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## Models of Little Higgs and Electroweak Precision Tests

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The little Higgs idea is an alternative to supersymmetry as a solution to the gauge hierarchy problem. In this note, I review various little Higgs models and their phenomenology with emphases on the precision electroweak constraints in these models.

*Keywords:* Electroweak symmetry breaking; Higgs; radiative corrections

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### 1. Introduction

The Standard Model (SM) requires a Higgs boson to explain the generation of fermion and gauge boson masses. The precise electroweak (EW) measurements at LEP suggest that the Higgs boson must be relatively light<sup>1</sup>, with  $m_H < 219$  GeV. A light Higgs is also required in order to unitarize the longitudinal  $W_L - W_L$  scattering amplitude<sup>2</sup>. In addition, the triviality bound indicates that the SM with a light Higgs can be a theory valid all the way up to the Planck scale. This simple assumption of the SM with a single Higgs doublet, however, has the theoretical problem that the Higgs boson mass is quadratically sensitive to any new physics which may arise at high energy scales. To cancel these quadratic divergences due to the SM particles, a set of new states, which are related to the SM particles by some symmetry, have to appear at the TeV scale<sup>a</sup>. Little Higgs (LH) models are a new approach to stabilizing the mass of the Higgs boson. These models have an expanded gauge structure at the TeV scale which contains the Standard Model  $SU(2) \times U(1)$  electroweak gauge groups. They are constructed such that there are multiple global symmetries that prohibit the Higgs boson from obtaining a quadratically divergent mass. It is only when *all* these global symmetries are broken can a quadratic contribution to the scalar potential arises, which is at least at the two loop order.

There is generally a tension between the solution to the gauge hierarchy problem and the EW precision fit, nevertheless. The deviations from the SM predictions due

<sup>a</sup>It has been realized recently in twin Higgs models that this needs not be the case<sup>3</sup>.

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to the presence of new interactions above the SM cutoff scale,  $\Lambda$ , can be parameterized by a set of higher dimensional operators which have been classified in Ref. 4. Among these operators, the most stringent bounds are those on the coefficients of the dim-6 operator,  $\frac{1}{\Lambda^2}(H^\dagger D_\mu H)^2$ , which breaks the custodial  $SU(2)$  symmetry, and  $\frac{1}{\Lambda^2}(D^2 H^\dagger D^2 H)$ , and thus contributes to the S-parameter. These bounds indicate that the cutoff scale  $\Lambda$  has to be above 5 TeV. Thus the form of possible new Physics, including the little Higgs models, which have to appear at the TeV scale to solve the gauge hierarchy problem, is severely constrained.

This review is organized as follows. In Sec. 2, I introduce the basic idea of the little Higgs models, show explicitly how the quadratic divergences are cancelled in various sectors of the littlest Higgs model, and briefly review other existing little Higgs models. Sec. 3 is devoted to the precision electroweak constraints in these models, followed by Sec. 4 in which other issues such as implicit fine-tuning and UV completion are briefly discussed. Sec. 5 concludes this review.

## 2. Little Higgs Models

The idea of Higgs boson being a pseudo-Goldstone boson arising from the breaking of some approximate global symmetry was proposed<sup>5</sup> in the early 80's. Because the Higgs boson mass is generated radiatively, this therefore provides a natural way to understand why the Higgs boson is so light. In its early realizations, the quadratic contributions to the Higgs boson mass arise at one-loop, leading to a Higgs mass that is still too heavy because it is only suppressed by the one-loop factor. The new ingredient in the little Higgs models is the so-called collective symmetry breaking. The idea is to choose the gauge and Yukawa interactions in such a way that by turning off some part of these interactions, the model has enhanced global symmetries that forbid a quadratic contribution to the Higgs mass. As a result, the quadratic contributions to the Higgs mass can arise only when two or more operators are involved, which can occur only at the two loop level or beyond. This leads to a Higgs mass  $\mu^2$  of the order of  $\mu^2 \sim \frac{f^2}{16\pi^2}$ . Note that, the logarithmic contributions to the Higgs mass can still appear at one-loop. These models have the following general structure: at the electroweak scale,  $v \sim \frac{g_2 f}{4\pi} \sim 200$  GeV, there are one or two Higgs doublets and possibly a few additional scalar fields; at the scale  $g \cdot f \sim 1$  TeV, there exist new gauge bosons and fermions; above the cut-off scale of the non-linear sigma model,  $\Lambda \sim 4\pi f \sim 10$  TeV, the model becomes strongly interacting. Various realizations of the little Higgs idea are described below<sup>b</sup>.

### 2.1. Original Littlest Higgs-like Models

#### 2.1.1. Littlest Higgs Model

The minimal realization of the little Higgs idea is the littlest Higgs model<sup>7</sup>, which is a non-linear sigma model based on  $SU(5)/SO(5)$ . The  $SU(5)$  global symmetry

<sup>b</sup>For recent reviews on the little Higgs models, see Ref. 6.

in the model is broken down to  $SO(5)$  by the VEV of the sigma field,  $\langle \Sigma \rangle$ , which transforms as an adjoint under  $SU(5)$ , where,

$$\langle \Sigma \rangle = \Sigma_0 = \begin{pmatrix} & & \mathbf{I}_{2 \times 2} \\ & 1 & \\ \mathbf{I}_{2 \times 2} & & \end{pmatrix}. \quad (1)$$

The sigma field can be expanded around the VEV in terms of the Goldstone modes,  $\Pi \equiv \pi^a X^a$ , as,

$$\Sigma = e^{2i\frac{\Pi}{f}} \Sigma_0 = \Sigma_0 + 2i\frac{\Pi}{f} \Sigma_0 + \dots \quad (2)$$

where  $X^a$  correspond to the broken  $SU(5)$  generators. The gauge subgroup is chosen to be  $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$ . It is broken down to its diagonal subgroup,  $[SU(2) \times U(1)]_{\text{SM}}$ , which is identified as the SM gauge group. The kinetic term of the sigma field can be written as

$$\mathcal{L}_\Sigma = \frac{1}{8} f^2 \text{Tr}[(D\Sigma^\dagger)(D\Sigma)], \quad (3)$$

where the covariant derivative is,

$$D_\mu \Sigma = \partial_\mu \Sigma - i \sum_{j=1,2} \left[ g_j W_j^a (Q_j^a \Sigma + \Sigma Q_j^{aT}) + g'_j B_j (Y_j \Sigma + \Sigma Y_j^T) \right]. \quad (4)$$

The generators of the two  $SU(2)$  gauge groups,  $Q_1^a$  and  $Q_2^a$  for  $(a = 1, 2, 3)$ , are,

$$Q_1^a = \begin{pmatrix} \frac{1}{2}\sigma^a & 0_{2 \times 3} \\ 0_{3 \times 2} & 0_{3 \times 3} \end{pmatrix}, \quad Q_2^a = \begin{pmatrix} 0_{3 \times 3} & 0_{3 \times 2} \\ 0_{2 \times 3} & -\frac{1}{2}\sigma^{a*} \end{pmatrix}, \quad (5)$$

and the two  $U(1)$  generators,  $Y_1$  and  $Y_2$ , are,

$$Y_1 = \frac{1}{10} \text{diag}(-3, -3, 2, 2, 2), \quad Y_2 = \frac{1}{10} \text{diag}(-2, -2, -2, 3, 3). \quad (6)$$

When the gauge coupling constants  $g_2$  and  $g'_2$  are turned off, a global symmetry  $SU(3)_1$ , which acts on the last three indices, is restored,

$$\left( \begin{array}{c|c} & \\ \hline & SU(3)_1 \end{array} \right). \quad (7)$$

Similarly, when  $g_1$  and  $g'_1$  are turned off, there is an enhanced  $SU(3)_2$  global symmetry acting on the first three indices,

$$\left( \begin{array}{c|c} SU(3)_2 & \\ \hline & \end{array} \right). \quad (8)$$

Each of these two  $SU(3)$  symmetries individually forbids a quadratic contribution to the Higgs potential. Thus a quadratic contribution to the Higgs potential can arise only when both global symmetries are broken, which is possible only at the two loop level.

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The 14 Goldstone bosons resulting from the breaking of the global symmetry can be decomposed in the following way,

$$14 = 4 \oplus 10 = 1_0 \oplus 3_0 \oplus 2_{\pm 1/2} \oplus 3_{\pm 1}, \quad (9)$$

where the subscripts in the above equation denote the hypercharges. The components  $1_0$  and  $3_0$  are eaten and become the longitudinal degrees of freedom of the heavy gauge bosons,  $A_H$ ,  $Z_H$  and  $W_H$ . In the low energy spectrum, there are two scalar fields: the component  $2_{\pm 1/2}$  is identified as the complex doublet Higgs boson,  $h$ , of the SM, while  $3_{\pm 1}$  is an additional complex  $SU(2)_L$  triplet Higgs,  $\Phi$ . In terms of these low energy degrees of freedom, the  $\Pi$  field can be written as,

$$\Pi = \begin{pmatrix} 0 & \frac{h^\dagger}{\sqrt{2}} & \Phi^\dagger \\ \frac{H}{\sqrt{2}} & 0 & \frac{H^*}{\sqrt{2}} \\ \Phi & \frac{h^T}{\sqrt{2}} & 0 \end{pmatrix}, \quad (10)$$

where

$$H = (h^+ \ h^0), \quad \Phi = \begin{pmatrix} \phi^{++} & \frac{\phi^+}{\sqrt{2}} \\ \frac{\phi^+}{\sqrt{2}} & \phi^0 \end{pmatrix}. \quad (11)$$

Expanding the kinetic terms of the  $\Sigma$  field in Eq. (3), one finds,

$$\begin{aligned} \mathcal{L}_\Sigma &\rightarrow \frac{f^2}{8} \text{Tr} \left| \sum_{j=1,2} \left[ g_j W_j (Q_j \Sigma_0 + \Sigma_0 Q_j^T) + g'_j B_j (Y_j \Sigma_0 + \Sigma_0 Y_j^T) \right] \right|^2 \\ &\rightarrow \frac{f^2}{8} \left\{ \left[ g_1^2 W_1 W_1 - 2g_1 g_2 W_1 W_2 - 2g_1 g_2 W_1 W_2 + g_2^2 W_2 W_2 \right] \right. \\ &\quad \left. + \frac{1}{5} \left[ g_1'^2 B_1 B_1 - 2g_1' g_2' B_1 B_2 + g_2'^2 B_2 B_2 \right] \right\}. \end{aligned} \quad (12)$$

This gives the following mass matrices for the gauge bosons,

$$M(W) = (W_1 \ W_2) \begin{pmatrix} g_1^2 & -g_1 g_2 \\ -g_1 g_2 & g_2^2 \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \end{pmatrix} \quad (13)$$

$$M(B) = (B_1 \ B_2) \begin{pmatrix} g_1'^2 & -g_1' g_2' \\ -g_1' g_2' & g_2'^2 \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}, \quad (14)$$

and the mass eigenstates of these mass matrices are

$$W_L = sW_1 + cW_2, \quad \text{with} \quad M_{W_L} = 0, \quad (15)$$

$$W_H = -cW_1 + sW_2, \quad \text{with} \quad M_{W_H} = \frac{1}{2} \sqrt{g_1^2 + g_2^2} f, \quad (16)$$

$$B_L = s'B_1 + c'B_2, \quad \text{with} \quad M_{B_L} = 0, \quad (17)$$

$$B_H = -c'B_1 + s'B_2, \quad \text{with} \quad M_{B_H} = \sqrt{\frac{g_1'^2 + g_2'^2}{20}} f, \quad (18)$$

where the mixing angle  $s$  is  $s = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}$  and  $s' = \frac{g_2'}{\sqrt{g_1'^2 + g_2'^2}}$  while  $c = \sqrt{1 - s^2}$  and  $c' = \sqrt{1 - s'^2}$ . The two massless eigenstates,  $W_L$  and  $B_L$  are identified as

the weak gauge bosons in the SM. The two massive eigenstates,  $W_H$  and  $B_H$ , are the additional gauge bosons having masses of the order of  $f$ . The gauge coupling constants of the unbroken subgroup,  $SU(2)_L$  and  $U(1)_Y$ , are given by,

$$g = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}, \quad g' = \frac{g'_1 g'_2}{\sqrt{g_1'^2 + g_2'^2}}. \quad (19)$$

The quartic couplings of the Higgs boson to the gauge bosons arise from the next-to-leading order terms in the expansion in Eq. (3),

$$\begin{aligned} \mathcal{L}_\Sigma &\rightarrow \frac{1}{2} \text{Tr} \left| \sum_{j=1,2} \left[ g_j W_j (Q_j \Pi \Sigma_0 + \Pi \Sigma_0 Q_j^T) + g'_j B_j (Y_j \Pi \Sigma_0 + \Pi \Sigma_0 Y_j^T) \right] \right|^2 \quad (20) \\ &\rightarrow \frac{1}{4} (g_1 g_2 W_1 W_2 + g'_1 g'_2 B_1 B_2) H^\dagger H + \dots \\ &= \frac{1}{4} \left[ g^2 (W_L W_L - W_H W_H) + g'^2 (B_L B_L - B_H B_H) \right] H^\dagger H + \dots \end{aligned}$$

Thus the quartic couplings  $H^\dagger H W_L W_L$  and  $H^\dagger H W_H W_H$  are of equal magnitude but opposite signs. The opposite signs come about because the  $W_L$  and  $W_H$  gauge bosons are orthogonal to each other. The cancellation of quadratic divergences among diagrams shown in Fig. 1 at one loop thus ensues.

In the fermion sector, to cancel the top loop contribution to the radiative corrections to the Higgs boson mass, one needs to introduce a vector-like pair of the color triplet and iso-singlet heavy tops,  $\tilde{t}$  and  $\tilde{t}'$ . The field  $\tilde{t}$  then form a triplet, together with  $(b t)$ , under the  $SU(3)_1$  global symmetry,  $\chi^T = (b t T)$ . The Yukawa interactions take the following form,

$$\mathcal{L}_{Yuk} = \frac{1}{2} \lambda_1 f \epsilon_{ijk} \epsilon_{xy} \chi_i \Sigma_{jx} \Sigma_{ky} u'_3 + \lambda_2 f \tilde{t} \tilde{t}' + h.c. . \quad (21)$$

The first term in this Yukawa Lagrangian preserves the  $SU(3)_1$  global symmetry and breaks the  $SU(3)_2$  global symmetry, while the mass term of the vector-like quarks preserves the  $SU(3)_2$  and breaks  $SU(3)_1$ . Due to the  $SU(3)_1$  global symmetry, the couplings of  $t_3$  to  $h^0 u'_3$  and  $\tilde{t} u'_3$ , and the quartic coupling  $h^0 h^{0*} \tilde{t} u'_3$  are related,

$$\mathcal{L}_{Yuk} \rightarrow -i \lambda_1 \left[ \sqrt{2} h^0 t_3 + i f \tilde{t} - i h^0 h^{0*} \frac{\tilde{t}}{f} \right] u'_3 + h.c. . \quad (22)$$

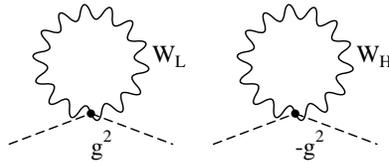


Fig. 1. The cancellation of the quadratic contributions to Higgs mass square at one loop in the gauge sector.

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These relations lead to the cancellation of the quadratic divergences among diagrams shown in Fig. 2 in the fermion sector at one-loop.

Because of the non-abelian transformation,  $h \rightarrow h + \epsilon$ , the quadratic contributions to  $m_H$  can only arise at the two-loop level, due to the following scalar and fermion interactions,

$$\mathcal{L}_s = \frac{a^2}{2} f^4 \left\{ g_j^2 \sum_a \text{Tr} \left[ \left( Q_j^a \Sigma \right) + \left( Q_j^a \Sigma \right)^* \right] + g_j'^2 \text{Tr} \left[ \left( Y_j \Sigma \right) + \left( Y_j \Sigma \right)^* \right] \right\} \quad (23)$$

$$\mathcal{L}_f = -\frac{a'}{4} \lambda_1^2 f^4 \epsilon^{wx} \epsilon_{yz} \epsilon^{ijk} \epsilon_{kmn} \Sigma_{iw} \Sigma_{jx} \Sigma^{*my} \Sigma^{*nz} , \quad (24)$$

where the parameters  $a$  and  $a'$  are the coefficients that parametrize the unknown UV physics. The scalar potential arises by integrating out the heavy top,  $T$ , as well as the heavy gauge bosons,

$$V_{\text{CW}} = \lambda_{\Phi^2} f^2 \text{Tr}(\Phi^\dagger \Phi) + \lambda_{H\Phi H} f(H\Phi^\dagger H) - \mu^2 H H^\dagger + \lambda_{H^4} (H H^\dagger)^2 , \quad (25)$$

where

$$\mu^2 \sim a \frac{f^2}{16\pi^2} \quad (26)$$

$$4\lambda_{H^4} = \lambda_{\Phi^2} = \frac{a}{2} \left[ \frac{g^2}{s^2 c^2} + \frac{g'^2}{s'^2 c'^2} \right] + 8a' \lambda_1^2 \quad (27)$$

$$\lambda_{H\Phi H} = -\frac{a}{4} \left[ \frac{g^2(c^2 - s^2)}{s^2 c^2} + \frac{g'^2(c'^2 - s'^2)}{s'^2 c'^2} \right] + 4a' \lambda_1^2 . \quad (28)$$

The complete Feynman rules of the littlest Higgs model has been presented in Ref. 8. Various collider phenomenology of the littlest Higgs model has been discussed extensively in Ref. 8, 9, and the flavor sector in this model has been studied in Ref. 10, including the generation of fermion masses<sup>11</sup>.

### 2.1.2. Littlest Higgs with $T$ -parity

As we will see in the next section, the littlest Higgs model is severely constrained by the precision electroweak data. The most stringent one comes from the tree level contributions to the  $\rho$  parameter, due to the presence of the  $W_L W_H H H$  coupling and the tri-linear  $H^T \Phi^\dagger H$  coupling, which breaks the tree level custodial symmetry explicitly. In Ref. 12, Cheng and Low found that these operators can be forbidden

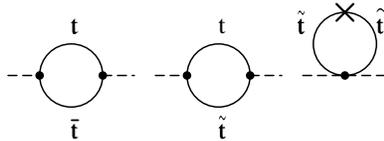


Fig. 2. The cancellation of the quadratic contributions to Higgs mass square at one loop in the top quark sector.

by imposing a discrete  $Z_2$  symmetry, called the T-parity. Under the T-parity, all new particles, except the heavy top partner,  $\tilde{t}$ , that are responsible for canceling the SM contributions to the one-loop quadratic divergences in the Higgs potential are odd, while all the other particles are T-even<sup>c</sup>. As a result, the heavy particle contributions to the observables involving only the SM particles in the external states are forbidden at the tree level. These contributions of the new particles to precision EW observables can arise only at the loop levels in this model, and thus the constraints in this model is not as stringent as in the littlest Higgs model. Due to T-parity, all T-odd particles have to be pair-produced, and the lightest stable particle can be the candidate of the weak scale dark matter<sup>14</sup>. Various collider phenomenology and flavor constraints in this model has been discussed in Ref. 15 and 16, respectively. It was found that the collider signatures in this model mimic that of the MSSM.

### 2.1.3. $SU(6)/Sp(6)$ Little Higgs Model

In Ref. 17, the global symmetry of the model is chosen to be  $SU(6)$  which is broken down to  $Sp(6)$ . This has the advantage that the pseudo-Goldstone multiplet does not contain a  $SU(2)$  triplet component. This thus weakens the constraints from precision data. The gauged subgroup of  $SU(6)$  is  $[SU(2) \times U(1)]^2$ , which is broken down to its diagonal subgroup,  $[SU(2) \times U(1)]_{\text{SM}}$ . There are fourteen Goldstone bosons ( $35 - 21 = 14$ ) resulting from the global symmetry breaking. Four of them are eaten due to gauge symmetry breaking. Thus at low energy spectrum, there are two complex doublet scalar fields. At the TeV scale, there are one complex singlet scalar, a few pairs of vector-like colored fermions and an extra copy of the  $SU(2) \times U(1)$  gauge bosons. By turning off either one of the  $SU(2)$  gauge coupling constants, there is an enhanced  $SU(4)$  global symmetry which forbids a square mass term for the Higgs. Thus the quadratic divergences to the Higgs mass can only arise at two loop. The issue of vacuum stability in the presence of the anti-symmetric condensate of the  $\Sigma$  field in this model has also been investigated<sup>17</sup>.

## 2.2. Models with Simple Group

### 2.2.1. Little Higgs Model from A Simple Group

In the model proposed in Ref. 18, the global symmetry is  $[SU(4)]^4$  which breaks down to  $[SU(3)]^4$ . And the gauge subgroup is  $SU(4) \times U(1)$  which breaks down to  $SU(2) \times U(1)$ . There are  $(15 - 8) \times 4 = 28$  NGB's: 12 of them are eaten due to gauge symmetry breaking and the remaining 16 real components are decomposed into two complex doublets, three complex  $SU(2)$  singlets and two real singlet scalars. By embedding the electroweak  $SU(2)_L$  gauge group into a simple group such as  $SU(3)$  or  $SU(4)$ , this model has a nice feature that the cancellation of one-loop quadratic

<sup>c</sup>See also Ref. 13.

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divergences from the gauge and perturbatively coupled fermion loops is automatic. Generation of neutrino masses in this model has been discussed in Ref. 19.

### 2.2.2. *A Simple Model of Two Little Higgses*

The coset of the model proposed in Ref. 20 is  $SU(9)/SU(8)$  while the electroweak gauge symmetry of this model is expanded to  $SU(3) \times U(1)$  which is embedded into  $SU(9)$ . Due to the enlarged gauge group,  $SU(3) \times U(1)$ , there is no mixing induced by the VEV of the Higgs boson between the light and the heavy gauge bosons in this model, and it has only one additional gauge boson, the  $Z'$ .

## 2.3. *Moose Models*

### 2.3.1. *Minimal Moose Model*

In moose models, the electroweak sector of the SM is embedded into a theory with a product global symmetry,  $G^N$ , which is broken by a set of condensates transforming as bi-fundamental representations under  $G_i \times G_j$  for pair (i,j). Some subgroup of  $G^N$  which contains the SM is being gauged for each site; it is then broken down to the SM gauge group  $SU(2) \times U(1)$  by the bi-fundamental condensate at the TeV scale. The minimal moose model proposed in Ref. 21 has  $[SU(3)]^8$  as global symmetry. The gauge subgroup of the model is  $[SU(3) \times SU(2) \times U(1)]$ . The spectrum at low energy contains two complex Higgs doublets, a complex triplet and a singlet.

### 2.3.2. *SO(5) Moose Model*

By enlarging the global group in the minimal moose model to  $[SO(5)]^8 = [SO(5)_L]^4 \times [SO(5)_R]^4$  and the gauge group to  $SO(5) \times SU(2) \times U(1)$ , it has been shown in Ref. 22 that the model preserves an approximate custodial  $SU(2)$  symmetry. As a generic feature of the moose models, it is a two-Higgs doublet model at low energy. At the TeV scale, there is a colored Dirac fermion, a triplet Higgs and extra gauge bosons in the spectrum.

### 2.3.3. *SO(9) Moose Model*

The mass splitting among the triplet components give large contributions to the  $\rho$  parameter, which occurs in the  $SO(5)$  moose model. This problem can be alleviated by expanding the global symmetry to  $SO(9)$  as shown in Ref. 23. In the  $SO(9)$  moose model, the coset is  $SO(9)/[SO(5) \times SO(4)]$ , while the embedded gauge symmetry is  $SU(2)_L \times SU(2)_R \times SU(2) \times U(1)$ . At the TeV scale, there are three triplet Higgses present whose VEVs preserve an approximate custodial symmetry. This is an essential feature that reduces the contributions to the  $\rho$  parameter from the scalar sector in this model.

### 3. Precision Electroweak Constraints

As the electroweak sector of the SM has been tested to a very high accuracy, an important test of the validity of any new models is through the agreement between the predictions of these models with the precision data. A SM-like renormalization procedure with three input parameters in the gauge sector is valid as long as the models has tree level custodial symmetry and thus  $\rho = 1$  at tree level. Examples of models with  $\rho = 1$  at tree level include the SM augmented by additional Higgs doublets or singlets, additional fermion families and MSSM. On the other hands, in many extension of the SM, the tree level custodial symmetry is no longer a good symmetry of the model, *i.e.*  $\rho \neq 1$  already at the tree level. Models of this type include the left-right symmetric model based on  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ , models with additional  $SU(2)_L$  triplet Higgses (which might be relevant to generation of neutrino masses<sup>24,25</sup>), and various little Higgs models, to name a few.

In the gauge sector of the SM, there are only three *independent* parameters, the  $SU(2)_L$  and  $U(1)_Y$  gauge coupling constants,  $g$  and  $g'$ , as well as the VEV of the Higgs doublet,  $v$ . Once these three parameters and their counter terms are fixed by the experimental data, all other physical observables in the gauge sector can then be predicted in terms of these three input parameters<sup>d</sup>. A special feature of the SM with the assumption of one Higgs doublet is the validity of the tree level relation,  $\rho = 1 = \frac{M_W^2}{M_Z^2 c_\theta^2}$  due to the tree level custodial symmetry. There is thus a definite relation between the W-boson mass and the Z-boson mass. Of course, one can equivalently choose any three physical observables as the input parameters in the gauge sector. If we choose  $G_\mu$ ,  $M_Z$  and  $\alpha$  as the three input parameters in the gauge sector, the W-boson mass,  $M_W$ , then is predicted in the usual way via muon-decay,

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu s_\theta^2} \left[ 1 + \Delta r \right], \quad (29)$$

where  $\Delta r$  summarizes the one-loop radiative corrections, and it is given in terms of the gauge boson self-energy two point functions as,

$$\begin{aligned} \Delta r &= -\frac{\delta G_\mu}{G_\mu} - \frac{\delta M_W^2}{M_W^2} + \frac{\delta\alpha}{\alpha} - \frac{\delta s_\theta^2}{s_\theta^2} \\ &= \frac{\Pi^{WW}(0) - \Pi^{WW}(M_W)}{M_W^2} + \Pi^{\gamma\gamma'}(0) + 2\frac{s_\theta}{c_\theta} \frac{\Pi^{\gamma Z}(0)}{M_Z^2} - \frac{\delta s_\theta^2}{s_\theta^2}. \end{aligned} \quad (30)$$

The counter term for the weak mixing angle  $s_\theta$  which is defined through the W- and Z-boson mass ratio,  $s_\theta^2 = 1 - \frac{M_W^2}{M_Z^2}$ , is given by,

$$\frac{\delta s_\theta^2}{s_\theta^2} = \frac{c_\theta^2}{s_\theta^2} \left[ \frac{\Pi^{ZZ}(M_Z)}{M_Z^2} - \frac{\Pi^{WW}(M_W)}{M_W^2} \right]. \quad (31)$$

<sup>d</sup>In addition to these three input parameters in the gauge sector, there are additional input parameters in the fermion and scalar sectors. These can be chosen to be the fermion and scalar masses.

Both of the two point functions,  $\Pi^{WW}(M_W)$  and  $\Pi^{WW}(0)$ , have identical leading quadratic  $m_t$  dependence,  $\frac{\sqrt{2}G_\mu}{16\pi^2}3m_t^2(1 + 2\ln\frac{Q^2}{m_t^2})$ , and thus their difference is only logarithmic. The two-point function,  $\Pi^{\gamma\gamma}(0)$ , is also logarithmic in  $m_t$ . However, the difference between  $\Pi^{WW}(M_W)$  and  $\Pi^{ZZ}(M_Z)$  has quadratic dependence in  $m_t$ . Thus the prediction for  $M_W$  is quadratic in  $m_t$ .

In the presence of a  $SU(2)_L$  triplet Higgs, on the other hand, a tri-linear coupling between the doublet and the triplet Higgs,  $H^T\Phi^\dagger H$ , is allowed by the gauge symmetry  $SU(2)_L \times U(1)_Y$ . So unless one imposes a discrete symmetry to forbid such a tri-linear interaction, the VEV of the triplet is non-zero,  $\langle v' \rangle \neq 0$ . This thus leads to the need for a fourth input parameter in the gauge sector, with the fourth parameter being the VEV of the triplet Higgs,  $v'$ . Many of the familiar predictions of the Standard Model are drastically changed by the need for this extra input parameter<sup>26</sup>. One can equivalently choose the effective leptonic mixing angle,  $s_\theta$ , as the fourth input parameter, where  $s_\theta$  is defined through the ratio of the vector to axial vector parts of the  $Ze\bar{e}$  coupling,  $4s_\theta^2 - 1 = \frac{\text{Re}(g_V^e)}{\text{Re}(g_A^e)}$ . The counter term for  $s_\theta^2$  is formally related to the wave function renormalizations for  $\gamma$  and  $Z$  and it is given by,

$$\frac{\delta s_\theta^2}{s_\theta^2} = \text{Re} \left\{ \frac{c_\theta}{s_\theta} \left[ \frac{\Pi^{\gamma Z}(M_Z)}{M_Z^2} - \frac{v_e}{2s_\theta c_\theta} \left( \frac{a_e^2 - v_e^2}{a_e v_e} \Sigma_A^e(m_e^2) + \frac{\Lambda_V^{Zee}(M_Z)}{v_e} - \frac{\Lambda_A^{Zee}(M_Z)}{a_e} \right) \right] \right\}, \quad (32)$$

where  $\Sigma_A^e$  is the axial part of the electron self-energy and  $\Lambda_{V,A}^{Zee}$  are the vector and axial vector parts of the  $Ze\bar{e}$  vertex corrections. Contrary to the SM case in which the  $m_t$  dependence is quadratic, in models with a triplet Higgs the dominant contribution in Eq. 32,  $\Pi^{\gamma Z}(M_Z)$ , depends on  $m_t$  only logarithmically. Due to this logarithmic dependence, the constraint on the model is weakened. On the other hand, the scalar contributions become important as they are quadratic due to the lack of the tree level custodial symmetry, as pointed out in Ref. 26, 27, 28, 31, 32.

### 3.1. *Littlest Higgs Model*

In many little Higgs models, the  $\rho$  parameter also differs from one already at the tree level, due to the presence of the triplet Higgs which acquires a non-vanishing VEV. (For analyses based on tree level constraints can be found in Ref. 29 and those including the heavy top effects at one loop can be found in Ref. 30.) A consistent renormalization scheme thus requires an additional input parameter in the gauge sector. In Ref. 31, the authors choose the muon decay constant  $G_\mu$ , the physical Z-boson mass  $M_Z^2$ , the effective lepton mixing angle  $s_\theta^2$  and the fine-structure constant  $\alpha(M_Z^2)$  as the four independent input parameters in the renormalization procedure. The  $\rho$  parameter, defined as,  $\rho \equiv M_{W_L}^2/(M_Z^2 c_\theta^2)$ , and the W-boson mass, which is defined through muon decay, are then derived quantities. Since the loop factor occurring in radiative corrections,  $\frac{1}{16\pi^2}$ , is similar in magnitude to the expansion parameter,  $\frac{v^2}{f^2}$ , of the chiral perturbation theory for  $f$  of a few TeV, the

one-loop radiative corrections can be comparable in size to the next-to-leading order contributions at tree level. Both types of corrections are of the order of a few percent.

The effective leptonic mixing angle is defined through the ratio of the vector to axial vector parts of the  $Zee$  coupling,

$$4s_\theta^2 - 1 = \frac{\text{Re}(g_V^e)}{\text{Re}(g_A^e)}, \quad (33)$$

which differs from the naive definition of the Weinberg angle in the littlest Higgs model,  $s_W^2 = g'^2/(g'^2 + g^2)$ , by,

$$\Delta s_\theta^2 \equiv s_W^2 - s_\theta^2 = -\frac{1}{2\sqrt{2}G_\mu f^2} [s_\theta^2 c^2 (c^2 - s^2) - c_\theta^2 (c'^2 - s'^2)(-2 + 5c'^2)]. \quad (34)$$

The W-boson mass is defined through muon decay,

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu s_\theta^2} [1 + \Delta r_{\text{tree}} + \Delta r'], \quad (35)$$

where  $\Delta r_{\text{tree}}$  summarize the tree level corrections due to the change in definition in the weak mixing angle as well as the contributions from exchange of the heavy gauge bosons,

$$\Delta r_{\text{tree}} = -\frac{\Delta s_\theta^2}{s_\theta^2} + \frac{c^2 s^2}{\sqrt{2}G_\mu f^2}. \quad (36)$$

The one-loop radiative corrections are collected in  $\Delta r'$ ,

$$\begin{aligned} \Delta r' &= -\frac{\delta G_\mu}{G_\mu} - \frac{\delta M_W^2}{M_W^2} + \frac{\delta\alpha}{\alpha} - \frac{\delta s_\theta^2}{s_\theta^2} \\ &= \frac{1}{M_W^2} [\Pi^{WW}(M_W) - \Pi^{WW}(0)] + \Pi^{\gamma\gamma}(0)' - \frac{c_\theta}{s_\theta} \frac{\Pi^{\gamma Z}(M_Z)}{M_Z^2}. \end{aligned} \quad (37)$$

The predictions for  $M_{W_L}$  with and without the one-loop contributions for  $f = 2$  TeV is given in Fig. 3, which demonstrates that a low value of  $f$  ( $f \sim 2$  TeV) is allowed by the experimental restrictions from the  $W$  and  $Z$  boson masses,<sup>31</sup> due to cancellations among the tree-level and one-loop corrections. This shows the importance of a full one-loop calculation in placing the electroweak precision constraints.

### 3.2. Constraints in Other Little Higgs Models

Except for the littlest Higgs model with T-parity, where  $f$  can be as low as 500 GeV<sup>15</sup>, all other models discussed in the previous section receive tree level contributions to the EW observables. The precision EW constraint in the  $SU(6)/Sp(6)$  model has been analyzed in Ref. 33. Due to the absence of the triplet Higgs, the contribution to the  $T$  parameter is minimal, and the scale for  $f$  in this model can be 1 TeV<sup>33</sup>. In the simple group model based on  $[SU(4)]^4/[SU(3)]^4$ , the precision EW fit has been performed in Ref. 34, and the bound on  $f$  is found to be 4.2 TeV. In the  $SU(9)/SU(8)$  simple group model, the bound on  $f$  is slightly improved to be

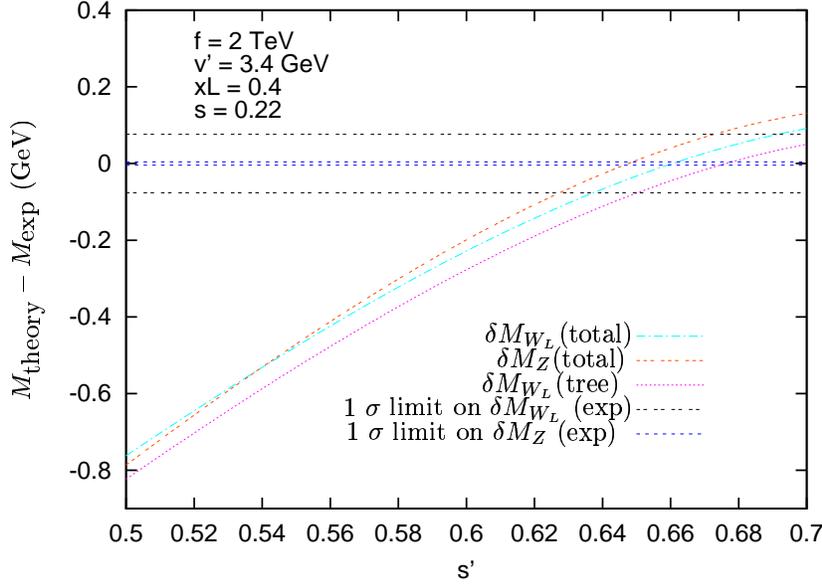


Fig. 3. Prediction for  $M_{W_L}$  as a function of the mixing angle  $s'$  at the tree level and the one-loop level. Also plotted is the correlation between  $M_Z$  and  $s'$  for fixed  $s$ ,  $v'$  and  $f$ . The cutoff scale  $f$  in this plot is  $2 \text{ TeV}$ , the  $SU(2)$  triplet VEV  $v' = 3.4 \text{ GeV}$ , the mixing angle  $s = 0.22$ , and  $x_L = 0.4$ . (Figure taken from Ref. 31)

$3.3 \text{ TeV}$ <sup>20,35</sup>. The precision EW constraints in the minimal moose model have been investigated in Ref. 36, in which  $f$  is found to be very severely constrained. Due to the approximate custodial symmetry in the  $SO(5)$  and  $SO(9)$  moose models, this bound can be relaxed<sup>37</sup>.

#### 4. Other Issues

It has been pointed out that in many new models with Physics beyond the Standard Model, there is generally an implicit fine-tuning among the model parameters which may be over-looked at first glance but show up in a systematic analysis<sup>38</sup>. Such implicit fine-tuning, which is needed to render the models phenomenologically viable, has been quantified by Barbieri and Giudice in Ref. 38.

In Ref. 39, Casas, Espinosa and Hidalgo examine this issue in the little Higgs models and find that the degree of such implicit fine-tuning is usually much more substantial than the rough estimate. If we define the amount of fine-tuning in the Higgs VEV in the model *a la* Barbieri and Giudice<sup>38</sup> as,  $v^2 = v^2(p_1, p_2, \dots)$ , where  $p_i$  for  $(i = 1, \dots)$  are input parameters of the model, then the amount of fine-tuning associated with parameter  $p_i$  is given by,

$$\frac{\delta M_Z^2}{M_Z^2} = \frac{\delta v^2}{v^2} = \Delta_{p_i} \frac{\delta p_i}{p_i}, \quad (38)$$

and the total amount of fine-tuning in the model is defined as,

$$\Delta \equiv \sqrt{\sum_i \Delta_{p_i}^2} . \quad (39)$$

Fig. 4 shows the fine-tuning that exists in various versions of the little Higgs models as well as in the SM and MSSM. Such implicit fine-tuning in the little Higgs models has been found to be more severe compared to the fine-tuning required in MSSM<sup>39</sup>. (See also Ref. 40.)

Since the little Higgs models are effective theories valid only up to the cut-off scale,  $\Lambda \sim 4\pi f \sim 10$  TeV, an important question one has to address is what lies beyond this cut-off scale<sup>41</sup>. A few possible UV completions have been speculated. In Ref. 42, the little Higgs model is UV completed into a model in which the little Higgs is composite at 10 TeV and the model has a matter parity,  $(-1)^{(2S+3B+L)}$ , similar to the R-parity in SUSY. In Ref. 43 and 44, the little Higgs model is completed by another little Higgs model. This thus postpones the onset of the strong coupling regime to  $\sim 100$  TeV. In Ref. 45 an alternative was proposed in which the littlest Higgs model is UV completed by a five-dimensional Anti-de Sitter space where the global SU(5) symmetry in 4D littlest Higgs model corresponds to an SU(5) gauge symmetry in the 5D bulk.

The issue of vacuum stability in various little Higgs models has been investigated in Ref. 46, while their finite temperature effects have been studied in Ref. 47. An attempt to supersymmetrize the little Higgs model has been made in Ref. 48, and

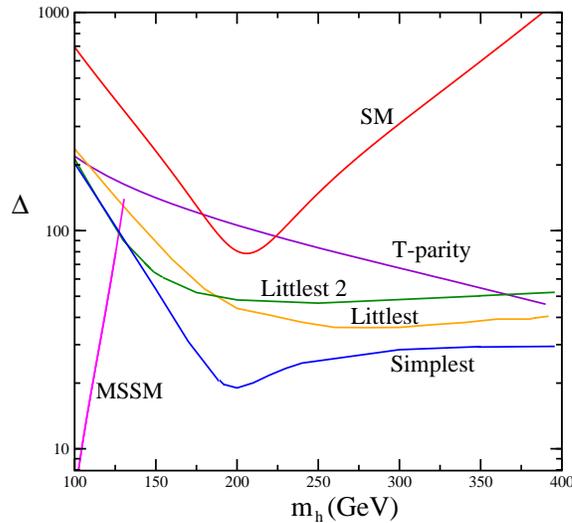


Fig. 4. Comparison of fine-tuning in SM (with a cutoff of 10 TeV), MSSM and various Little Higgs models. (Figure taken from Ref. 39)

it was found that the model can be embedded into an  $SU(6)$  GUT.

## 5. Conclusions

Little Higgs models are a new approach to stabilizing the Higgs mass. The Higgs boson in these models arises as a pseudo-Nambu-Goldstone boson resulting from the spontaneous breaking of some global symmetry. No interaction in these models alone can break the complete global symmetries that prohibit the quadratic contribution to the Higgs potential. Thus only when these interactions act collectively can the complete global symmetries be broken to give a quadratic mass term to the Higgs boson. The quadratically divergent contributions to the Higgs potential can therefore arise only at the two-loop level. In this note, I review various little Higgs models and the precision EW constraints placed in them. In most of these models, the parameter space is severely constrained by the precise electroweak data, with the exception of the littlest Higgs model with T-parity, in which a low cutoff scale is allowed. The importance of a full one-loop calculation is also emphasized and demonstrated explicitly in the case of the littlest Higgs model. There are other important questions such as the UV completion, vacuum stability and the contributions to the electroweak precision observables from higher dimensional operators at the cutoff scale, which are either out of the scope of this review or have only been briefly mentioned. More careful studies on these issues are clearly needed.

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