Determining the Neutrino Mass Hierarchy and CP–Violation in NO\(\nu\)A with a Second Off-Axis Detector

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We consider a Super-NO\(\nu\)A-like experimental configuration based on the use of two detectors in a long-baseline experiment as NO\(\nu\)A. We take the far detector as in the present NO\(\nu\)A proposal and add a second detector at a shorter baseline. The location of the second off-axis detector is chosen such that the ratio \(L/E\) is the same for both detectors, being \(L\) the baseline and \(E\) the neutrino energy. We consider liquid argon and water-Cherenkov techniques for the second off-axis detector and study, for different experimental setups, the detector mass required for the determination of the neutrino mass hierarchy, for different values of \(\theta\)\textsubscript{13}. We also study the capabilities of such an experimental setup for determining CP–violation in the neutrino sector. Our results show that by adding a second off-axis detector a remarkable enhancement on the capabilities of the current NO\(\nu\)A experiment could be achieved.

PACS numbers: 14.60.Pq

I. INTRODUCTION

During the last several years the physics of neutrinos has achieved a remarkable progress. The experiments with solar\(^7\), atmospheric\(^2\), reactor\(^8\) and recently also long-baseline accelerator\(^9\) neutrinos have provided compelling evidence for the existence of neutrino oscillations. The most significant recent contributions are given by the new Super-Kamiokande data on the \(L/E\) dependence of multi-GeV \(\mu\)-like atmospheric neutrino events\(^10\), \(L\) being the distance traveled by neutrinos and \(E\) the neutrino energy, and by the new more precise spectrum data of the KamLAND\(^11\) and K2K experiments\(^9\). For the first time these data show the oscillatory dependence on \(L/E\) of the probabilities of neutrino oscillations in vacuum\(^12\).

The existence of neutrino oscillations plays a crucial role in our understanding of neutrino physics as it implies non-zero neutrino masses and neutrino mixing. The present data requires\(^1\) two large (\(\theta\)\textsubscript{12} and \(\theta\)\textsubscript{23}) and one small (\(\theta\)\textsubscript{13}) angles in the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) neutrino mixing matrix\(^16\), and at least two mass square differences, \(\Delta m^2_{31} = m^2_3 - m^2_1\), with \(m_{1,3}\) the neutrino masses, one driving the atmospheric neutrino oscillations (\(\Delta m^2_{31}\)) and one the solar ones (\(\Delta m^2_{21}\)). The mixing angles \(\theta\)\textsubscript{12} and \(\theta\)\textsubscript{23} control the solar and the dominant atmospheric neutrino oscillations, while \(\theta\)\textsubscript{13} is the angle limited by the data from the CHOOZ and Palo Verde experiments\(^17,18\).

The Super-Kamiokande\(^7\) and K2K\(^9\) data are well described in terms of dominant \(\nu_\mu \rightarrow \nu_\tau\) (\(\nu_\tau \rightarrow \bar{\nu}_\tau\)) vacuum oscillations. The former require the following best fit values of the mass squared difference and mixing angle: \(|\Delta m^2_{31}| = 2.1 \times 10^{-3}\) eV\(^2\), \(\sin^2 2\theta_{23} = 1.0\)\(^7\), whereas the latter are best explained for \(|\Delta m^2_{31}| = 2.8 \times 10^{-3}\) eV\(^2\), \(\sin^2 2\theta_{23} = 1.0\)\(^9\). The 90\% C.L. allowed ranges of these parameters obtained by the Super-Kamiokande experiment read\(^7\):

\[
|\Delta m^2_{31}| = (1.5 - 3.4) \times 10^{-3}\text{eV}^2, \quad \sin^2 2\theta_{23} \geq 0.92.
\] (1.1)

\(^1\) We restrict ourselves to a three-family neutrino analysis. The unconfirmed LSND signal\(^13\) cannot be explained within this context and might require additional light sterile neutrinos or more exotic explanations (see, e.g. Ref.\(^14\)). The ongoing MiniBooNE experiment\(^15\) is going to test the oscillation explanation of the LSND result.
A new preliminary analysis \[19\] of Super-Kamiokande data, using a finer momentum binning of multi-GeV neutrinos, yields a slightly higher best fit value for the mass squared difference, \(\Delta m^2_{31} = 2.5 \times 10^{-3} \text{eV}^2\). The allowed 90\% C.L. interval reads \(|\Delta m^2_{31}| = (2.0-3.0) \times 10^{-3} \text{eV}^2\). The sign of \(\Delta m^2_{31}\), \(sgn(\Delta m^2_{31})\), cannot be determined with the existing data. The two possibilities, \(\Delta m^2_{31} > 0\) or \(\Delta m^2_{31} < 0\), correspond to two different types of neutrino mass ordering: normal hierarchy (NH), \(m_1 < m_2 < m_3\) \((\Delta m^2_{31} > 0)\), and inverted hierarchy (IH), \(m_3 < m_1 < m_2\) \((\Delta m^2_{31} < 0)\). In addition, information on the octant where \(\theta_{23}\) lies, if \(\sin^2 2\theta_{23} \neq 1.0\), is beyond the reach of present experiments.

The 2-neutrino oscillation analysis of the solar neutrino data, including the results from the complete salt phase of the Sudbury Neutrino Observatory (SNO) experiment \[6\], in combination with the recent KamLAND 766.3 ton-year spectrum data \[11\], shows that the solar neutrino oscillation parameters lie in the low-LMA (Large Mixing Angle) region, with best fit values \[6\]

\[
\Delta m^2_{21} = 8.0 \times 10^{-5} \text{eV}^2, \quad \sin^2 \theta_{12} = 0.31. \tag{1.2}
\]

A combined 3-neutrino oscillation analysis of the solar, atmospheric, reactor and long-baseline neutrino data gives \[20\] (see also Ref. \[21\]) constrains the third mixing angle to be:

\[
\sin^2 \theta_{13} < 0.041, \quad 3\sigma \text{ C.L.} \tag{1.3}
\]

The future goals in the study of neutrino properties will be to precisely determine the already measured oscillation parameters and to obtain information on the unknown ones, namely \(\theta_{13}\), the CP–violating phase \(\delta\) and the type of neutrino mass hierarchy (or equivalently \(sgn(\Delta m^2_{31})\)). A more accurate measurement of the leading neutrino oscillation parameters will be achieved by the MINOS \[22\], OPERA \[23\] and ICARUS \[24\] experiments and future atmospheric and solar neutrino detectors \[27\]. The determination of the \(\theta_{13}\) angle is crucial as it opens up the possibility of the experimental measurement of the CP– (or T–) violating phase \(\delta\), and to establishing the type of neutrino mass hierarchy. A wide experimental program is under discussion in order to achieve these goals.

The mixing angle \(\theta_{13}\) controls the \(\nu_{\mu} \rightarrow \nu_{e}\) and \(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}\) conversions in long-baseline experiments and the \(\nu_{e}\) disappearance in short-baseline reactor experiments. Present and future conventional beams \[22\] , super-beams with an upgraded proton source and a more intense neutrino flux \[26\] , and future reactor neutrino experiments \[28\] have the possibility to measure, or set a stronger limit on, \(\theta_{13}\). Smaller values of this mixing angle could be achieved by very long baseline experiments such as neutrino factories \[22\] , \[30\], or by \(\beta\)-beams \[31\] , \[32\] , \[37\], \[41\], \[42\] which exploit neutrinos from boosted-ion decays, or by super-beams with Megaton detectors \[26\] , or by electron-capture facilities \[35\]. The mixing angle \(\theta_{13}\) also controls the Earth matter effects in multi-GeV atmospheric \[36\] , \[37\] , \[38\] , \[39\] , \[40\] and in supernova \[41\] neutrino oscillations.

The magnitude of the T-violating and CP-violating terms as well as matter effects in long baseline neutrino oscillations is controlled by \(\sin \theta_{13}\) \[32\]. If the value of \(\theta_{13}\) is sizable and at the reach of future experiments, it would be possible to search for CP-violation in the lepton sector and to determine the type of neutrino mass hierarchy by establishing \(sgn(\Delta m^2_{31})\) \[2\]. Typically, the proposed experiments have a single detector and plan to run with the beam in two different modes, neutrinos and antineutrinos. In principle, by comparing the probability of neutrino and antineutrino flavor conversion, the values of the CP–violating phase \(\delta\) and of \(sgn(\Delta m^2_{31})\) can be extracted. Different sets of values of CP–conserving and violating parameters, \((\theta_{13}, \theta_{23}, \delta, sgn(\Delta m^2_{31}))\), lead to the same probabilities of neutrino and antineutrino conversion and provide a good description of the data at the same confidence level. This problem is known as the problem of degeneracies in the neutrino parameter space \[31\] , \[32\] , \[33\] , \[34\] , \[35\] and severely affects the sensitivities to these parameters in future long-baseline experiments. Many strategies have been advocated to resolve this issue. Some of the degeneracies might be eliminated with sufficient energy or baseline spectral information \[30\]. However, statistical errors and realistic efficiencies and backgrounds limit considerably the capabilities of this method. Another detector \[52\] , \[56\] , \[57\] , \[58\] , \[59\] , \[60\] , \[61\] or the combination with another experiment \[62\] , \[63\] , \[64\] , \[65\] , \[66\] , \[67\] , \[68\] , \[69\] would thus, be necessary \[3\].

The use of only a neutrino beam could help in resolving the type of hierarchy when two different long-baselines are considered \[60\] , \[61\] , \[62\] . The most favorable case is keeping the same \(L/E\) at the two different baselines. In particular, it was shown that just one experiment, named Super-NO\(\nu\)A \[60\] , which runs in the neutrino mode and

\[2\] Information on the type of neutrino mass hierarchy might be obtained in future atmospheric neutrino experiments \[33\] , \[35\] , \[36\] , \[37\] . If neutrino are Majorana particles, next generation of neutrinoless double \(\beta\)-decay experiments could establish the type of neutrino mass spectrum \[45\] (see also Refs. \[46\] , \[47\] , \[48\] , \[49\] ) and, possibly, might provide some information on the presence of CP-violation in the lepton sector due to Majorana CP–violating phases \[12\] (see also Refs. \[13\] , \[14\] , \[15\] , \[16\] ).

\[3\] New approaches which exploit other neutrino oscillations channels such as muon neutrino disappearance have been proposed \[66\] for determining the type of hierarchy. They require very precise neutrino oscillation measurements. Under certain rather special conditions it might be determined also in experiments with reactor \(\bar{\nu}_e\) \[10\].
uses two detectors at different distances and different off-axis angles, could determine the type of hierarchy for values of \( \sin^2 \theta_{13} \) as small as 0.02. This method is free of degeneracies. It uses the fact that, comparing the probability of conversion at the two sites, the vacuum oscillation term cancels out, leaving as dominant the term sensitive to matter effects. Hence the leading CP-violating term cancels and CP-violation gives only a subdominant correction. Off-axis neutrino beams have a very narrow neutrino spectra, and their peak energy can be tuned by displacing the detector out of the main beam axis. We notice that an off-axis beam can be obtained by either putting the detector a few km away from the location of an on-axis surface detector, or by placing it on the vertical of the beam-line but at a much shorter distance. In such a way, a single beam could do the job of two beams with different energies. In Ref. 54, the case of the NuMI beam and two 50 kton liquid argon TPC detectors, one at the NO\( \nu \)A proposed site 27 and the second at a shorter baseline, 200 and 434 km, was studied in detail. It was shown that the hierarchy can be determined at 95 \% C.L., regardless of the value of \( \delta \), for \( \sin^2 \theta_{13} \geq 0.05 \) for a conventional beam and for \( \sin^2 \theta_{13} \geq 0.02 \) with a proton driver. This should be compared with the sensitivity of the proposed NO\( \nu \)A experiment 27. At 95 \% C.L., only for less than 40 \% of the values of \( \delta \), the type of neutrino mass hierarchy can be resolved if \( \sin^2 2\theta_{13} \leq 0.10 \) in a 3 neutrino plus 3 antineutrino running 27.

Here, we follow the Super-NO\( \nu \)A strategy 60 but we consider a more realistic scenario. We consider the use of the proposed NO\( \nu \)A detector, a 30 kton low-Z calorimeter, at the far site. We add a second off-axis detector, either a liquid argon or a water-\( \bar{C} \)erenkov detector, at a shorter baseline of 200 km, keeping \( L/E \) constant at the two sites. In addition, we provide a sequencing for the construction of the experiment, assuming that the second detector will be constructed at a second stage. If the mixing angle \( \theta_{13} \) is very small, then adding the second detector would increase the statistics for constraining this parameter but would not provide any useful information on the type of neutrino mass hierarchy. On the contrary, if \( \theta_{13} \) is within the sensitivity of the NO\( \nu \)A experiment and a positive signal is found in the first years of running, the construction of the second off-axis detector would enormously enhance the capabilities for the determination of the type of neutrino mass hierarchy, (almost) free of degeneracies. Let us stress that the unique contribution of the NuMI neutrino program and of the NO\( \nu \)A experiment would be establishing the type of hierarchy \(^4\). Therefore we focus on this issue and we study in detail the requirements for the second detector for achieving this remarkable result. In addition, we also study the possibility of the measurement of CP-violation within this experimental setup. In order for this to be possible, the running with antineutrinos is completely necessary. Hence, in our sequencing of the experimental setup, we also add some years of antineutrino data.

We start by reviewing the relevant formalism and the power of the method in Sec. II. In Sec. III we describe the two different experimental configurations we consider. We show in Sec. IV the results for different possibilities for the 200 km detector and we depict how the sensitivity changes for different values of \( |\Delta m_{31}^2| \). In Sec. V we compare different possible choices for the location of the second off-axis detector and argue why the choice we make here is preferable. Finally, in Section VI we summarize and conclude.

II. FORMALISM

In the present study we focus on the capabilities of reducing degeneracies of a long-baseline neutrino experiment with two off-axis detectors. It is well known that an off-axis neutrino beam is characterized by a well peaked spectrum, so for sake of analytical understanding we can take it as approximately monochromatic. It was shown in Refs. 60, 62, 63 that a particularly suitable configuration for the determination of the neutrino mass hierarchy is such that the ratio \( L/E \), where \( L \) is the baseline and \( E \) is the neutrino energy, is the same for both detectors. For this particular configuration, the use of just a neutrino running could be sufficient to determine the type of neutrino mass spectrum, depending upon the actual value of \( \sin^2 2\theta_{13} \). We review below the analytical expressions illustrating this result.

For neutrino energies \( E \gtrsim 1 \) GeV, \( \theta_{13} \) within the present bounds 21, 21, and baselines \( L \lesssim 1000 \) km 54, 5, the oscillation probability \( |\mathcal{P}|(L) \) can be expanded in the small parameters \( \theta_{13} \), \( \Delta_{12}/\Delta_{13} \), \( \Delta_{12}/A \) and \( \Delta_{12}L \), where \( \Delta_{12} \equiv \Delta m_{21}^2/(2E) \) and \( \Delta_{13} \equiv \Delta m_{31}^2/(2E) \) 31 (see also Ref. 72):

\[
|\mathcal{P}|(L) \approx \sin^2 \theta_{23} \sin^2 2\theta_{13} \left( \frac{\Delta_{14}}{\Delta_{13}} \right)^2 \sin^2 \left( \frac{\Delta A_{13} \lambda L}{2} \right) \cos \left( \frac{\Delta A_{13} \lambda L}{2} \right) \cos (\Delta_{13}L \pm \delta) \\
+ \cos \theta_{13} \sin \theta_{21} \sin 2\theta_{12} \sin (\Delta \lambda L) \sin \left( \frac{\Delta A_{13} \lambda L}{2} \right) \sin (\Delta_{21}L) \sin (\Delta_{31}L) \cos (\Delta_{13}L \pm \delta) \\
+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \left( \frac{\Delta \lambda L}{\Delta_{13}} \right)^2 \sin^2 \left( \frac{\Delta_{13} \lambda L}{2} \right). \tag{2.1}
\]

\(^4\) The proposed atmospheric neutrino experiment INO 27 could achieve a similar result on the same timescale if finally approved.

\(^5\) For \( E \gtrsim 0.6 \) GeV we have checked that the analytical expansion is accurate for \( L < 500 \) km within the present bounds of \( \theta_{13} \) and \( \Delta m_{31}^2/\Delta m_{21}^2 \) 62.
Here, we use the constant density approximation for the index of refraction in matter $A \equiv \sqrt{2} G_F \bar{n}_e(L)$. We define $\bar{n}_e(L) = 1/L \int_0^L n_e(L')dL'$ as the average electron number density, with $n_e(L)$ the electron number density along the baseline. In addition, taking into account that for the baselines and energies of interest the matter effects are small ($A \ll \Delta m^2_{31}$ and $AL \ll 1$) a further expansion can be carried out. It was shown in Ref. \[60\] that for this experimental configuration of the detectors, at first order, the $\nu_\mu \rightarrow \nu_e$ conversion probability in vacuum is the same at both locations, so the dominant term of the normalized difference of the oscillation probabilities computed at the near and far baselines, $D \equiv P(L_N)-P(L_F)$, depends linearly on the matter term. Up to terms of order $O(A \Delta_2/\Delta_1)$, $O(A \theta^2_{13})$ and $O(A^2)$, the expression for $D$ reads \[60\]:

$$D \simeq \frac{A_N L_N - A_F L_F}{2} \left( \frac{1}{(\Delta_{13} L/2)} - \frac{1}{\tan(\Delta_{13} L/2)} \right) \left( 1 - \frac{\Delta_{12} \cos \theta_{13}}{\Delta_{13} \tan \theta_{23}} \frac{\sin 2\theta_{12}}{\sin 2\theta_{13}} \frac{\Delta_{13} L/2}{\sin(\Delta_{13} L/2)} \cos \left( \frac{\Delta_{13} L}{2} \right) - \frac{\sin^2 2\theta_{13}}{2} \right)$$

$$+ \frac{1}{2} \frac{A_N^2 L_N^2 - A_F^2 L_F^2}{4} \left( \frac{1}{(\Delta_{13} L/2)^2} - \frac{1}{\sin^2(\Delta_{13} L/2)} \right). \tag{2.2}$$

As shown in this formula, the corrections due to the CP–violating terms are small as far as $\sin^2 \theta_{13} \gtrsim 0.02$ because the dominant CP–violating terms in $P(L_N)$ and $P(L_F)$, which depend upon the vacuum terms, cancel out. As we can clearly see from Eq. \[2.2\], by considering both detectors with the same vacuum oscillation phase, the dependence on the matter potential dominates, with little contamination from other parameters. Thus, the normalized asymmetry is sensitive to the type of hierarchy as the sign of the matter potential term is a direct indication of the sign of the atmospheric mass squared difference. This is the power of the method, for corrections due to the rest of parameters can modify slightly its value, but cannot change its sign.

In Ref. \[60\], the determination of $\text{sgn}(\Delta m^2_{31})$ was considered and studied in detail with a configuration based on the (old) NO\(\nu\)A proposal \[23\] with a single running in the neutrino mode. Although, as was shown there, this could be the right way to proceed for the resolution of the mass hierarchy, running in antineutrinos is necessary in order to search for effects due to the CP–violating phase $\delta$. Hence, in the present study we consider the possibility to run also in the antineutrino channel and we analyze to what extent this opens the possibility to search also for CP–violation in the lepton sector. In an analogous way as for the previous case, we can compute the quantity $D_{CP} \equiv \frac{P(L_N)-P(L)}{P(L_N)+P(L)}$, i.e., the CP asymmetry for neutrinos and antineutrino detected at a distance $L$. Up to second order, $D_{CP}$ is given by:

$$D_{CP} \simeq AL \left( \frac{1}{(\Delta_{13} L/2)} - \frac{1}{\tan(\Delta_{13} L/2)} \right) + \frac{\Delta_{12} \cos \theta_{13}}{\Delta_{13} \tan \theta_{23}} \frac{\sin 2\theta_{12}}{\sin 2\theta_{13}} \frac{\Delta_{13} L}{2} \left[ 2 \sin \delta - \frac{1}{\Delta_{13} L/2} - \frac{1}{\tan(\Delta_{13} L/2)} \right] \frac{1}{\tan(\Delta_{13} L/2)} \cos \left( \frac{\Delta_{13} L}{2} \right). \tag{2.3}$$

The main feature of Eq. \[2.3\] lies on the fact that, for the energies and baselines of interest, the first two terms in the right-hand-side, the dominant matter effect term and the CP–violating one, are of the same order, with the latter becoming more important as $\sin^2 \theta_{13}$ decreases. This typically introduces an important degeneracy and limits the sensitivity to CP–violation in long-baseline experiments and is the main reason why having two beams of neutrino with different $L$ and $E$, but the same $L/E$, gives better sensitivity to $\text{sgn}(\Delta m^2_{31})$ than one neutrino and one antineutrino beam at the same $L$ and $E$. Hence, by placing the second detector in such a configuration, as it will be shown below, the fact that the type of hierarchy could be determined by using the neutrino channel only, solves this degeneracy and improves the sensitivity to CP–violation when running with antineutrinos.

### III. EXPERIMENTAL CONFIGURATION

As has been mentioned above, the idea of exploiting the capabilities of two off-axis detectors, with the same $L/E$ at a single experiment, was presented in Ref. \[60\] for the first time by using the proposed configuration of the NO\(\nu\)A experiment. In that study it was assumed the use of two liquid argon detectors of 50 kton each, one placed off-axis at 810 km (the proposed site for NO\(\nu\)A) and the other one off-axis at 200 km (or 434 km), which was named Super-NO\(\nu\)A. It was shown that the best capabilities for determining the type of neutrino mass hierarchy were achieved for the case of a near off-axis detector placed at the shortest baseline, i.e., 200 km, for which the differences in the matter effects with respect to the far detector were larger, and consequently so was the sensitivity to this parameter. Although the development of the liquid argon technology is already quite advanced and has been successfully proven to work in small prototypes \[23\], it is reasonable to think that the timelines for the NO\(\nu\)A experiment might not allow
to build such a detector at the far site. Hence, in the present study, keeping the essence of Super-NOνA, we consider
the use of the currently proposed detector \(27\) at the far site, that is, a low density tracking calorimeter of 30 kton
with an efficiency of 24\%, and the possibility of building a second off-axis detector at 200 km.

For the detector technologies we consider water-Čerenkov and liquid argon and study the case of different sizes.
For the purposes of this study, we consider flat efficiencies with no energy dependence, \(\epsilon\), and backgrounds coming
only from the beam. Consequently, one of the relevant statistical parameters is what we define as the efficient mass of
the detector, \(M_{\text{efc}} = M_{\text{det}} \times \epsilon\), where \(M_{\text{det}}\) is the fiducial mass of the detector. The proposed NOνA detector
has \(M_{\text{det}} = 7.2\) kton. We will consider the medium energy configuration of the NuMI beam and \(6.5 \times 10^{20}\) protons
on target per year (pot/yr) \(27\). In addition, we propose to sequence the experiment such that the first phase uses
only the far detector as in the NOνA proposal \(27\). In the second phase, the second detector, the near off-axis one,
is already constructed and starts taking data. The choice of adding the second detector should be guided by the
knowledge of the value of the mixing angle \(\theta_{13}\). Such information can be provided by the NOνA experiment itself
in the first years of running \(74\) and by other experiments which are expected to run previously \(25\) or at the same
time \(26\). If the angle \(\theta_{13}\) is beyond the reach of the NOνA experiment, adding a second detector would not be useful
in determining the type of neutrino mass hierarchy or CP-violation. Conversely, for a sizable value of \(\theta_{13}\), we show
that constructing a near off-axis detector with the same \(L/E\) would allow to determine \(\text{sgn}(\Delta m_{31}^2)\), (almost) free of
degeneracies.

The NOνA experiment itself has the capability to probe values of \(\theta_{13}\) down to \(\sin^2 \theta_{13} \sim 0.02\), within the first 3
years of running \(74\). Therefore, we consider first only the NOνA far detector and 3 years of neutrino run plus 3
years of antineutrino run. Assuming that a positive signal for \(\nu_\mu \rightarrow \nu_e\) conversion is found after the first neutrino run,
signaling \(\theta_{13}\) in the interesting range of values, we assume that the second detector is constructed and starts taking
data immediately after the antineutrino running period.

Following these general guidelines we have considered two different scenarios. In scenario I (SciI), without proton
driver, after the first 6 years with only the NOνA detector, we assume other 6 years of neutrino run plus 2 years of
antineutrino run. We do this for different values of \(M_{\text{efc}} = 17.5, 35, 70, 13.5, 27, 45\) kton, where the first three values
would account for a water-Čerenkov detector (70\% efficiency) of 25, 50 and 100 kton, whereas the last three values
would account for a liquid argon detector (90\% efficiency) of 15, 30 and 50 kton. In scenario II (SciII), the beam
is upgraded from the beginning with a proton driver (\(25 \times 10^{20}\) pot/yr), which is equivalent to multiply \(M_{\text{efc}}\) by \(\sim 3.85\). During the first 6 years, only the NOνA detector would be taking data. Afterwards, both near and far detectors
would take data for 3 years of neutrino run plus 1 year of antineutrino run. We use the same values of \(M_{\text{efc}}\) as for
SciI. The total number of years in SciI (SciII) is of 14 (10).

The question that we would like to address is what is the minimum efficient mass of the near off-axis detector
required to ensure the resolution of the neutrino mass hierarchy for the full range of values of the CP-violating
parameter \(\delta\) for a given value of \(\theta_{13}\). We have thus performed a \(\chi^2\) analysis for both \(\theta_{13}\) and \(\delta\). For a given value
of the oscillation parameters, we have computed the expected number of electron events, \(N_{\ell,}\) detected at the possible
locations \(\ell = N, F\) (near,far sites). The observable that we exploit, \(N_{\ell,}\), is given by

\[
N_{\ell,\pm} = \int_{E_{\text{min}}}^{E_{\text{max}}} \Phi_{\ell,\nu}(E_{\nu}, L) \sigma_{\nu}(E_{\nu}) P_{\nu}(E_{\nu}, L, \theta_{13}, \delta, \Delta m_{31}^2, \alpha) \, dE_{\nu}
\]

(3.1)

where the sign (+ (−)) applies for the normal (inverted) hierarchies and \(\alpha\) is the set of remaining oscillation parameters:
\(\theta_{23}, \theta_{12}, \Delta m_{21}^2\) and the matter parameter \(A\) (which depends on the baseline under consideration), which are taken to
be known; \(\Phi_{\ell,\nu}\) denotes the neutrino flux, \(\sigma_{\nu}\) the relevant cross sections and \(P_{\nu}\) the \(\nu_\mu \rightarrow \nu_e\) conversion probability.
The convolution of these three magnitudes are then integrated over a narrow energy window of 1 GeV, where \(E_{\text{min}}\)
and \(E_{\text{max}}\) refer to the lower and upper energy limits, respectively. We have divided the total number of events in two
bins of equal width \(\Delta E_{\nu} = 0.5\) GeV \(6\).

For our analysis, unless otherwise stated, we will use a representative value of \(|\Delta m_{31}^2| = 2.4 \times 10^{-3}\) eV\(^2\), which lies
within the best-fit values for the Super-Kamiokande \(5\) and K2K \(4\) experiments. However, we will also present how the
\(\chi^2\) analysis results depend on the value of the atmospheric mass difference \(|\Delta m_{31}^2|\). For the remaining oscillation
parameters, \(\theta_{23}, \Delta m_{21}^2\) and \(\theta_{12}\), we will use the best fit values quoted in the introduction.

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\(6\) This is a very conservative estimate for the neutrino energy resolution, which in the range of interest is \(\Delta E_{\nu}/E_{\nu} \sim 50\%\).
FIG. 1: 90%, 95% and 99% C.L. contours resulting from the simultaneous extraction of $\sin^2 2\theta_{13}$ and $\delta$. The true value that we have assumed is $\sin^2 2\theta_{13} = 0.09$ and $\delta = 285^\circ$ (denoted by a star). The top panel shows the results for the analysis of data from the NO$\nu$A far detector. The blue lines denote the 90%, 95% and 99% C.L. contours resulting from the analysis of the data assuming normal hierarchy. If the data analysis is performed with the opposite sign of the atmospheric mass splitting, fake solutions associated to the wrong choice of the neutrino hierarchy appear, shown as cyan contours. The medium panel shows the results from the analysis of the data at a 15 kton liquid argon detector located at 200 km from the neutrino source with magenta contours. Again fake solutions are depicted with cyan contours. The bottom panel shows the results from the combination of the data from the two detectors with black contours. The case considered is that of scenario I (ScI) for which the far detector is assumed to take data during 9 years of neutrino running and 5 years of antineutrino running, while the near detector is assumed to take data during 6 years of neutrino running and 2 years of antineutrino running. The total number of years would be 14. Let us notice that the choice of the true value of the $\delta$ phase is the least favorable one. For different values of $\delta$ a better sensitivity would be achieved.

IV. DETERMINATION OF THE TYPE OF HIERARCHY AND OF CP–VIOLATION

For the progress in the studies of neutrino physics it is of fundamental importance measuring the value of the small $\theta_{13}$ angle, determining the type of neutrino mass hierarchy and searching for possible CP–violation in the neutrino sector. Possibly, the only experiments available to be sensitive to all these three parameters in the near future are long-baseline experiments. However, the chances for them to succeed depend upon the value of $\theta_{13}$. If the mixing angle $\theta_{13}$ is out of their sensitivity reach, then next generation of neutrino experiments would be needed, such as neutrino factories [29, 30], $\beta$-beams or super-beams [31, 32, 33, 34] or electron-capture facilities [35]. In this study we assume that the value of $\theta_{13}$ is within the sensitivity of a long-baseline experiment like NO$\nu$A and of future reactor neutrino experiments [28], that is $\sin^2 2\theta_{13} \gtrsim 0.01$. With this assumption, we tackle the problem of resolving the type of neutrino mass hierarchy and determining CP–violation by considering the experimental setup described above. We show that the improvement with respect to the current NO$\nu$A proposal is truly remarkable.

We first explore what is the minimal scenario which could provide the mass hierarchy resolution for different values
of $\sin^2 2\theta_{13}$. For doing this, we have performed a $\chi^2$ analysis of the data in the $(\sin^2 2\theta_{13}, \delta)$ plane. We assume Nature has chosen the normal or inverted hierarchy and we attempt to fit the data to the expected number of events for the opposite hierarchy. Generically one expects two fake solutions associated with the wrong choice of the hierarchy at fixed neutrino energy and baseline. The $\chi^2$ function in the combination of two baselines and of the neutrino and antineutrino channels reads

$$\chi^2_{\ell\ell'} = \sum_{\ell\ell'} \sum_{\nu=e^+,e^-} (N_{\ell,\pm} - N_{\ell',\pm}) C^{-1}_{\ell\ell'} (N_{\ell',\pm} - N_{\ell',\pm}),$$

where the $+$ ($-$) sign refers to normal (inverted) hierarchy and $C$ is the covariance matrix, which for the particular analysis considered in the present study, contains only statistical errors. Systematic errors due to the solar neutrino parameters can be safely neglected, while those for the atmospheric mixing parameters $\Delta m^2_{31}$ and $\sin^2 2\theta_{23}$ will be at the level of 5% and 2% respectively when NO$\nu$A is expected to start taking data. On the other hand, the input particle production spectra from which the neutrino ones at the detectors are simulated are only known to about the 20% level at present. Before NO$\nu$A starts taking data, we expect the MIPP experiment to improve this knowledge to about the 5% level [27]. By using the two detectors, whose flux can be related by simple kinematics in a Monte Carlo simulation, this systematic error would be further reduced. Our knowledge of CC cross sections represents another important source of systematic errors. For the energy range of interest, the uncertainties amount to about 20-30% at present. However, during the next few years, experiments like K2K [77], MiniBooNE [78] and Minerva [79] will reduce all these errors substantially, to the 5% level. Systematic errors related to the detectors are expected to be smaller [80]. The impact of the uncertainties related to the beam and cross sections has been found analytically to be negligible compared to the uncertainties in the oscillation parameters. All in all, a detailed analysis with a full simulation of these errors has not been yet performed [27] and would be required. Nevertheless, we are confident on the fact that the impact of systematic errors should not change our basic conclusions.
FIG. 3: The same as in Fig. 1 but for \( \sin^2 2\theta_{13} = 0.058 \) and \( \delta = 300^\circ \). The near detector has been upgraded to a 50 kton water-Cherenkov detector.

The experimental “data”, \( N_{\ell,\pm} \), are given by

\[
N_{\ell,\pm} = \langle N_{\ell,\pm} + N_{b\ell} \rangle - N_{b\ell,\pm},
\]

where we have considered that the efficiencies are flat in the visible energy window, \( N_{b\ell} \) are the background events and \( \langle \rangle \) means a Gaussian/Poisson smearing (according to the statistics).

We start the analysis by assuming that the true value of \( \sin^2 2\theta_{13} \), that is, the value that Nature has chosen, is close to its present upper bound. We depict in Fig. 1 the 90%, 95% and 99% C.L. contours resulting from the simultaneous extraction of \( \sin^2 2\theta_{13} \) and \( \delta \). The true values that we have assumed are \( \sin^2 2\theta_{13} = 0.095 \) and \( \delta = 285^\circ \) and the point in the \((\sin^2 2\theta_{13}, \sin \delta)\) plane is denoted by a star. The top panel shows the results for the analysis of data from only the NO\( \nu \)A far detector. For this particular value of \( \sin^2 2\theta_{13} \) we have considered the scenario I (ScI), with no proton driver. Therefore, the statistics at the far detector corresponds to 9 years of neutrino plus 5 years of antineutrino running. The blue lines denote the 90%, 95% and 99% C.L. contours resulting from the analysis to the data assuming NH. If the data analysis is performed with IH, a fake solution associated to the wrong choice of the neutrino mass hierarchy appears. We have depicted these sign(\( \Delta m^2_{31} \))-degeneracies by cyan contours. Notice, from the results depicted if Fig. 1 that the future NO\( \nu \)A experiment (with just one far detector) might not be able to determine the neutrino mass hierarchy if Nature has chosen the central values in Fig. 1 even if the statistics is increased by more than a factor of two.

The medium panel shows the results from the analysis of the data at a near detector located at 200 km from the neutrino source with magenta contours. The off-axis location of the near detector is chosen to ensure the same \( L/E \) at the near and at the far detectors. The detector considered in this first example is a modest one, i.e., a 15 kton liquid argon TPC and we assume 6 years of neutrino and 2 years of antineutrino data taking in the near detector. All in all, the total number of years of neutrino and antineutrino running would be 14, 6 with only the far detector plus 8 with both detectors (ScI). The degeneracies related to the wrong choice of the atmospheric mass squared difference are
FIG. 4: The same as in Fig. 1 but for $\sin^2 2\theta_{13} = 0.043$ and $\delta = 315^\circ$. The near detector has been upgraded to a 100 kton water-Čerenkov detector.

again depicted with cyan contours. Notice that the second detector by itself does not give any better results than the far detector. Synergy effects when combining the data of both detectors have a remarkable outcome. This is shown in the bottom panel, where this combination is depicted with black contours. It is clear from the results shown in Fig. 1 that the combination of these two setups (NO$\nu$A as the far detector plus a 15 kton liquid argon as the near detector with the same $L/E$ at both sites) could resolve the neutrino mass hierarchy if $\sin^2 2\theta_{13} > 0.09$. Let us notice that in Fig. 1 we have only illustrated the case for $\delta = 285^\circ$ because this is the value of $\delta$ which gives the worst resolution of $\text{sgn}(\Delta m^2_{31})$ for the value of $\sin^2 2\theta_{13}$ assumed. Hence, it is important to realize that, when one explores the full range of $\delta$, there exist some values of the CP–violating phase $\delta$ for which the sign of the atmospheric mass difference could be resolved also in a more modest scenario, i.e., a scenario with fewer years of data taking, or a scenario with a smaller detector, or a combination of both cases. It might certainly happen that Nature is kind enough to have chosen a different value for the CP–violating phase. In that case, the determination of the type of neutrino mass hierarchy would be much easier. Here we present the most pessimistic case. On the other hand, another experiment which may improve the results depicted in Fig. 1 is MINOS, which could be sensitive to $\sin^2 2\theta_{13} > 0.08$ if the total number of protons on target achieves a value of $16 \times 10^{20}$.

The next step in the study we are presenting here is to assume that the true value of $\sin^2 2\theta_{13}$ is smaller than the one considered in Fig. 1 but still within the sensitivity reach of the NO$\nu$A experiment. We show in Fig. 2 the results for the same combination of two detectors with the same $L/E$ (that is, NO$\nu$A far detector and a 15 kton liquid argon TPC near detector) for $\sin^2 2\theta_{13} = 0.076$. The plots are qualitatively very similar to those of Fig. 1 but

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7 The value of $\delta$ assumed in the rest of the figures is the one which gives the worst resolution of the type of neutrino mass hierarchy for each value of $\sin^2 2\theta_{13}$. 
FIG. 5: The same as in Fig. 4 but for \( \sin^2 2\theta_{13} = 0.019 \) and \( \delta = 300^\circ \). The near detector has been upgraded to a 100 kton water-Čerenkov and the case exploited is that of scenario II (ScII) with a proton driver. The far detector is assumed to take data during 6 years of neutrino running and 4 years of antineutrino running, while the near detector is assumed to take data during 3 years of neutrino running and 1 year of antineutrino running. The total number of years equals 10.

as is evident from the bottom panel, in this case, when combining the data at the two detectors, the fake solution remains present, although only at the 99\% C.L. In this case, the value of \( \delta \) for which the resolution of the type of neutrino mass hierarchy is most difficult is \( \delta = 300^\circ \). Again, let us point out that this happens for this specific value of \( \delta \). Hence, for any other value of the CP–violating phase, the results would be even better.

If \( \sin^2 2\theta_{13} < 0.07 \), in order to eliminate the fake solutions, it is necessary to increase the mass of the near detector. We show the results for a true value of \( \sin^2 2\theta_{13} = 0.058 \) in Fig. 3 and \( \delta = 300^\circ \), where we have upgraded the modest 15 kton near detector to a 50 kton water-Čerenkov detector \( (M_{efc} = 13.5 \) and 35 kton, respectively) and for \( \sin^2 2\theta_{13} = 0.043 \) and \( \delta = 315^\circ \) in Fig. 4 where the near detector has been upgraded to a 100 kton water-Čerenkov detector \( (M_{efc} = 70 \) kton). We should notice here that for other values of the CP–violating phase \( \delta \) a smaller detector would provide the statistics required for resolving the type of hierarchy. This will be illustrated below by the use of exclusion plots in the \( (\sin^2 2\theta_{13}, \delta) \) plane.

As we have summarized in the description of the experimental configuration, ScI does not include a proton driver. It turns out that for values of \( \sin^2 2\theta_{13} < 0.04 \) the efficient mass of the detector required would exceed \( M_{efc} = 70 \) kton which might be a too demanding solution. A more practical approach to resolve the \( sgn(\Delta m^2_{31}) \)-degeneracies would be to upgrade the experimental setup from ScI to scenario II (ScII, with a Proton Driver), which might constitute a more feasible option that would provide the statistics required for small values of \( \sin^2 2\theta_{13} \). We present the results for \( \sin^2 2\theta_{13} = 0.019 \) and \( \delta = 300^\circ \) in Fig. 5. The top panel shows the results of the data at the far detector, which within the ScII scheme would take data for 6 years in the neutrino plus 4 years in the antineutrino channel. The medium panel shows the results for the near detector, a 100 kton water Čerenkov detector, which would take data for 3 years in the neutrino channel plus 1 year in the antineutrino channel. The total number of years of running in the ScII strategy would be 10. We can see from the bottom panel that for this scenario, \( sgn(\Delta m^2_{31}) \) could be
determined at 99% C.L. for $\sin^2 2\theta_{13} \simeq 0.02$ for any value of the CP–violating phase $\delta$. Finally, we show in Fig. 5 the same combination of near and far detectors as in Fig. 5 and with a proton driver, but for $\sin^2 2\theta_{13} = 0.011$ and $\delta = 300^\circ$. It gives similar results, but as can be noticed from the bottom pannel, when adding the data at both detectors, the fake solution remains at the 99% C.L.

We summarize these results with the exclusion plots in Figs. 7 and 8, where we show the fraction of $\delta$ as a function of $\sin^2 2\theta_{13}$, for which the sign of the atmospheric mass squared difference can be determined at 95% C.L. in ScI and ScII, respectively. The left panel of both figures shows the results for liquid argon detectors for three different masses: 15 kton (upper solid magenta line), 30 kton (dotted magenta line) and 50 kton (dashed magenta line). The right panel shows the results for the case of three water-Čerenkov detectors: 25 kton (upper solid blue line), 50 kton (dotted blue line) and 100 kton (dashed blue line). In each panel it is also shown what would be the result if there was no second detector and all data was collected by the proposed NOνA far detector (lower black curve). It is evident from the plots that adding the second detector helps considerably and that even a 15 kton liquid argon detector would improve the capabilities of the experiment enormously. It is also clear from the figures that if $\sin^2 2\theta_{13} < 0.04 \sim 0.05$ the use of a proton driver is crucial. Running with a proton driver and with a second detector, placed at the same $L/E$ as the far detector, would make the determination of the type of mass hierarchy feasible for values of $\sin^2 2\theta_{13}$ as small as 0.02 and for a substantial range of values of the CP–violating phase $\delta$.

Once we have studied in detail the extraction of the type of the neutrino mass spectrum in our approach, we analyze the reach in searching for CP–violation. In this case, we have only considered ScII, as a larger neutrino flux is required. As can be seen by comparing the top and middle panels with the bottom panel in Figs. 1, 4, resolving the mass hierarchy removes one of the existing degeneracies, increasing the sensitivity to CP–violation. Whereas without the second detector almost none of the values of $\delta$ can be excluded (making the determination of CP–violation challenging) by adding the second detector, only the region related to the true hierarchy is left. To be quantitative, we show in Fig. 9 as a function of $\sin^2 2\theta_{13}$, the minimum value of $\delta$ at which the error at 95% C.L. reaches $\delta = 0$, and therefore the CP–violating case is indistinguishable from the CP-conserving one, i.e., $\delta = 0$. Below the curves
FIG. 7: The fraction of values of $\delta$ as a function of $\sin^2 2\theta_{13}$ for which the resolution of the neutrino mass hierarchy at 95% C.L. is achieved, assuming that Nature has chosen $\Delta m_{31}^2$ to be positive but the analysis of the data is performed with the opposite sign. The number of protons on target per year is $6.5 \times 10^{20}$ and the statistics considered here consists of 9 years of neutrino running plus 5 years of antineutrino running for the far detector and 6 years of neutrino running plus 2 years of antineutrino running for the near site (Sci). The total number of years of running would be 14. The lower solid black line corresponds to the case of the proposed NO$\nu$A far detector alone. (a) For liquid argon detectors in the near site: 15 kton (upper solid magenta line), 30 kton (dotted magenta line) and 50 kton (dashed magenta line). (b) For water-Čerenkov detectors at the near site: 25 kton (upper solid blue line), 50 kton (dotted blue line) and 100 kton (dashed blue line).

it is impossible to tell the difference between the CP-violating and the CP-conserving cases, whereas above them CP-violation could be stated at 95% C.L. As in Figs. 7 and 8, the left panel depicts the results for three liquid argon detectors and in the right panel, those for the case of three water-Čerenkov detectors are plotted. In each panel it is also shown what would be the result if there was no second detector and all data was collected by the proposed NO$\nu$A far detector (upper black curve). As an indication, we have only shown the results for the first quadrant of $\delta$. From the figure we see that for values of $\sin^2 2\theta_{13} < 0.04$ the sensitivity to CP-violation decreases quite fast, which is related to the fact that the mass hierarchy cannot be resolved for a larger range of values of $\delta$. Nevertheless, it is evident from the figure that if $\theta_{13}$ is large enough, by using the proposed configuration, CP-violation could be established, e.g., for the modest case of a 15 kton liquid argon second detector if the true $\delta \gtrsim 45^\circ$ and $\sin^2 2\theta_{13} \gtrsim 0.04$.

Finally, let us comment that throughout this work we are assuming maximal mixing in the atmospheric sector. If this turns out not to be the true solution, another degeneracy would show up due to the inability to distinguish the octant where the mixing angle $\theta_{23}$ lies. This is due to the fact that current atmospheric and long-baseline neutrino experiments are sensitive to $\sin^2 2\theta_{23}$, but not to $\sin^2 \theta_{23}$. This degeneracy can be broken by future atmospheric neutrino experiments [31] or by neutrino factories making use of both golden and silver channels [30, 58]. We expect this degeneracy not to change substantially our conclusions with respect to the determination of the type of neutrino mass hierarchy, but we do expect a reduction in the sensitivity to CP-violation. In any case, the study of this degeneracy is beyond the scope of this work.

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8 The exclusion lines are symmetric in $\delta$ in the vacuum case.
The same as Fig. 7 but for scenario II (ScII), with a proton driver. The number of protons on target per year in the proton driver scenario is $2.5 \times 10^{20}$. We have considered 6 years of neutrino running and 4 years of antineutrino running in the proposed NOνA far detector and 3 years of neutrino running and 1 year of antineutrino running in the near detector. The total number of years of running would be 10.

A. Dependence on $|\Delta m^{2}_{31}|$

Throughout this work we have been assuming a fixed value for $|\Delta m^{2}_{31}| = 2.40 \times 10^{-3}$ eV$^2$. However, the value of this parameter is known currently with a precision of $\sim 30\%$ at 90% C.L. In addition, from Eq. 2.2, we can see that the value of the asymmetry increases monotonically as the vacuum oscillation phase increases, and therefore the asymmetry is larger for larger $|\Delta m^{2}_{31}|$. This correspondingly means better sensitivity to the type of mass hierarchy the larger the atmospheric mass square difference is. Hence, although a precision at the level of $\sim 5\%$ is expected to be achieved by the time this experiment could turn on, it is very important to investigate the effect of a different value for $|\Delta m^{2}_{31}|$ on the results presented above.

We have performed such a study, in a similar way to Ref. 60. In Fig. 11, we depict the sensitivity to the sign of $\Delta m^{2}_{31}$ for different values of $|\Delta m^{2}_{31}| = (2.0; 2.4; 3.0) \times 10^{-3}$ eV$^2$. We have assumed ScI with a 25 kton water-Cerenkov detector at 200 km. We depict the fraction of $\delta$ for which the type of neutrino mass hierarchy can be determined as a function of $\sin^2 2\theta_{13}$, for three different values of $|\Delta m^{2}_{31}| = 2.0 \times 10^{-3}$ eV$^2$ (dotted line); $2.4 \times 10^{-3}$ eV$^2$ (solid line) and $3.0 \times 10^{-3}$ eV$^2$ (dashed line). As anticipated, the larger the value of $|\Delta m^{2}_{31}|$, the better the sensitivity to $\text{sgn}(\Delta m^{2}_{31})$. As for comparison, for a value of $|\Delta m^{2}_{31}| = 2.0 \times 10^{-3}$ eV$^2$, the results are only slightly better than what is achieved with just the far NOνA detector if $|\Delta m^{2}_{31}| = 2.4 \times 10^{-3}$ eV$^2$. Conversely, if $|\Delta m^{2}_{31}| = 3.0 \times 10^{-3}$ eV$^2$, the sensitivity is at the level of that for $|\Delta m^{2}_{31}| = 2.4 \times 10^{-3}$ eV$^2$ with four times more statistics, i.e., a 100 kton water-Cerenkov detector instead. On the other hand, let us point out that since the CHOOZ bound is weaker for small values of $|\Delta m^{2}_{31}|$, the loss in range for $\theta_{13}$ is not as large as one would naively think from Fig. 11.

Hence, if future atmospheric and long-baseline experiments determine that the actual value of $|\Delta m^{2}_{31}|$ happens to be smaller than the one assumed for this work, a different solution must be considered in order to achieve a comparable sensitivity. A possible solution would be to adopt a larger L/E, which could be accomplished either by considering longer baselines or larger off-axis distances, i.e., smaller energies. Another possibility would be to consider running with the low-energy configuration of the NuMI beam. Nevertheless, these modified experimental setups would imply a reduction of the neutrino flux at the detectors, which would require a detailed analysis to evaluate their actual capabilities. On top of this, if $\theta_{13}$ is very small and $|\Delta m^{2}_{31}|$ is also small, then the construction of the proton driver and possibly a longer neutrino running would be necessary.
FIG. 9: Sensitivity to the determination of CP-violation at 95% C.L. as a function of $\sin^2 2\theta_{13}$ for the first quadrant of $\delta$. It is shown the minimum value of $\delta$ at which the error at 95% C.L. reaches $\delta = 0$, and therefore the CP-violating case is indistinguishable from the CP-conserving one, i.e., $\delta = 0$. (a) For liquid argon detectors: 15 kton (lower solid magenta line), 30 kton (dotted magenta line) and 50 kton (dashed magenta line). (b) For water-Cherenkov detectors at the near site: 25 kton (lower solid blue line), 50 kton (dotted blue line) and 100 kton (dashed blue line). Also depicted is the result if there was no second detector and all data was collected by the proposed at NOνA far detector (upper solid black line).

V. COMPARING DIFFERENT LOCATIONS FOR THE SECOND DETECTOR

In the previous section we have studied the capabilities for determining the type of neutrino mass hierarchy and of CP violation by adding a second detector to the already proposed far detector for the NOνA configuration. The results obtained are for a Super-NOνA-like configuration [60], that is with the second detector placed such that (the peak energy) has the same $L/E$ of the far detector. This choice is based on the findings of Refs. [62, 63] and the study of Ref. [60], where it was shown that such a configuration is specially sensitive to matter effects even with only the neutrino run.

Interestingly, in the NOνA proposal [27], the possibility to add a second detector is pointed out. The approach there consists in placing the second detector at the second oscillation maximum, where the matter effect is smaller by a factor of three because the energy is smaller by the same factor. However, for such a configuration there is no cancellation between the vacuum terms at both detectors, and the CP-violating effects are also different (larger by a factor of three in the second detector for the same reason as above). These facts imply that the matter-dependent terms are of the same order of those carrying information on CP-violation, which makes much more difficult to disentangle the type of mass hierarchy from CP-violating effects. Therefore, a measurement using only a neutrino run would not be as sensitive to the type of neutrino mass spectrum as in the case of Super-NOνA and a running also in the antineutrino mode would be needed in order to achieve a comparable sensitivity. In addition, going to the second oscillation maximum means going more off-axis, and hence losing flux. In this case the loss in number of events amounts to about a factor of 15 with respect to the first detector [9].

In Fig. 11 we compare these two possibilities for placing the second detector. For illustrative purposes, we have assumed for this comparison the same type of detectors as for Super-NOνA [60], i.e., two 50 kton liquid argon TPC. The results depict the cases without proton driver, where solid lines are for 5 years of neutrino running and dashed lines are for 5 years of neutrino plus 5 years of antineutrino running. As was anticipated above, the configuration for

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9 Compare with the Super-NOνA case where in the second detector the number of events is just about half that at the far detector [60].
FIG. 10: Sensitivity to $\text{sgn}(\Delta m^2_{31})$ for different values of $|\Delta m^2_{31}| = 2.0 \times 10^{-3}$ eV$^2$ (dotted line), $2.4 \times 10^{-3}$ eV$^2$ (solid line), and $3.0 \times 10^{-3}$ eV$^2$ (dashed line). We have exploited the data from two off-axis detectors located at 200 km and 810 km by assuming ScI with a 25 kton water-Čerenkov detector at the near site and the proposed NO$\nu$A detector at the far site.

which both detectors are located such that the vacuum oscillation phase is the same (thick solid magenta lines), i.e., the same $L/E$, is much better when running only with neutrinos than that with the second detector at the second oscillation maximum (thin solid blue lines). In order to achieve comparable sensitivities for the determination of the type of neutrino mass hierarchy, the antineutrino run is necessary for the case of NO$\nu$A plus a second detector located at the second oscillation maximum (thin dashed blue line). On the other hand, in the case of Super-NO$\nu$A the antineutrino information does not add any synergy, but just statistics (thick dashed magenta line). Hence, it is important to note that in order to determine $\text{sgn}(\Delta m^2_{31})$ with the second detector at the same $L/E$, the antineutrino run is not really needed.

As it is well known, provided $\sin^2 2\theta_{13}$ is in the range accessible to conventional neutrino beams, the unique contribution of the NuMI neutrino program, and in particular of the NO$\nu$A experiment [27], will be the resolution of the type of neutrino mass hierarchy. Thus, from the results presented in Fig. 11 we learn that the best approach in case the second detector is needed would be the one studied in Ref. [60] and throughout the present paper.

VI. CONCLUSIONS

Determining the type of neutrino mass hierarchy, whether normal or inverted, constitutes one of the fundamental questions in neutrino physics. Future long-baseline experiments aim at addressing this fundamental issue but suffer typically from degeneracies with other CP–conserving and CP–violating parameters, namely $\theta_{13}$, $\delta$ and $\theta_{23}$. The presence of such degeneracies limits the sensitivity to the type of hierarchy as well as to CP–violation. Many studies focus on resolving such degeneracies, e.g. by combining more than one experiment [62, 63, 64, 65, 66, 74], using more than one detector [52, 56, 57, 58, 59, 60], or using information from atmospheric neutrino data [67].

In the present article, we follow the strategy defined in Ref. [60], in which the type of hierarchy is determined free of degeneracies in only one neutrino experiment with two off-axis detectors and by using the neutrino beam alone. The off-axis configurations are chosen in such a way that at the two detectors the ratio $L/E$ is the same, and so the vacuum oscillation phases. Comparing the probabilities of neutrino conversion at the two distances, the vacuum term (hence the dominant CP–violating one) cancels out in the difference [60, 62]. The normalized difference $D$ depends only on matter effects and its sign is determined by the type of neutrino mass hierarchy. CP–violating terms can give relevant contributions only for small values of $\sin^2 2\theta_{13}$. This implies that the determination of the type of neutrino
FIG. 11: Comparison of the capabilities for determining $\text{sgn}(\Delta m^2_{31})$ for two different setups for the second detector: at the same \( L/E \) as the far detector (thick lines) and at the second oscillation maximum (thin lines). For illustrative purposes, we have assumed two 50 kton liquid argon TPC detectors, both at the far and at the near sites (see Ref. [60]) without proton driver. We have considered 5 years of neutrino running (solid lines) and 5 years of neutrino plus 5 years of antineutrino running (dashed lines).

mass spectrum, exploiting this method, suffers no degeneracies from other parameters. In the Super-\(\nu\)A proposed configuration, two off-axis 50 kton liquid argon detectors, one at a far site and one at a shorter distance \( \sim 200 \text{ km} \), were used. Even if the construction of small liquid argon detectors has proven to be possible, it might be difficult to build the required large liquid argon detectors on the \(\nu\)A timescale. Here we have considered a more realistic proposal, in which at the far site the \(\nu\)A currently proposed detector [27] is used. The second detector should be added off-axis at about 200 km. We study both liquid argon and water-\(\bar{C}\)erenkov techniques and different sizes for the detectors. Even if establishing the type of hierarchy with this setup does not require antineutrinos, here we consider both neutrino and antineutrino running modes and we analyze the capabilities of determining CP–violation as well.

We have proposed to sequence the experiment. In the first phase, the experiment would have just the far off-axis detector as in the \(\nu\)A proposal [27]. In the second phase, the near off-axis detector is added at 200 km. We have considered two different scenarios. In scenario I (ScI) the experiment runs with the NuMI beam for 6 years as in the \(\nu\)A proposal and then with a second off-axis detector 6 years in the neutrino mode and 2 with antineutrinos. In scenario II (ScII) the beam is upgraded by the use of a proton driver and the number of years of data taking with both detectors is reduced to 3 and 1 for neutrinos and antineutrinos, respectively. We have performed an \(\chi^2\) analysis of the simulated data on the \((\sin^2 2\theta_{13}, \delta)\) plane. In Figs. 1–6 for different values of \(\sin^2 2\theta_{13}\) and different type and mass of detectors, we show the capabilities of different experimental setups to measure \(\sin^2 2\theta_{13}\), \(\delta\) and determine the type of hierarchy. The values of \(\delta\) chosen are those that give the worst results, so they represent the most pessimistic case. In particular, for each set of parameters, we depict the sensitivity when each detector, far and near off-axis, is considered alone, and we show how the degeneracies can be lifted when the data at the two locations are combined. Obviously, the smaller the value of \(\sin^2 2\theta_{13}\), the larger the required effective mass of the second detector. In Figs. 7 and 8 we compare the reach of different choices for the second detector, for ScI and ScII, respectively. We depict the fraction of values of \(\delta\) as a function of \(\sin^2 2\theta_{13}\) for which the type of hierarchy can be determined at 95% C.L. For ScI, we see that adding a modest detector, such as a 15 kton liquid argon or 25 kton water-\(\bar{C}\)erenkov detector, would yield a substantial improvement with respect to the performance with just the proposed \(\nu\)A detector. For example, the fraction of \(\delta\) values is \(\sim 85\% (\sim 65\%)\) at \(\sin^2 2\theta_{13} = 0.1(0.06)\), while in the case of the \(\nu\)A configuration alone such fraction decreases to \(\sim 40\% (\sim 25\%)\). The construction of a larger detector, such as a 100 kton water-\(\bar{C}\)erenkov one, would have even a more dramatic impact, raising the fraction of \(\delta\) to 100% for \(\sin^2 2\theta_{13} \gtrsim 0.075\) and to 70% even...
for $\sin^2 2\theta_{13} \gtrsim 0.04$. In ScII, the higher flux would allow the determination of $sgn(\Delta m_{23}^2)$, independently of $\delta$, for $\sin^2 2\theta_{13} \gtrsim 0.06$ even for a small detector. More remarkable results would be obtained for larger detectors or longer data-taking time.

Thanks to the years of antineutrino run, the discussed experimental setup would also allow to search for CP-violation. We show the sensitivity to CP-violation at 95% C.L. in Fig. 10 for different types of second off-axis detector. Obviously, also in the case of CP-violation, the sensitivity increases with statistics and improves greatly with respect to the proposed NO$\nu$A experiment.

From our analysis it is clear that the choice of building the second detector should be guided by the knowledge of the value of $\sin^2 2\theta_{13}$, provided by NO$\nu$A itself and by other experiments [22, 24, 25]. If $\sin^2 2\theta_{13} < 0.01$–0.02, no sensitivity on the type of mass spectrum could be reached and the construction of a second detector would not improve it significantly. On the contrary, if $\sin^2 2\theta_{13} \gtrsim 0.02$, a sufficiently large second detector would guarantee the determination of the type of hierarchy (almost) independently of the value of $\delta$. The actual type and mass of the second off-axis detector should be chosen on the basis of the measured value of $\sin^2 \theta_{13}$. As the sensitivity to $sgn(\Delta m_{23}^2)$ depends also on the unknown value of $\delta$, the best strategy would be to build a detector for which the determination of the type of hierarchy can be achieved for a large fraction of values of the CP-violating phase, e.g. 60%–70%. In case we are lucky and Nature has chosen a favorable value of $\delta$, the above discussed number of years of data taking, if not less, would allow to answer this question. Otherwise additional years of neutrino run would be needed to resolve this issue.

In addition, as discussed in Ref. [60], the reach of this experimental setup depends on the value of $|\Delta m_{31}^2|$. In Fig. 11 we also show the sensitivity to $sgn(\Delta m_{31}^2)$ for three values of $|\Delta m_{31}^2| = (2.0; 2.4; 3.0) \times 10^{-3}$ eV$^2$. The sensitivity decreases for lower values of $|\Delta m_{31}^2|$ and larger statistics might be required if $|\Delta m_{31}^2|$ lies in the low side of the presently allowed range [6], depending on the true value of $\delta$.

On the other hand, in the recent NO$\nu$A proposal the possibility of adding a second off-axis detector at the second maximum has been considered. Let us notice that the flux at the second location is greatly suppressed by the large off-axis angle, though. In our proposal the reduction in flux is not as dramatic as the second detector should be located at a much shorter baseline, $\sim 200$ km. In Fig. 11 we compare the capabilities of the two experimental setups in determining the type of hierarchy. Considering 5 years of only neutrino running, the Super-NO$\nu$A-like configuration discussed in the present article could resolve the neutrino mass ordering for any value of $\delta$ for $\sin^2 2\theta_{13} \gtrsim 0.04$. The other experimental setup, with the second detector at second oscillation maximum, cannot fully lift the degeneracies for $\sin^2 2\theta_{13} < 0.095$. The reach of the two setups become comparable only when additional 5 years of antineutrino running are considered. Having in mind that the unique contribution of the NO$\nu$A experiment is to determine the type of neutrino mass spectrum, we consider that the the best approach in case the second detector is needed would be the one studied in Ref. [60] and throughout the present paper.

In conclusion, following the strategy illustrated in ref. [60], we have studied different experimental setups for a sequenced off-axis experiment which could achieve considerably better sensitivity to the type of neutrino mass hierarchy with respect to the proposed NO$\nu$A experiment. The problem of degeneracies is weakened and for a sufficiently large second off-axis detector completely solved. The capabilities of measuring the value of the CP-violating phase $\delta$ have also been analyzed in detailed and have been shown to be greatly improved. Let us stress again that, for $\sin^2 2\theta_{13}$ in the range accessible to conventional neutrino beams, the main breakthrough of the NuMI neutrino program, and in particular of the NO$\nu$A experiment [27], will be the resolution of the type of neutrino mass hierarchy. This goal should be achieved in the most efficient and fast possible way. Here we have presented a very powerful approach to this remarkable problem.

VII. ACKNOWLEDGMENTS

We would like to thank A. Para, S. Parke and A. Rubbia for discussions. SPR thanks the Theory Division at CERN and the Department of Theoretical Physics of the University of Valencia for hospitality during his stay. SP is also grateful for the hospitality of the Fermilab Theoretical Physics Department during her visit. Our calculations made extensive use of the Fermilab General-Purpose Computing Farms [32]. SPR is supported by NASA Grant ATP02-0000-0151 and by the Spanish Grant FPA2002-00612 of the MCT. Fermilab is operated by URA under DOE contract DE-AC02-76CH03000.
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