

# Production of a $Z$ Boson and Two Jets with One Heavy-Quark Tag

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## Abstract

We present a next-to-leading-order calculation of the production of a  $Z$  boson with two jets, one or more of which contains a heavy quark ( $Q = c, b$ ). We show that the cross section with only one heavy-quark jet is larger than that with two heavy-quark jets at both the Fermilab Tevatron and the CERN LHC. These processes are the dominant irreducible backgrounds to a Higgs boson produced in association with a  $Z$  boson, followed by  $h \rightarrow b\bar{b}$ . Our calculation makes use of a heavy-quark distribution function, which resums collinear logarithms and makes the next-to-leading-order calculation tractable.

# 1 Introduction

The discovery of new physics at hadron colliders often relies on a detailed understanding of standard-model background processes. Prominent among these is the production of weak bosons ( $W, Z$ ) in association with jets, one or more of which contains a heavy quark ( $Q = c, b$ ). The prime example is the discovery of the top quark at the Fermilab Tevatron, which required a thorough understanding of the  $W$ +jets background, with one or more heavy-quark jets [1, 2, 3]. The discovery of single-top-quark production via the weak interaction requires an even more sophisticated understanding of this background [4, 7].

In this paper we present a next-to-leading-order (NLO) calculation of the production of a  $Z$  boson in association with two or more jets, one or more of which contains a heavy quark. This is the dominant irreducible background to the production of a Higgs boson in association with a  $Z$  boson, followed by  $h \rightarrow b\bar{b}$  [5, 6]. This signal is currently being sought at the Tevatron with either one or both  $b$  jets tagged [8]. The calculation we present provides the irreducible background at NLO for both cases.

The production of a  $Z$  boson plus two jets with at least one heavy-quark jet is also a background to the production of a Higgs boson in association with one or more  $b$  jets, which is a discovery mode for a supersymmetric Higgs boson at large values of  $\tan\beta$  [9, 10, 11].<sup>1</sup> This background process is also a benchmark for this Higgs discovery channel. The search for a supersymmetric Higgs boson via this mode is underway at the Tevatron [12, 13], and will be vigorously pursued at the CERN Large Hadron Collider (LHC) [14, 15].

When one considers the production of a heavy quark at a hadron collider, one's first thought is usually of a virtual gluon splitting into a final-state  $Q\bar{Q}$  pair, as shown in Fig. 1(a), or via gluon fusion, as shown in Fig. 1(b). However, initial gluons splitting into a  $Q\bar{Q}$  pair is just as important a source at the Tevatron, and even more important at the LHC, when only one heavy quark is observed at high transverse momentum ( $p_T$ ). In that situation, it is advantageous to think of the initial gluon as splitting into a collinear  $Q\bar{Q}$  pair, with one heavy quark remaining at low  $p_T$  while the other heavy quark participates in the hard scattering and emerges at high  $p_T$ . The heavy quark that participates in the hard scattering can be treated as part of the proton sea, with a parton distribution function that is calculated perturbatively from the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [16, 17]. The production of a  $Z$  boson with two jets, one of which contains a heavy quark, then proceeds as shown in Fig. 2.

The reasons for using a heavy-quark distribution function are twofold. First, it resums collinear logarithms of the form  $\ln Q/m_Q$  to all orders, where  $Q$  is the scale of the hard scattering. Second, it simplifies the leading-order process, which makes a higher-order calculation tractable.

This paper completes the NLO calculation of  $Z$ +jets, with one or more heavy quarks, up to two jets [18]. The production of a  $Z$  plus one jet, with one or more heavy quarks, was presented in Ref. [19], and agrees well with data from the Tevatron [20]. The inclusive production of a  $Z$  with one or more heavy quarks was presented in Ref. [21].

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<sup>1</sup>The minimal supersymmetric standard model requires two Higgs doublets; the ratio of their vacuum expectation values is  $\tan\beta \equiv v_2/v_1$ .

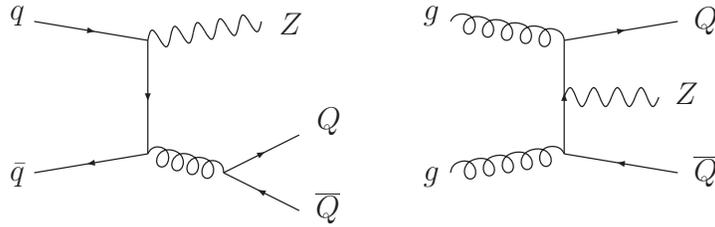


Figure 1: Diagrams contributing to the associated production of a  $Z$  boson and two high- $p_T$  heavy quarks ( $Q = c, b$ ).

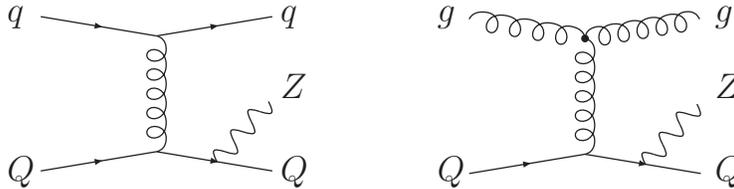


Figure 2: Diagrams contributing to the associated production of a  $Z$  boson and two high- $p_T$  jets, one of which contains a heavy quark ( $Q = c, b$ ).

## 2 $ZQj$ at NLO

The leading-order (LO) processes for  $Z$  plus two jets, one or more of which contains a heavy quark, are  $q\bar{q}(gg) \rightarrow ZQ\bar{Q}$  (Fig. 1) and  $Qq(g) \rightarrow ZQq(g)$  (Fig. 2). The LO cross sections are given in Table 1, with the jets satisfying the conditions  $p_T > 15$  GeV,  $|\eta| < 2$  (2.5 at the LHC), and  $\Delta R_{jj} > 0.7$ . At the Tevatron, the leading-order cross sections for these two classes of processes are comparable; at the LHC, the latter dominates. The processes  $Qq(g) \rightarrow ZQq(g)$  are relatively more important at the LHC than at the Tevatron because they are initiated by a heavy sea quark, whose distribution function rises at small values of  $x$ . Furthermore,  $Qg \rightarrow ZQg$  is larger than  $Qq \rightarrow ZQq$  at the LHC, while they are comparable at the Tevatron, since the former involves the gluon distribution function, which is large at small values of  $x$ . For the same reason,  $gg \rightarrow ZQ\bar{Q}$  is dominant at the LHC compared with  $q\bar{q} \rightarrow ZQ\bar{Q}$ , while the latter is more important at the Tevatron.

As is often the case, the distinction between the various processes is lost once one goes beyond leading order. Fig. 3(a) shows a Feynman diagram which ostensibly contributes to the NLO correction to  $q\bar{q} \rightarrow ZQ\bar{Q}$ , while the Feynman diagram in Fig. 3(b) ostensibly contributes to the NLO correction to  $Qq \rightarrow ZQq$ , and the diagram in Fig. 3(c) to the NLO correction to  $gg \rightarrow ZQ\bar{Q}$ . However, all three diagrams contribute to the same amplitude, and therefore interfere. Thus one cannot uniquely identify them with any of the leading-order processes.

The next-to-leading-order calculations in this paper were performed with the Monte-Carlo code MCFM [23]. The leading-order calculations were performed both with this code and with MadEvent [24]. The code MCFM is general enough to provide results for final states from  $\gamma^*, Z^* \rightarrow \ell^+\ell^-$  (including interference); for this paper we specialize to the case of a real  $Z$  boson.

Table 1: Leading-order cross sections (pb) for  $Z$  boson plus two jets, one or two of which contains a heavy quark, at the Tevatron ( $\sqrt{s} = 1.96$  TeV  $p\bar{p}$ ) and the LHC ( $\sqrt{s} = 14$  TeV  $pp$ ). A jet lies in the range  $p_T > 15$  GeV and  $|\eta| < 2$  (2.5 at the LHC), with  $\Delta R_{jj} > 0.7$ . No branching ratios or tagging efficiencies are included. The labels on the columns have the following meaning:  $ZQj$  = exactly two jets, one of which contains a heavy quark;  $ZQ\bar{Q}$  = exactly two jets, both of which contain a heavy quark. The CTEQ6L1 parton distribution functions are used [22], with the factorization and renormalization scales chosen as  $\mu_F = \mu_R = M_Z$ . Also given, in square brackets, is the LO cross section obtained without using a heavy-quark distribution function, from  $gq(g) \rightarrow ZQ\bar{Q}q(g)$ .

Process	$\sigma$ (pb)			
	Tevatron		LHC	
	$ZQj$	$ZQ\bar{Q}$	$ZQj$	$ZQ\bar{Q}$
$bq \rightarrow Zbq + bg \rightarrow Zbg$	0.89+1.29=2.18 [1.91]	–	76+276=352 [191]	–
$q\bar{q} \rightarrow Zb\bar{b} + gg \rightarrow Zb\bar{b}$	–	1.89+0.58=2.47	–	13+96=109
$cq \rightarrow Zcq + cg \rightarrow Zcg$	1.37+1.83=3.20 [3.26]	–	98+345=443 [271]	–
$q\bar{q} \rightarrow Zc\bar{c} + gg \rightarrow Zc\bar{c}$	–	1.89+0.45=2.34	–	12+75=87

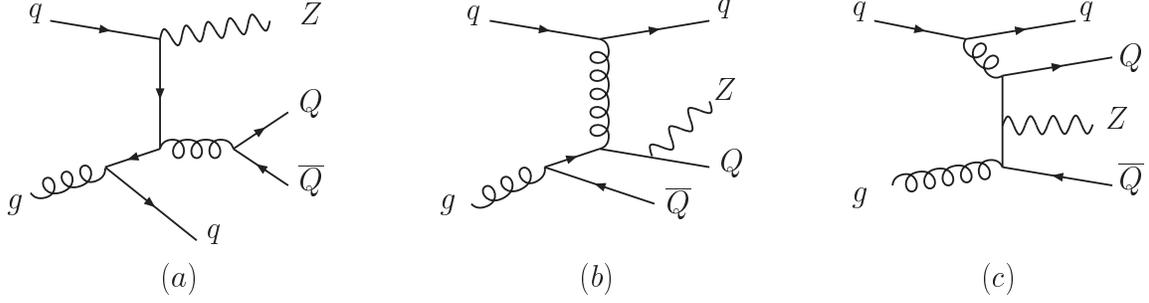


Figure 3: Diagrams contributing to the NLO correction to the associated production of a  $Z$  boson and two high- $p_T$  jets, one or more of which contains a heavy quark ( $Q = c, b$ ).

The processes involved in the calculation are as follows:

- $q\bar{q} \rightarrow ZQ\bar{Q}$  at tree level (Fig. 1) and one loop
- $gg \rightarrow ZQ\bar{Q}$  at tree level (Fig. 1) and one loop
- $Qq \rightarrow ZQq$  at tree level (Fig. 2) and one loop
- $Qg \rightarrow ZQg$  at tree level (Fig. 2) and one loop
- $q\bar{q} \rightarrow ZQ\bar{Q}g$  at tree level
- $gg \rightarrow ZQ\bar{Q}g$  at tree level
- $Qg \rightarrow ZQgg$  at tree level
- $Qq \rightarrow ZQqq$  at tree level
- $gq \rightarrow ZQ\bar{Q}q$  at tree level (Fig. 3)
- $Qg \rightarrow ZQq\bar{q}$  at tree level

We also include the processes  $QQ' \rightarrow ZQQ'$ ,  $Q\bar{Q}' \rightarrow ZQ\bar{Q}'$ , and  $Q\bar{Q} \rightarrow ZQ'\bar{Q}'$  at LO. These processes are already small at LO, so it is safe to neglect their NLO corrections. For charm final states, we only take  $Q = Q' = c$ ; for bottom,  $Q, Q' = c, b$ .

The results of our calculation are presented in Tables 2 and 3. The columns  $ZQj$  and  $ZQ\bar{Q}$  contain the leading-order cross sections (in parentheses) for the processes  $Qq(g) \rightarrow ZQq(g)$  and  $q\bar{q}(gg) \rightarrow ZQ\bar{Q}$ , respectively, taken from Table 1. We require the jets to satisfy the conditions  $p_T > 15$  GeV,  $|\eta| < 2$  (2.5 at the LHC), and  $\Delta R_{jj} > 0.7$ . The heavy-quark mass is neglected here and throughout the calculation (except where noted). To obtain the NLO cross sections for  $ZQj$  and  $ZQ\bar{Q}$ , which involve the radiation of additional partons (e.g., Fig. 3), two partons are combined into a single jet by adding their four-momenta if  $\Delta R_{jj} < 0.7$ . If the combined partons are both heavy quarks, then the process contributes to the column labeled  $Z(Q\bar{Q})j$ . This is a  $Z + 2j$  event in which one jet contains two heavy

Table 2: Cross sections (pb) for  $Z$ -boson plus two (or more) jets, one or more of which contains a heavy quark, at the Tevatron ( $\sqrt{s} = 1.96$  TeV  $p\bar{p}$ ). A jet lies in the range  $p_T > 15$  GeV and  $|\eta| < 2$ . Two final-state partons are merged into a single jet if  $\Delta R_{jj} < 0.7$ . No branching ratios or tagging efficiencies are included. Numbers in parenthesis are leading-order results. The labels on the columns have the following meaning:  $ZQj$  = exactly two jets, one of which contains a heavy quark;  $ZQ\bar{Q}$  = exactly two jets, both of which contain a heavy quark;  $Z(Q\bar{Q})j$  = exactly two jets, one of which contains a heavy-quark pair;  $ZQ\bar{Q}j$  = exactly three jets, two of which contain a heavy quark;  $ZQjj$  = exactly three jets, one of which contains a heavy quark. For the last set of processes, the labels mean:  $Zjj$  = exactly two jets, including heavy quarks;  $Zjjj$  = exactly three jets, including heavy quarks. For  $ZQj$  and  $ZQ\bar{Q}$ , both the leading-order (in parentheses) and next-to-leading-order cross sections are given. The CTEQ6M parton distribution functions are used throughout, except for the LO cross sections in parentheses, where CTEQ6L1 is used [22]. The factorization and renormalization scales are chosen as  $\mu_F = \mu_R = M_Z$ .

Tevatron	$\sigma$ (pb)					
	$ZQj$	$ZQ\bar{Q}$	$Z(Q\bar{Q})j$	$ZQ\bar{Q}j$	$ZQjj$	
bottom	(2.18) 5.23	(2.47) 3.07	0.634	0.672	0.326	
charm	(3.20) 7.49	(2.34) 2.75	2.00	0.621	0.495	
	$Zjj$			$Zjjj$		
$Z$ +jets	(163) 182			22.9		

quarks, which changes the tagging probability for that jet [25]. It is calculated with a finite heavy-quark mass ( $m_c = 1.4$  GeV,  $m_b = 4.75$  GeV) in order to regulate the divergence present when the heavy quarks are collinear. If all three partons are well separated, then the process contributes to either  $ZQ\bar{Q}j$  or  $ZQjj$ .

We checked that the effect of the heavy-quark mass is negligible by comparing  $ZQ\bar{Q}$  at tree level with and without a finite quark mass. Similarly, we found that the heavy-quark mass is also negligible for  $ZQ\bar{Q}j$  at tree level, as expected.

The NLO correction to  $ZQj$  is quite sizable at the Tevatron, more than 100%. One of the reasons is that the process  $q\bar{q} \rightarrow ZQ\bar{Q}g$  (where one of the heavy quarks is outside the acceptance) makes a relatively large contribution. This is not really a NLO correction to  $Qq(g) \rightarrow ZQq(g)$ , but rather a new channel. This contribution is about 1.2 pb for both bottom and charm at the Tevatron. In contrast, this process makes a relatively small contribution to  $ZQj$  at the LHC, only 10 pb for bottom and 4 pb for charm.

We also list in Tables 2 and 3 the LO (in parentheses) and NLO cross sections for  $Zjj$  [18, 26, 27], and the LO cross section for  $Zjjj$ . In these cross sections we have included the contribution from light partons as well as heavy quarks. Thus, for example, the fraction of  $Z + 2j$  events in which only one of the jets contains heavy quarks is given by  $[ZQj +$

Table 3: Same as Table 2, except at the LHC ( $\sqrt{s} = 14$  TeV  $pp$ ). A jet lies in the range  $p_T > 15$  GeV and  $|\eta| < 2.5$ .

LHC	$\sigma$ (pb)					
	$ZQj$	$ZQ\bar{Q}$	$Z(Q\bar{Q})j$	$ZQ\bar{Q}j$	$ZQjj$	
bottom	(352) 421	(109) 92.1	23.5	60.8	92.1	
charm	(443) 623	(87) 75.2	58.6	49.3	123	
	$Zjj$			$Zjjj$		
$Z$ +jets	(6090) 4840			1810		

$Z(Q\bar{Q})j]/Zjj$ .

In Tables 4 and 5 we list the results in an inclusive manner, and also include the NLO result for  $Z$  plus one heavy-quark jet [19]. In addition, we give the NLO cross section for  $Zj$  [19], including both light and heavy partons. Thus  $ZQ + X$  is the cross section for a  $Z$  plus at least one heavy-quark jet; it is the sum of the columns labeled  $ZQ$ ,  $ZQj$ , and  $ZQ\bar{Q}$  in Tables 1 and 2 of Ref. [19], including all contributing processes.  $Z(Q\bar{Q})$  contains one jet which contains a heavy-quark pair. It is calculated at LO, including the heavy-quark mass in order to regulate the collinear divergence present in  $q\bar{q} \rightarrow Z(Q\bar{Q})$ .  $ZQj + X$  has at least two jets, one of which contains a heavy quark; it is the sum of the columns labeled  $ZQj$  and  $ZQjj$  in Tables 2 and 3.  $ZQ\bar{Q} + X$  has at least two jets, two of which contain a heavy quark; it is the sum of the columns labeled  $ZQ\bar{Q}$  and  $ZQ\bar{Q}j$  in Tables 2 and 3. Finally,  $Z(Q\bar{Q})j$  contains two jets, one of which contains a heavy-quark pair; it is calculated at LO with a finite quark mass.

We estimate the uncertainties in the NLO inclusive cross sections by varying the renormalization scale, the factorization scale, and the parton distribution functions independently. The renormalization scale is varied over  $\mu_R = (0.5-2)M_Z$ , with  $\mu_F = M_Z$ . Similarly, we vary  $\mu_F = (0.5-2)M_Z$  with  $\mu_R = M_Z$ . The uncertainties due to scale variation are significantly reduced at NLO in comparison with the LO calculation. The parton distribution functions are varied over the 41 different sets contained in CTEQ6M [22].

As an example of the use of these tables, consider the inclusive cross section for  $Z + 2$  jets with one heavy-quark tag. This cross section is obtained from

$$\sigma = \epsilon_Q \sigma_{ZQj+X} + 2\epsilon_Q(1 - \epsilon_Q) \sigma_{ZQ\bar{Q}+X} + \epsilon_{Q\bar{Q}} \sigma_{Z(Q\bar{Q})j} \quad (1)$$

where  $\epsilon_Q$  is the tagging probability for a heavy-quark jet, and  $\epsilon_{Q\bar{Q}}$  is the tagging probability for a jet containing a heavy-quark pair.

We show in Figures 4 and 5 the transverse-momentum spectrum of the  $Z$  boson in events with at least two jets, one of which contains a bottom quark, at the Tevatron and the LHC. Both the LO and NLO distributions are shown. The radiative corrections do not significantly

Table 4: Inclusive cross sections (pb) for  $Z$  boson plus one or two jets, one or two of which contains a heavy quark, at the Tevatron ( $\sqrt{s} = 1.96$  TeV  $p\bar{p}$ ). A jet lies in the range  $p_T > 15$  GeV and  $|\eta| < 2$ . Two final-state partons are merged into a single jet if  $\Delta R_{jj} < 0.7$ . No branching ratios or tagging efficiencies are included. Numbers in parenthesis are leading-order results. The labels on the columns have the following meaning:  $ZQ + X$  = at least one jet, at least one of which contains a heavy quark;  $Z(Q\bar{Q})$  = one jet which contains a heavy-quark pair;  $ZQj + X$  = at least two jets, one of which contains a heavy quark;  $ZQ\bar{Q} + X$  = at least two jets, two of which contain a heavy quark;  $Z(Q\bar{Q})j$  = two jets, one of which contains a heavy-quark pair. The last row gives the inclusive cross section for jets containing both light and heavy partons. The CTEQ6M parton distribution functions are used throughout, except for the LO cross sections in parentheses, where CTEQ6L1 is used [22]. The factorization and renormalization scales are chosen as  $\mu_F = \mu_R = M_Z$ . The uncertainties are from the variation of the renormalization scale, the factorization scale, and the parton distribution functions, respectively.

Tevatron	$\sigma$ (pb)				
	$Z + 1 \text{ jet} + X$		$Z + 2 \text{ jets} + X$		
	$ZQ + X$	$Z(Q\bar{Q})$	$ZQj + X$	$ZQ\bar{Q} + X$	$Z(Q\bar{Q})j$
bottom	(8.23) 18.1	2.1	(2.19) $5.56_{-0.9}^{+1.2} {}_{-0.05}^{+0.07} {}_{-0.5}^{+0.5}$	(2.49) $3.74_{-0.45}^{+0.45} {}_{-0.12}^{+0.12} {}_{-0.15}^{+0.15}$	$0.63_{-0.16}^{+0.26} {}_{-0.06}^{+0.06} {}_{-0.05}^{+0.05}$
charm	(11.3) 27.5	6.6	(3.21) $8.23_{-1.4}^{+1.7} {}_{-0.26}^{+0.15} {}_{-0.8}^{+0.8}$	(2.35) $3.47_{-0.37}^{+0.44} {}_{-0.87}^{+0.0} {}_{-0.14}^{+0.14}$	$2.08_{-0.53}^{+0.85} {}_{-0.16}^{+0.22} {}_{-0.15}^{+0.15}$
all jets	(898) 1070		(163) $205_{-19}^{+19} {}_{-2}^{+7} {}_{-5}^{+5}$		

Table 5: Same as Table 4, except at the LHC ( $\sqrt{s} = 14$  TeV  $pp$ ). A jet lies in the range  $p_T > 15$  GeV and  $|\eta| < 2.5$ .

LHC	$\sigma$ (pb)				
	$Z + 1 \text{ jet} + X$		$Z + 2 \text{ jets} + X$		
	$ZQ + X$	$Z(Q\bar{Q})$	$ZQj + X$	$ZQ\bar{Q} + X$	$Z(Q\bar{Q})j$
bottom	(826) 1060	25	(353) $513_{-58}^{+84} {}_{-35}^{+44} {}_{-25}^{+25}$	(111) $153_{-20}^{+20} {}_{-2}^{+2} {}_{-9}^{+9}$	$24_{-6}^{+10} {}_{-0.3}^{+0.3} {}_{-1.2}^{+1.2}$
charm	(989) 1430	50	(443) $746_{-110}^{+110} {}_{-46}^{+0} {}_{-45}^{+45}$	(90) $125_{-17}^{+17} {}_{-2}^{+2} {}_{-8}^{+8}$	$59_{-15}^{+23} {}_{-2}^{+2} {}_{-3}^{+3}$
all jets	(15300) 18400		(6090) $6650_{-500}^{+470} {}_{-50}^{+170} {}_{-240}^{+240}$		

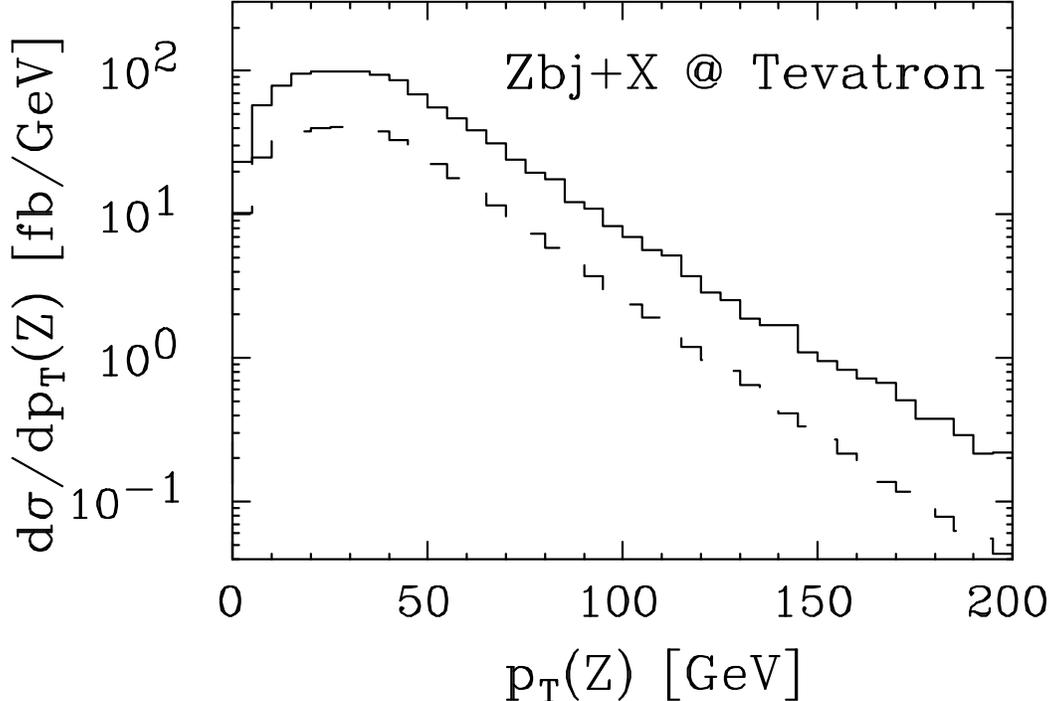


Figure 4: Transverse-momentum distribution of the  $Z$  boson in events with at least two jets, one of which contains a bottom quark, at the Tevatron ( $\sqrt{s} = 1.96$  TeV  $p\bar{p}$ ). The jets satisfy the conditions  $p_T > 15$  GeV,  $|\eta| < 2$ , and  $\Delta R_{jj} > 0.7$ . Both LO (dashed) and NLO (solid) distributions are shown.

change the shape of the distribution, as is often the case when the quantity plotted is not affected by the change in the kinematics due to additional radiation.

### 3 Conclusions

In this paper we present a NLO calculation of the production of a  $Z$  boson plus two jets, one or more of which contains a heavy quark. This greatly improves the accuracy with which this fundamental background is known at the Tevatron and the LHC. We provide our results in both an exclusive (Tables 2 and 3) and an inclusive manner (Tables 4 and 5). The NLO cross section is significantly greater than the LO cross section for the default scale choice  $\mu_F = \mu_R = M_Z$ .

Our calculation makes use of the heavy quarks present in the proton sea, which are perturbatively calculable. This makes the LO calculation simpler, thus allowing a higher-order calculation to be tractable. We showed that the processes  $Qq(g) \rightarrow ZQq(g)$  are a significant source of  $ZQj$  events at the Tevatron, and the dominant source at the LHC.

Alternatively, one could eschew the heavy quarks in the proton sea, and regard the proton as containing only light quarks and gluons. In that approach, the LO processes for  $ZQj$  are  $gq(g) \rightarrow ZQ\bar{Q}q(g)$ , where one of the heavy quarks is outside the acceptance of the detector.

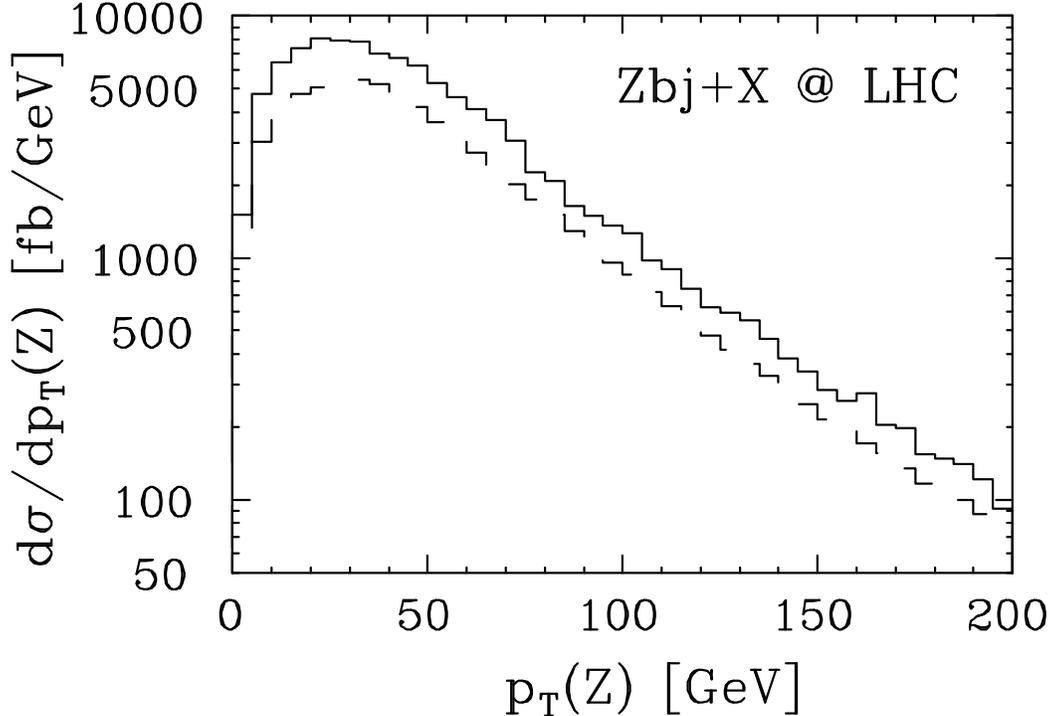


Figure 5: Same as Fig. 4, but at the LHC ( $\sqrt{s} = 14$  TeV  $pp$ ). The jets satisfy the conditions  $p_T > 15$  GeV,  $|\eta| < 2.5$ , and  $\Delta R_{jj} > 0.7$ .

We give the results of this calculation in square brackets in Table 1, using CTEQL1 with  $\mu_F = \mu_R = M_Z$ . The results are quite consistent with the LO results of the heavy-quark approach at the Tevatron, but at the LHC they lie somewhat below, though within a factor of two. In any case, the most accurate cross sections are the NLO results presented in this paper, based on the heavy-quark approach.

An important application of these results is to the search for the Higgs boson via  $q\bar{q} \rightarrow Zh$ , followed by  $h \rightarrow b\bar{b}$ , where the signal is  $Z + 2j$  with one or two  $b$  tags [8]. We provide NLO results for the backgrounds with either one or two heavy-quark jets. We have shown that the majority of such events have only one heavy-quark jet. We see from Tables 4 and 5 that  $Zbj + X$  is nearly twice as big as  $Zb\bar{b} + X$  at the Tevatron, and more than three times as large at the LHC. Similarly,  $Zcj + X$  is about thrice  $Zc\bar{c} + X$  at the Tevatron, and about six times as large at the LHC.

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