However, since the sum of the partition numbers for 10 MeV protons is ~0.36, it is difficult to obtain simultaneous transverse and longitudinal cooling. (Equal cooling has g ≅ 0.12 for x, y, z.)

The initial scenario parameters are not very favorable for transverse damping, because the betatron function at the absorbers is relatively large (~1m). At $\beta_\perp$ = 1m and g_t = 1, the equilibrium rms normalized emittance is $\varepsilon_{N,rms}$= 0.000267m. The geometric rms emittance is $\varepsilon_t = \varepsilon_{Nrms}/(\beta\gamma)$ = 0.0018m at 10MeV, and the equilibrium rms beam size is ~4.3cm at $\beta_\perp$=1m. A 3σ aperture requirement is then ~13cm, somewhat larger than in the initial design. This equilibrium assumes g_t = 1; if g_t is reduced to obtain longitudinal cooling then the aperture requirement is greater by $1/\sqrt{g_t}$. Therefore, the apertures of the baseline example are not large enough to accommodate 3-D cooling.

With the listed longitudinal motion parameters, the energy width of the rf bucket is:

$$\delta E = \pm m_p c^2 \sqrt{\frac{2\beta^2\gamma V_{rf}\left(2\cos\phi_s - (\pi - 2\phi_s)\sin\phi_s\right)}{2\pi\alpha_p h m_p c^2}} ,$$

which is ~±0.76MeV at the baseline parameters. The momentum width of this acceptance is δP/P ≅ ±3.75%, which is less than the desired full momentum acceptance of δP/P ≅ ±10%. Obtaining ±10% would require ~1MV of rf.

Cooling/heating of beam in material

The simplified (rms) cooling equations can be integrated numerically or analytically. We consider first the case in which only x and z motions are coupled with the dispersion/wedge and the y motion is completely decoupled.

A critical parameter is the aperture of the machine, which could be redesigned to make the scenario more favorable. In the initial example of ref. 1, the physical aperture in the bend dipole is ~±20cm horizontal and ±8cm vertical. At these points the focusing functions are $\beta_x \cong$ 1.0m, β_y = 1.0m, η ≅ 0.6m. One assumes the initial beam is relatively small, we assume a matched transverse size of $\sigma_x = \sigma_y$ =1cm ($\varepsilon_{N,rms}$ =0.000015m) and σ_E = 0.1MeV (δp/p=0.5%). We assume that the aperture limit in tracking of cooling is $\sigma_x = \sigma_y$ =8.3cm ($\varepsilon_{N,rms}$ =0.001m), and that the aperture limit in σ_E is 1.0MeV (δp/p=5%).

At these parameters the rms cooling equations are:

$$\frac{d\varepsilon_{N,rms}}{dn} = -1.81 \cdot 10^{-3} g_{\perp} \varepsilon_{N,rms} + 4.84 \cdot 10^{-7} \quad \text{m/turn}$$

$$\frac{d(\delta E)^2}{dn} = -1.81 \cdot 10^{-3} g_L (\delta E)^2 + 0.0000325 \quad (\text{MeV})^2/\text{turn}$$

where n is the number of turns. In the energy $(\delta E)^2$ increase equation, we assume that there is sufficient bunching rf to maintain the beam in an rf bucket (mixing δE and ϕ). Without bunching, the growth rate for $(\delta E)^2$ is twice that of the above expressions.

If we assume no wedge/dispersion cooling, then $g_x = g_y = 1$ and $g_L = -1.63$. The rms emittance converges to 0.00027 (below the reference value), while the energy limit of 1MeV is reached at 1300 turns. If the wedge angle mixing is set to obtain $g_x = 0$, $g_L = -0.63$, the energy limit is reached at 2900 turns, while the emittance limit is reached at ~2000 turns. (see Fig.) An optimum for these aperture conditions is obtained at $g_x = 0.1$, $g_L = -0.73$, when both limits are reached at 2500 turns. In this example an optimum choice of wedge angle would provide twice as many storage turns as a straight absorber. (This is in reasonable agreement with the discussion of Okabe and Mori.) More detailed simulations (ICOOL/DPGeant, etc.)[9, 10] are needed to quantify this improvement, and to verify that the ring dynamic aperture is adequate.

With no x-y mixing, the beam is cooled vertically with an equilibrium emittance of ~0.00027m, but at these optimized parameters the beam would fill the vertical aperture, with the physical vertical aperture at ~$2\sigma_{y,equil.}$. The beam would not be effectively cooled horizontally and longitudinally, but the longitudinal heating rate could be reduced by a factor of ~2, while horizontal losses would also occur. The beam lifetime would be approximately doubled.

The example would also serve as a demonstration of many aspects of ionization cooling. The transverse cooling terms and multiple scattering effects should be measurable. The longitudinal energy spread increase (due to $\partial(dE/ds)/\partial E$) would also be measurable, and the use of wedge absorbers to reduce it would be measurable. The energy straggling term is probably too small (compared to other heating effects) at these energies to be measurable. It would probably not be practical to set the wedge at a large enough angle to get longitudinal cooling, because the transverse heating would become too large.

Variations and Improvements

Some changes in the parameters could possibly greatly improve the cooling scenario and even ring performance. If β^* at the absorber were smaller, the multiple scattering emittance growth and the equilibrium transverse emittance would be proportionately less. The wedge could be adjusted to obtain less longitudinal heating, or even cooling. The vertical β^* need not be reduced as much as the horizontal, since the vertical cooling is not mixed with the longitudinal, and vertical cooling could be adequate. As an example we consider a value of β^* = 0.2m, which reduces the multiple scattering emittance growth by a factor of 5, and therefore the equilibrium emittance by that factor. In a reoptimization of cooling/heating, the maximum number of storage turns could be redoubled to ~4000 turns.

In the above examples, x and y motions are decoupled, and emittance exchange is limited to x-δE exchanges. Obtaining 3-D cooling would require mixing y motion into the emittance exchange. In the lattices of the ERIT example, the vertical motion is relatively restricted, and reduced β_y^*, with increased physical and dynamic apertures, would be needed. However, if the x and y emittance cooling factors are reduced by mixing and the longitudinal factors matched to obtain $g_x = g_y = g_L = 0.12$, and the low β^* matched to 0.2m in x and y, we could obtain simultaneous cooling in x, y, and L, with equilibrium emittances of $\varepsilon_{N,rms} \cong 0.0004$m, and $\delta E_{rms} \cong 0.4$MeV.

Mori[11] has proposed setting up an initial demonstration experiment in the 20MeV KURRI FFAG ring.[12] In that experiment, a wedge absorber would be placed in the beam, and ionization cooling effects observed over multiturn operation. While I do not have a detailed design at present, the 20MeV ring has similar parameters to the ERIT concept ($C \cong 11$m, $\nu_x = 2.2$, $\nu_y = 1.7$) so β^* would be ~1m and the dispersion $\eta \cong 0.5$m. The higher energy shifts the parameters so that $1/p \cdot dp/ds$ is a factor of 4 smaller, and the equilibrium normalized emittance is $\sim 2^{1/2} \times$ larger, but the equilibrium beam size is ~ the same ($\sigma \cong 5$cm). This may make the vertical beam size too large, since this ring may have a smaller vertical aperture. The wedge parameters could be varied to increase transverse heating while reducing longitudinal heating, which may increase beam lifetime. The results can be compared with simulation to verify that the transverse and longitudinal heating and cooling effects agree with simulation. The study would provide an interesting test, provided the physical and dynamic apertures are adequate.

ACKNOWLEDGMENTS

I thank Y. Mori and K. Okabe for initial discussions, and developing this innovative application of ionization cooling.

REFERENCES

[1] A. N. Skrinsky and V.V. Parkhomchuk, Sov. J. Nucl. Physics 12, 3 (1981)
[2] E. A. Perevedentsev and A. N. Skrinsky, Proc. 12th Int. Conf. on High Energy Accel., 485 (1983).
[3] D. Neuffer, Particle Accelerators 14, 75 (1983)
[4] D. Neuffer, Proc. 12th Int. Conf. on High Energy Accelerators, 481 (1983)
[5] D. Neuffer, Nucl. Inst. and Meth. A350, 27 (1994).
[6] D. Neuffer, Nucl., Inst. and Meth. A532, p.26 (2004).
[7] K. Okabe, “Design of FFAG-ERIT”, presented at FFAG05, Osaka, Japan, December 2005.
[8] Y. Mori, “Neutron Source for BNCT FFAG-ERIT Scheme”, FFAG05, Osaka, Japan, December 2005.
[9] R. Fernow, “ICOOL, a Simulation Code for Ionization Cooling of Muon Beams”, Proc. 1999 Particle Accelerator Conference, New York, p. 3020 (1999), see http://pubweb.bnl.gov/people/fernow/.
[10] T. Roberts et al., G4Beamline, release 1.01, see http://www.muonsinc.com/g4beamline/index.html.
[11]Y. Mori, unpublished discussion (2006).
[12]M. Tanigaki et al., “Status of FFAG Complex at KURRI”, presented at FFAG05, Osaka, Japan, December 2005.

Table 1: Reference parameters of the ERIT Ring

Parameter	**Symbol**	**Ref. Value**	**Units**
Beam Kinetic Energy	E_p	10	MeV
Beam Momentum	P_p	137.4	MeV/c
Beam velocity	β=v/c	0.145	
Beam current	I_p	40	mA
Ring Circumference	C	11.3	m
Ring tunes	ν_x, ν_y	1.89,1.34	
Betatron function	$\langle\beta_\perp\rangle$	0.95,	m
Maximum betatron functions	$\beta_{x,max}$, $\beta_{y,max}$	1.48,2.03	m
Dispersion (at wedge)	η_0	0.6	m
Transition gamma	γ_t	1.7	
Energy loss (Be) at ref. energy	dE/ds	36	MeV/cm
Sum of partition numbers (at E_p)	Σ_g	0.37	
Absorber central thickness	δz	5	μ
Rf voltage	V_{rf}	200	kV/turn
Rf harmonic	h	5	
Rf frequency	f_{rf}	19.25	MHz
Longitudinal focusing function	β_ϕ	2.1	Radians/MeV

Figure 2: Geometric representation of a wedge absorber. δ_0 is the wedge width at the center of the beam (at reference energy) and θ is the wedge opening angle.

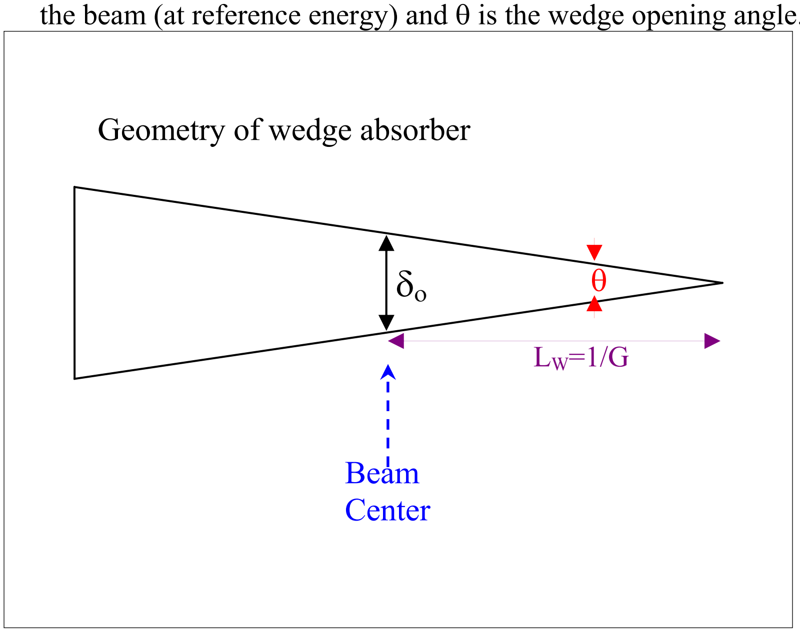


Figure 3. Design view of a spiral-sector storage storage ring considered for ERIT-FFAG. (from ref. 8.)

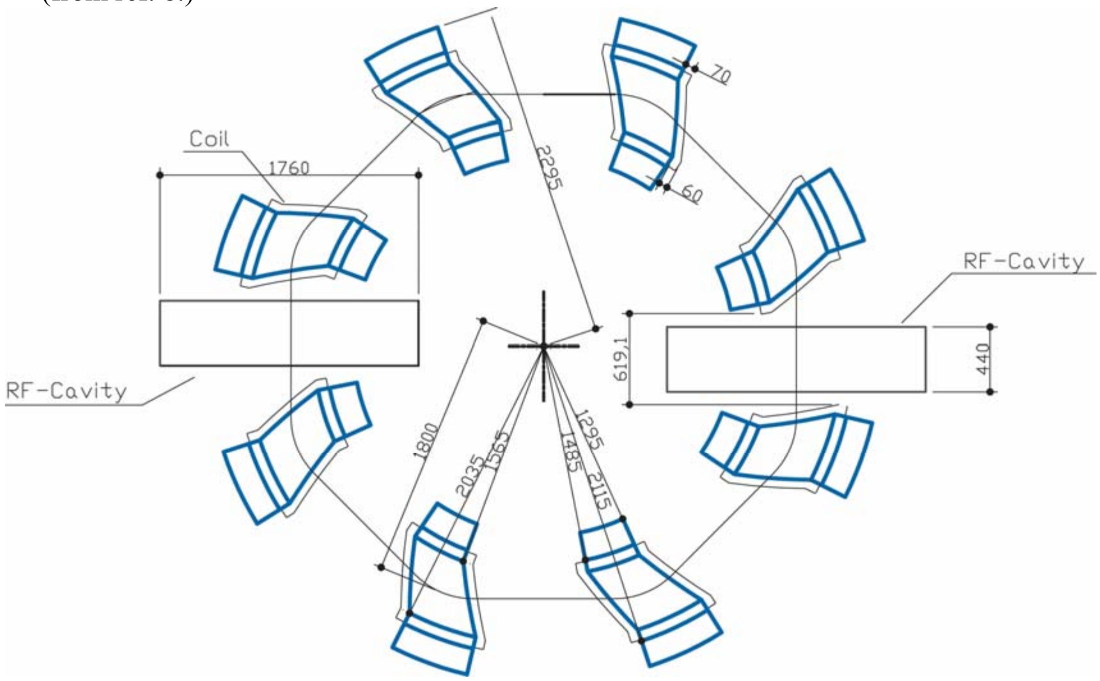
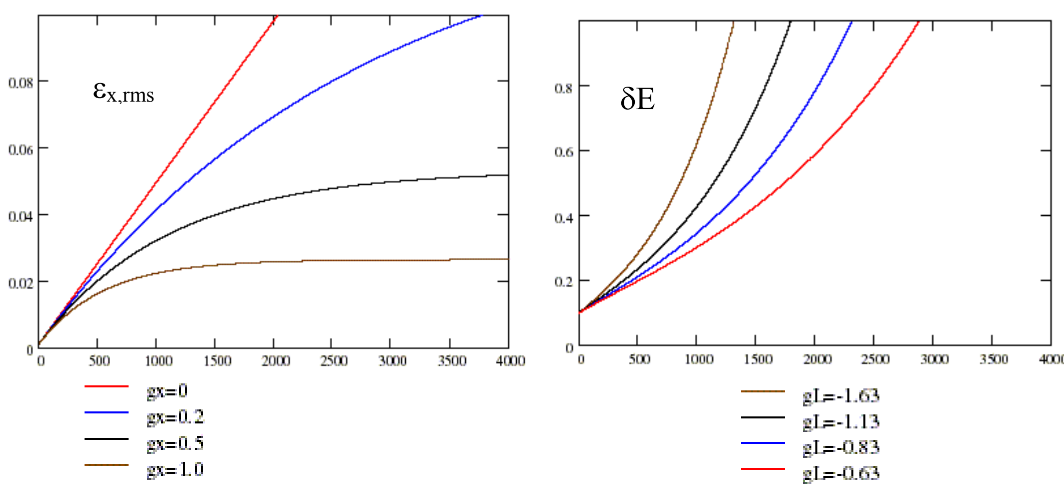


Figure 4: a) b)



Growth of transverse (x) emittance $\varepsilon_{N,rms}$ (left) and rms energy spread δE (right) for various values of g_x and g_L (g_x = 1., 0.5, 0.2, 0.0), while (g_L = -1.63, -1.13, -0.83, -0.63). The horizontal scale in each graph is number of turns (0 to 4000). The vertical scale in a) is emittance in cm (from 0 to 0.01cm) while in b) δE is in MeV (0 to 1.0). Initial values are $\varepsilon_{x,rms}$=0.0015cm and δE =0.1Mες.