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ZFITTER: a semi-analytical program for fermion pair production in e^+e^- annihilation, from version 6.21 to version 6.42

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Abstract

ZFITTER is a Fortran program for the calculation of fermion pair production and radiative corrections at high energy e^+e^- colliders; it is also suitable for other applications where electroweak radiative corrections appear. ZFITTER is based on a semi-analytical approach to the calculation of radiative corrections in the Standard Model. We present a summary of new features of the ZFITTER program version 6.42 compared to version 6.21. The most important additions are: (i) some higher-order QED corrections to fermion pair production, (ii) electroweak one-loop corrections to atomic parity violation, (iii) electroweak one-loop corrections to $\bar{\nu}_e\nu_e$ production, (iv) electroweak two-loop corrections to the W boson mass and the effective weak mixing angle.

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New version summary

Title of program: ZFITTER version 6.42 (18 May 2005)

Authors of original program: D. Bardin, P. Christova, M. Jack, L. Kalinovskaya, A. Olchevski, S. Riemann, T. Riemann

Program obtainable from:
http://www-zeuthen.desy.de/theory/research/zfitter/ (main web site),
/afs/cern.ch/user/b/bardindy/public/ZF6_42

Reference for ZFITTER version 6.21 D. Bardin et al., Comput. Phys. Commun. 133 (2001) 229–395

Operating system: UNIX/LINUX, program tested under, e.g., HP-UX and PC/Linux

Programming language used: FORTRAN 77

High speed storage required: < 2 MB

No. of cards in combined program and test deck: about 26,200

Keywords: Quantum electrodynamics (QED), Standard Model, electroweak interactions, heavy boson Z, e^+e^--annihilation, fermion pair production, radiative corrections, initial state radiation (ISR), final state radiation (FSR), QED interference, SLD, LEP, ILC.

Nature of the physical problem: Fermion pair production is an important reaction for precision tests of the Standard Model, at LEP/SLC and future linear colliders at higher energies. For this purpose, QED, electroweak and QCD radiative corrections have to be calculated with high precision, including higher order effects. Multi parameter fits used to extract model parameters from experimental measurements require a program of sufficient flexibility and high calculational speed. ZFITTER combines these two aspects by employing analytical integrations of matrix elements and at most one one dimensional numerical integration, as well as a variety of flags defining the physics content used. The calculated predictions are typically at the per mille precision level, sometimes better.

Method of solution: Numerical integration of analytical formulae.

Restrictions on the complexity of the problem: Fermion pair production is described below the top quark pair production threshold. Photonic corrections are taken into account with simple cuts on photon energy, or the energies and acollinearity of the two fermions, and *one* fermion production angle. The treatment of Bhabha scattering is less advanced.

Typical running time: On a Pentium IV PC installation (2.8 GHz) using

g77 under Linux 2.4.21, approximately 23 sec are needed to run the standard test of subroutine `ZFTEST`. This result is for a *default/recommended* setting of the input parameters, with *all* corrections in the Standard Model switched *on*. `ZFTEST` computes 12 cross-sections and cross-section asymmetries for 8 energies with 5 interfaces, i.e. about 360 cross-sections in 23 seconds.

1 Introduction

The Fortran program ZFITTER is based on a semi-analytical approach to the calculation of fermion pair production in e^+e^- annihilation at a wide range of centre-of-mass energies, including SLC/LEP1, LEP2, and ILC energies below the $t\bar{t}$-threshold. ZFITTER allows the calculation of several quantities needed for precision studies of the Standard Model:

- M_W – the W boson mass;
- Γ_Z, Γ_W, – total (and also partial) Z and W boson decay widths;
- $d\sigma/d\cos\vartheta$ – differential cross-sections;
- σ_T – total cross-sections;
- A_{FB} – forward-backward asymmetries;
- A_{LR} – left-right asymmetries;
- A_{pol}, A_{FB}^{pol} – final state polarisation effects for τ leptons.

All observables are calculated including radiative corrections using the (running) fine structure constant α, the muon decay constant G_μ, the Z mass M_Z as well as the fermion masses and the Higgs mass M_H as input.

ZFITTER version 6.21 and higher was mainly intended for the use at LEP1/SLC energies, and versions since 6.30 are also adapted to the LEP2 kinematics. Various interfacing subroutines (short: interfaces) allow the user to calculate observables, e.g., for fits to the experimental data with different sets of free parameters.

The Fortran package DIZET, a library for the calculation of electroweak radiative corrections, is part of the ZFITTER distribution. It can also be used in a stand-alone mode. On default, DIZET performs the following calculations:

- by call of subroutine DIZET: W mass, Z and W partial and total decay widths;
- by call of subroutine ROKANC: four weak neutral-current (NC) form factors, running electromagnetic and strong couplings needed for the calculation of effective NC Born cross sections for the production of massless fermions (however, the mass of the top quark appearing in the virtual state of loop diagrams for the process $e^+e^- \to f\bar{f}$ is not ignored);
- by call of subroutine RHOCC: the corresponding form factors and running strong coupling for the calculation of effective CC Born cross sections;
- by call of subroutine ZU_APV: $Q_W(Z,A)$ – the weak charge used for the description of parity violation in heavy atoms.

If needed, the form factors of cross sections may be made to contain the contributions from WW and ZZ box diagrams thus ensuring the correct kinematic behaviour over a larger energy range compared to the Z pole.

ZFITTER version 6.21 [1] was released in July 1999 and was described in [2]. Since then, there were several important developments of the program. The current release is that of ZFITTER version 6.42, dated 18 May 2005. This article describes the changes and additions in ZFITTER from version 6.21 to version 6.42. Other sources of information on ZFITTER and its use are given by the previous program description and the references therein, the ZFITTER webpage [3], the studies [4, 5], the 1999/2000 CERN LEP2 workshop proceedings [6], and the studies of the LEPEWGG (LEP electroweak working group) [7].

The essential changes from ZFITTER v. 6.21 to v. 6.42 in terms of physics topics are:

- Higher order QED corrections to fermion-pair production, of importance at energies off the Z boson peak;
- Electroweak corrections to the weak charge Q_W, describing the parity violation effects in atoms, of importance for so-called global Standard Model fits;
- Electroweak corrections to $\bar{\nu}_e\nu_e$ production, of importance for a precise description of $\bar{\nu}\nu\gamma$ production;
- Electroweak two-loop corrections to M_W and the effective weak mixing angle $\sin^{2,eff}\theta_W$, of importance for global Standard Model fits and for precise predictions of the Higgs mass M_H.

Further, an option to change the strength of the Wtb vertex, $|V_{tb}|$, was implemented in DIZET since version 6.30: $|V_{tb}|$ is now one of the parameters of subroutine DIZET. The default is the Standard Model calculation with $|V_{tb}| = 1$. Several new interface routines deal with this case, see Appendix C for the details.

This program description update is organized as follows. In Sections 2 to 5, we describe the improved or new physics issues. Section 6 contains a description of the input parameters and pseudo observables, and Section 7 reproduces reference outputs from a running of the sample program package. A Summary closes this update note. In Appendices A to C we collect, for the convenience of the user, updated ZFITTER and DIZET user guides and an overview of the user interfaces. The presentation assumes, of course, a familiarity of the reader with [2].

With the present update of ZFITTER and its description, the maintenance of the program has been migrated to a group of volunteers, the ZFITTER support group. The main webpage has been migrated to:
http://www-zeuthen.desy.de/theory/research/zfitter/
The decision to do so was taken together with the original authors of the program. Since we did not want to change the list of authors in view of the long-standing history of the program, the idea to create a ZFITTER support

group was considered to be an appropriate way to handle the maintenance situation requiring permanent care about the program. The ZFITTER support group is composed of authors of ZFITTER version 6.21, long-term users of the program, and colleagues who contributed substantially to its present state. We hope that a kernel of us will stay with ZFITTER as long as the code is needed by the community.

A correct citation of the ZFITTER package will include reference [2], together with the present update.

2 Higher order QED corrections

Measurements at LEP1/SLC were performed in the vicinity of the Z resonance so that many of the photonic corrections were suppressed. Cross sections at energies away from the Z resonance peak have a much stronger dependence on higher order photonic corrections and their inclusion is of numerical importance at LEP2. Several improvements in this respect have been performed since ZFITTER version 6.21 and will be described in the following subsections. They have also been discussed in [6].

2.1 Second order initial state fermion pair corrections

Since ZFITTER version 6.21 there is an improved treatment of second order corrections for angular distributions and A_{FB}. These corrections were applied in the leading logarithmic approximation as described in Ref. [2]. The option is accessed by a new flag:
FBHO $= 0$ – old treatment with photonic corrections only,
FBHO $= 1$ – leading log fermion pair corrections are added.
The non-singlet and singlet fermion pair contributions to the electron (positron) structure function of [8] (see Eq. (11) in [9] and Eq. (47) in [8]) are used (with different options according to the values of the flags ISPP, IPFC and IPSC). They are directly added to the corresponding photonic contributions [10, 11], which are governed by the flag FUNA. For pair corrections in general, the new value of flag IPTO $= -1$ is added. It allows to calculate pure virtual pair contributions separately. This option can be used for comparisons or in the case, when the contribution of real pair emission is taken from another program.

Since ZFITTER version 6.30, the second order initial state radiation (ISR) QED corrections in presence of angular cuts are improved [12]. A new option governed by a new flag FUNA is implemented, with:
FUNA=0 – old treatment,
FUNA=1 – new treatment.
The corrected treatment of these corrections takes the angular acceptance cuts ANGO, ANG1 into account. The corrections are relevant for the angular distribution and for the integrated forward-backward asymmetry. We use the leading logarithmic approximation by means of the electron structure function formalism [8,9]. In fact the differential angular distribution of the electron–positron annihilation process can be represented in a form analogous to that of the Drell–Yan process:

$$\frac{\mathrm{d}\sigma(s,c)}{\mathrm{d}c} = \int_0^1 \mathrm{d}x_1 \mathcal{D}(x_1,L_e) \int_0^1 \mathrm{d}x_2 \mathcal{D}(x_2,L_e) \frac{\mathrm{d}\hat{\sigma}(\hat{s},\hat{c})}{\mathrm{d}\hat{c}} \mathcal{J} \Theta(\hat{s}-s'), \qquad (2.1)$$

with $c = \cos\vartheta$ and

$$\hat{s} = x_1 x_2 s, \qquad (2.2)$$

$$\mathcal{J} = \frac{4x_1x_2}{[x_1+x_2-c(x_1-x_2)]^2}, \qquad (2.3)$$

where the structure functions $\mathcal{D}(x_i,L_e)$ give the probability to find an electron (positron) with a reduced energy fraction $x_{1(2)}$ in the initial electron (positron). The Born–level annihilation cross section $\mathrm{d}\hat{\sigma}(\hat{s},c)/\mathrm{d}\hat{c}$ is defined in the center–of–mass reference frame of electron and positron with reduced energy fractions. The Jacobian $\mathcal{J}$ is coming from the relation to the angles in the laboratory reference frame. The structure functions are taken in the leading logarithmic approximation keeping the first and second order photonic contributions:

$$\mathcal{D}(x,L_e) = \delta(1-x) + \frac{\alpha}{2\pi}(L_e-1)P^{(1)}(x) + \frac{1}{2}\left(\frac{\alpha}{2\pi}(L_e-1)\right)^2 P^{(2)}(x), \qquad (2.4)$$

with

$$L_e \equiv \ln \frac{s}{m_e^2}, \tag{2.5}$$

$$P^{(1)}(x) = \lim_{\Delta\to 0}\left\{\delta(1-x)\left(2\ln\Delta + \frac{3}{2}\right) + \Theta(1-\Delta-x)\frac{1+x^2}{1-x}\right\}, \tag{2.6}$$

$$\begin{aligned} P^{(2)}(x) = \lim_{\Delta\to 0}\Bigg\{ & \delta(1-x)\left[\left(2\ln\Delta + \frac{3}{2}\right)^2 - \frac{2\pi^2}{3}\right] \\ & + \Theta(1-\Delta-x)2\left[\frac{1+x^2}{1-x}\left(2\ln(1-x) - \ln x + \frac{3}{2}\right)\right. \\ & \left. + \frac{1+x}{2}\ln x - 1 + x\right]\Bigg\}. \end{aligned} \tag{2.7}$$

In order to avoid a double counting, we expanded formula (2.1) in α and take only the $\mathcal{O}(\alpha^2)$ terms, which are then added to the full first order corrections. Where possible, we performed integrations over the angle and the energy fractions analytically. This allows to get a relatively fast code for these corrections. The calculation is realized with subroutine `funang.f`. It is compatible with the use of `ICUT` = 1,2,3. We just mention that for the contribution to the total cross section the complete $\mathcal{O}(\alpha^2)$ formulae are used.

2.3 Higher order photonic corrections from the initial-final state interference

The exponentiation of photonic initial-final-state interference corrections was implemented according to Ref. [13]. It allows to take into account the most significant part of higher order corrections coming from the initial-final state interference. The corrections are relevant for the angular distribution and for the integrated forward-backward asymmetry. A combination of the one–loop initial–final interference corrections with the corresponding higher order effects from the exponentiation can be computed in `ZFITTER` by using flag `INTF` = 2. The old options are:
`INTF` = 0: the initial-final state interference in photonic corrections is omitted;
`INTF` = 1: the interference is taken in the one-loop approximation.
The numerical effect of the exponentiation was discussed in Ref. [6]. It was found to be close to the one obtained by a slightly different exponentiation procedure [14].

2.4 Final state fermion pair production corrections

Since `ZFITTER` version 6.30, the corrections from final state radiation (FSR) of fermion pairs are implemented [12], according to the formulae given in [15]. The option is governed by a new flag:

FSPP = 0 – without FSR pairs,
FSPP = 1 – with FSR pairs, additive,
FSPP = 2 – with FSR pairs, multiplicative.
For the best approximation, the FSR pair corrections should be treated multiplicative (FSPP = 2) with respect to the ISR photonic corrections (see Eq. (2.1) in [12]). The additive treatment of FSR pairs (FSPP = 1) can be used for a comparison. For the FSPP corrections, a cut on the invariant mass of the secondary pair is accessible. In order to accommodate this cut value, the variable SIPP of the
SUBROUTINE ZUCUTS(INDF,ICUT,ACOL,EMIN,S_PR,ANG0,ANG1,SIPP)
is now used. The dependence on this cut for realistic event selections was shown to be rather weak [12]. Therefore, the meaning of variable SIPP has been changed. Now it has nothing to do with cutting of the initial state pairs. In fact, there is no possibility to directly cut the initial state secondary pairs. The primary pair invariant mass cut (S_PR) is taken into account in the phase space of secondary pairs.

3 Improved Born approximation for $e^+e^- \to \bar{\nu}\nu$ in ZFITTER version 6.34

For a study of the reaction

$$e^+e^- \to \bar{\nu}\nu(n\gamma), \qquad \nu = \nu_e, \nu_\mu, \nu_\tau \tag{3.1}$$

one needs the effective Born approximation for

$$\frac{d\sigma}{d\cos\vartheta} = \sum_{i=e,\mu,\tau} \frac{d\sigma(e^+e^- \to \bar{\nu}_i\nu_i)}{d\cos\vartheta} = \; 3\;\sigma_s + \sigma_{st} + \sigma_t. \tag{3.2}$$

The specific property of this reaction is due to the interference of s channel Z boson and t channel W boson exchange for $\bar{\nu}_e\nu_e$ production. For an application in KKMC [16], the corresponding formulae have been derived [17] from a related study [18,19] and implemented in ZFITTER version 6.34 (05 Feb 2001). The changes in the program consist of a modification of the weak charged current form factor (variable ROW) in subroutine coscut in zfbib6_34.f. The addition is [1]:

$$\delta_{\mathtt{CC-NC}} = \mathtt{QED\_CC} - \mathtt{QED\_NC} = \frac{\alpha}{2\pi} Q_e^2 \left[\frac{3}{2} \ln\frac{M_W^2}{s} + \frac{1}{2}\ln^2\frac{t}{s} - 4\;\mathrm{Li}_2(1) + 2 \right]. \tag{3.3}$$

[1] In the Fortran code until version 6.42, the factor of 4 at the r.h.s. of Equation (3.3) is lacking.

The variables SIGST and SIGT are influenced by this, they correspond to σ_{st} and σ_t. Depending on the setting of flag ENUE there, part or all of them contribute to the prediction of $d\sigma/d\cos\vartheta$; see also Section B.2.

In ZFITTER version 6.42, the file zfbib6_34.f is replaced by zfbib6_40.f. The latter file contains a subroutine coscut where the described features are not accessible. The file zfEENN_34.f is no longer part of the distribution.

4 Atomic parity violation

The global precision tests of the Standard Model may include an experimental input from atomic parity violation measurements in heavy atoms. The observable quantity of interest is the so-called weak charge Q_W:

$$Q_W(Z,A) = -2\left[(2Z+N)\,C_{1u} + (Z+2N)\,C_{1d}\right]. \tag{4.1}$$

Here, Z and N are the numbers of protons and neutrons in the nucleus. The weak couplings involved are those which parameterize the electron-quark parity-violating Hamiltonian at zero momentum transfer:

$$\begin{aligned} H_{PV} = \frac{G_F}{\sqrt{2}}(&C_{1u}\bar{e}\gamma_\mu\gamma_5 e\bar{u}\gamma_\mu u + C_{2u}\bar{e}\gamma_\mu e\bar{u}\gamma_\mu\gamma_5 u + C_{1d}\bar{e}\gamma_\mu\gamma_5 e\bar{d}\gamma_\mu d \\ &+ C_{2d}\bar{e}\gamma_\mu e\bar{d}\gamma_\mu\gamma_5 d). \end{aligned} \tag{4.2}$$

In Born approximation, $C_{1u} = -\frac{1}{2}(1-8/3\sin^2\theta_W)$ and $C_{1d} = \frac{1}{2}(1-4/3\sin^2\theta_W)$; generally, $C_{1q} = 2a_e v_q$ and $C_{2q} = 2v_e a_q$, $q = u, d$. In [20], using the results of [21], the higher order corrections to atomic parity violation in the on-mass-shell renormalization scheme have been derived and the corresponding expressions were used for ZFITTER version 6.34 (26 Jan 2001) onwards. The formulae given in [22] were also reproduced.

In subroutine ZU_APV the corresponding expressions for $C_{1u}, C_{1d}, C_{2u}, C_{2d}$ are calculated. In fact, the sign conventions have been changed compared to [20] in order to share the definitions of the particle data group [2]. A sample use is prepared with subroutine ZF_APV, with $N = 78$ and $Z = 55$ for Caesium. Flag TUPV may be used for a study of the theoretical uncertainty, the default value is TUPV=1.

[2] We just mention that the signs of Equations (4) in [20] are not in accordance with Equation (3) there, while those of Equation (14) are.

5 Higher order electroweak corrections to Standard Model observables

In ZFITTER version 6.42, all known two- and three loop corrections of the Standard Model observables M_W, $\sin^2\theta_{\rm eff}$ and Γ_z have consistently been included. The corresponding changes began with version 6.33, with subsequent additions until version 6.42. They are described to some detail in this section.

5.1 The W boson mass

ZFITTER, version 6.36 (21 July 2001), contained only the leading and next-to-leading corrections to Δr, obtained through an expansion in the heavy top quark mass. They were applied according to [23–28] and have been implemented in the package m2tcor.f. The W boson mass was then evaluated with the relation:

$$M_W = \frac{M_z}{\sqrt{2}}\sqrt{1+\sqrt{1-\frac{4\pi\alpha}{\sqrt{2}M_z^2 G_\mu\,(1-\Delta r)}}}\,, \tag{5.1}$$

by an iterative procedure, since Δr also depends on M_W. More accurate results on the electroweak corrections to muon decay, which successively appeared in [29–38], were found in terms of exact one-dimensional integral representations at the order $\mathcal{O}(\alpha^2)$. However, since the computation of general two loop integrals is rather slow, only fitting formulas as published in [32, 37] were implemented in ZFITTER. Furthermore, since typical resummation prescriptions for Δr (see [39] and references therein) are problematic once the complete two-loop contributions are included, the new result has been directly applied to M_W.

With the option AMT4 $= 5$ the complete fermionic two loop corrections to the W boson mass [32] are used. Here the fermionic corrections encompass all two-loop contributions with at least one closed fermion loop. In addition to electroweak corrections, this setting also includes the leading $\mathcal{O}(\alpha\alpha_s)$ [40–46] and next-to-leading $\mathcal{O}(\alpha\alpha_s^2)$ [47–49] QCD corrections. Contrary to the settings AMT4 ≤ 4, the $\mathcal{O}(\alpha\alpha_s^2)$ corrections are incorporated exactly according to [49]. Previously, only the leading term in m_t^2 was used.

The assignment AMT4 $= 6$, which is used from Summer 2004 onwards, enables the calculation of M_W including complete (fermionic and bosonic) $\mathcal{O}(\alpha^2)$ corrections [34, 37], $\mathcal{O}(\alpha\alpha_s)$ [40–46] and $\mathcal{O}(\alpha\alpha_s^2)$ [49, 50] QCD corrections, as well as leading three-loop corrections in an expansion in m_t^2 of order $\mathcal{O}(\alpha^3)$ and $\mathcal{O}(\alpha^2\alpha_s)$ [51]. These contributions were implemented into a fitting formula of the following form [37]:

$$M_W = M_W^0 - c_1\,\mathrm{dH} - c_2\,\mathrm{dH}^2 + c_3\,\mathrm{dH}^4 + c_4(\mathrm{dh}-1) - c_5\,\mathrm{d}\alpha + c_6\,\mathrm{dt} \\ - c_7\,\mathrm{dt}^2 - c_8\,\mathrm{dH}\,\mathrm{dt} + c_9\,\mathrm{dh}\,\mathrm{dt} - c_{10}\,\mathrm{d}\alpha_\mathrm{s} + c_{11}\,\mathrm{dZ}, \tag{5.2}$$

where

$$\begin{aligned} \mathrm{dH} &= \log\left(\frac{M_H}{100\ \mathrm{GeV}}\right), & \mathrm{dh} &= \left(\frac{M_H}{100\ \mathrm{GeV}}\right)^2, \\ \mathrm{dt} &= \left(\frac{m_t}{174.3\ \mathrm{GeV}}\right)^2 - 1, & \mathrm{dZ} &= \frac{M_Z}{91.1875\ \mathrm{GeV}} - 1, \\ \mathrm{d}\alpha &= \frac{\Delta\alpha}{0.05907} - 1, & \mathrm{d}\alpha_\mathrm{s} &= \frac{\alpha_\mathrm{s}(M_Z)}{0.119} - 1, \end{aligned} \tag{5.3}$$

and the coefficients $M_W^0, c_1, \ldots, c_{11}$ take the following values

$$\begin{aligned} M_W^0 &= 80.3799\ \mathrm{GeV}, & c_1 &= 0.05429\ \mathrm{GeV}, & c_2 &= 0.008939\ \mathrm{GeV}, \\ c_3 &= 0.0000890\ \mathrm{GeV}, & c_4 &= 0.000161\ \mathrm{GeV}, & c_5 &= 1.070\ \mathrm{GeV}, \\ c_6 &= 0.5256\ \mathrm{GeV}, & c_7 &= 0.0678\ \mathrm{GeV}, & c_8 &= 0.00179\ \mathrm{GeV}, \\ c_9 &= 0.0000659\ \mathrm{GeV}, & c_{10} &= 0.0737\ \mathrm{GeV}, & c_{11} &= 114.9\ \mathrm{GeV}. \end{aligned} \tag{5.4}$$

With eq. (5.2) the full result for M_W is approximated to better than 0.5 MeV over the range of 10 GeV $\leq M_H \leq$ 1 TeV if all other experimental input values vary within their combined 2σ region around the central values, as being used in eq. (5.3).

For the new options `AMT4` $= 5, 6$ the algorithm used by `ZFITTER` for the estimation of the theory error due to the calculation of M_W, based on applying different resummation procedures (see eq. (2.104) of [2]) cannot be used anymore, since the next term in the m_t expansion, which is of order $\mathcal{O}(G_\mu^2 m_t^0 M_Z^2)$, is now known exactly. In contrast, for these options the uncertainties from unknown higher order contributions in the calculation of M_W have been estimated in [32,37] and are set here to ± 5 MeV and ± 4 MeV, respectively. They can be simulated by varying the flag `DMWW` between -1 and 1, which is the only relevant flag to simulate theoretical uncertainties for M_W for `AMT4` $= 5, 6$, whereas `EXPR`, `IHIGS`, `HIG2`, `SCRE` and `SCAL` are ignored. Furthermore, the use of `AMT4` $= 5, 6$ is not compatible with the setting `IMOMS` > 1.

5.2 The effective leptonic weak mixing angle

The amplitude for the decay of the Z boson into a pair of fermions is parameterized in `ZFITTER` as:

$$V_\mu^{Zf\bar{f}}\left(M_z^2\right) = \left(i\,(2\pi)^4\right)\, i\sqrt{\sqrt{2}G_\mu M_Z^2}\sqrt{\rho_Z^f}I_f^{(3)}\gamma_\mu\left[(1+\gamma_5) - 4|Q_f|s_W^2\kappa_Z^f\right]. \tag{5.5}$$

This formula allows to define the effective weak mixing angle as:

$$\sin^2\theta_{\rm eff}^f = {\rm Re}\Big(\kappa_Z^f\Big)s_W^2, \tag{5.6}$$

where the coefficients ρ_Z^f and κ_Z^f are called *effective couplings* of Z-decay.

Complete fermionic two-loop corrections to the leptonic effective weak mixing angle $\sin^2\theta_{\rm eff}^{\rm lept}$ [52–54] are included with the option `AMT4` $= 6$. They improve the realizations for `AMT4` ≤ 5 by including all electroweak two-loop contributions with at least one closed fermion loop and go beyond the leading [55–58] and next-to-leading [59] terms in an expansion in m_t^2. The corrections have been implemented directly using the numerical fitting formula as published in [52], thereby assuring a fast evaluation:

$$\begin{aligned}\sin^2\theta_{\rm eff}^{\rm lept} = {} & s_0 + d_1 L_H + d_2 L_H^2 + d_3 L_H^4 + d_4(\Delta_H^2 - 1) + d_5\Delta_\alpha \\ & + d_6\Delta_t + d_7\Delta_t^2 + d_8\Delta_t(\Delta_H - 1) + d_9\Delta_{\alpha_s} + d_{10}\Delta_Z,\end{aligned} \tag{5.7}$$

with

$$\begin{aligned} L_H &= \log\left(\frac{M_H}{100\ {\rm GeV}}\right), & \Delta_H &= \frac{M_H}{100\ {\rm GeV}}, \\ \Delta_\alpha &= \frac{\Delta\alpha}{0.05907} - 1, & \Delta_t &= \left(\frac{m_t}{178.0\ {\rm GeV}}\right)^2 - 1, \\ \Delta_{\alpha_s} &= \frac{\alpha_{\rm s}(M_Z)}{0.117} - 1, & \Delta_Z &= \frac{M_Z}{91.1876\ {\rm GeV}} - 1, \end{aligned} \tag{5.8}$$

and

$$\begin{aligned} &s_0 = 0.2312527, && d_1 = 4.729\times10^{-4}, && d_2 = 2.07\times10^{-5}, \\ &d_3 = 3.85\times10^{-6}, && d_4 = -1.85\times10^{-6}, && d_5 = 0.0207, \\ &d_6 = -0.002851, && d_7 = 1.82\times10^{-4}, && d_8 = -9.74\times10^{-6}, \\ &d_9 = 3.98\times10^{-4}, && d_{10} = -0.655. \end{aligned} \tag{5.9}$$

In addition to the electroweak one- and two-loop corrections, this formula also includes two-loop $\mathcal{O}(\alpha\alpha_s)$ [40–46] and three-loop $\mathcal{O}(\alpha\alpha_s^2)$ [49, 50] QCD corrections, as well as leading three-loop corrections in an expansion in m_t^2 of order $\mathcal{O}(\alpha^3)$ and $\mathcal{O}(\alpha^2\alpha_s)$ [51]. In contrast to the settings `AMT4` < 6, the three-loop QCD corrections are incorporated exactly according to [49], which goes beyond the leading term in m_t^2 used previously. Equation (5.7) reproduces the exact calculation with maximal and average deviations of 4.5×10^{-6} and 1.2×10^{-6}, respectively, as long as the input parameters stay within their 2σ ranges and the Higgs boson mass in the range 10 GeV $\leq M_H \leq$ 1 TeV.

With the new option `AMT4` $= 6$ the theory error in the calculation of $\sin^2\theta^{\rm lept}_{\rm eff}$ is no longer obtained through different resummation formulas (see eq. (2.134) of [2]). Instead, the value of $\pm 4.9 \times 10^{-5}$, as estimated in [52], is used. For this case the theory uncertainty can be simulated by varying the flag `DSWW` between -1 and 1.

5.3 Partial and total Z boson widths

The Z boson decay width is defined through the effective couplings, κ_Z^f and ρ_Z^f, of eq. (5.5). The form factor κ_Z^l is obtained from the effective weak mixing angle $\sin^2\theta^{\rm lept}_{\rm eff}$, as presented in subsection 5.2. For Z boson decays into a pair of fermions f, $f \neq l$, the two-loop corrections to κ_Z^f are currently implemented in an approximate way only, while for $\Gamma(Z \to b\bar{b})$, no two-loop corrections beyond the leading m_t^4 term are available. For the form factor ρ_Z^f, the two-loop corrections beyond the next-to-leading m_t^2 expansion are still missing. This quantity is therefore computed identically for the choices `AMT4` $= 4, 5, 6$.

5.4 Electroweak form factors for $e^+e^- \to f\bar{f}$

The radiative corrections to the cross-sections and asymmetries for the process $e^+e^- \to f\bar{f}$ are parameterized in `ZFITTER` by electroweak form factors, $\rho_{ef}, \kappa_e, \kappa_f$ and κ_{ef}, defined through a Z-boson exchange amplitude:

$$\begin{aligned}\mathcal{A}_Z(s,t) = i\,e^2\,4\,I_e^{(3)} I_f^{(3)} \frac{\chi_Z(s)}{s} \rho_{ef}(s,t) \Big\{ & \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu(1+\gamma_5) \\ & -4|Q_e| s_W^2 \kappa_e(s,t) \gamma_\mu \otimes \gamma_\mu(1+\gamma_5) - 4|Q_f| s_W^2 \kappa_f(s,t) \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu \\ & +16|Q_e Q_f| s_W^4 \kappa_{ef}(s,t) \gamma_\mu \otimes \gamma_\mu \Big\}, \end{aligned} \tag{5.10}$$

where

$$\chi_Z(s) = \frac{G_\mu}{\sqrt{2}} \frac{M_Z^2}{8\pi\alpha} \frac{s}{s - m_Z^2} \qquad \text{and} \qquad m_Z^2 = M_Z^2 - i M_Z \Gamma_Z(s). \tag{5.11}$$

In the leading pole approximation, the following relations hold:

$$\rho_{ef} = \sqrt{\rho_Z^e\, \rho_Z^f}, \qquad \kappa_e = \kappa_Z^e, \qquad \kappa_f = \kappa_Z^f, \qquad \kappa_{ef} = \kappa_Z^e \kappa_Z^f. \tag{5.12}$$

However, generally, ρ_{ef}, κ_e, κ_f, and κ_{ef} also include γ-Z interference effects, corrections from non-resonant Z and γ exchanges, and non-factorizable box

contributions. With the option AMT4 = 6 the coefficients κ_e and κ_f, for $f \neq b$, are calculated with all known two- and three-loop effects, as discussed in the previous sections. The corresponding factorizable vertex corrections to κ_{ef} are incorporated similarly. The electroweak box contributions to the form factors include the complete one-loop order correction, which is sufficient for present precision. For the form factor ρ_{ef}, complete two-loop corrections are still missing, and it is computed including two-loop corrections of the order $\mathcal{O}(G_\mu^2 m_t^2 M_Z^2)$ only for all options AMT4 = 4, 5, 6.

For the $Z \to b\bar{b}$ channel, no two-loop electroweak corrections beyond the leading m_t^4 terms are available so far. Furthermore, in ZFITTER versions before 6.42, a mismatch occurred in the treatment of the $b\bar{b}$ final state. The subroutine GDEGNL, which computes the leading two-loop corrections, is called at two places in the program: (*i*) in subroutine ZWRATE the effective couplings introduced in section 5.2 are used for the computation of Z partial widths; (*ii*) the interfaces ZUTHSM, ZUTPSM, ZULRSM and ZUATSM, on the other hand, calculate the cross-sections and asymmetries from the weak form factors $\rho_{ef}(s,t)$, $\kappa_e(s,t)$, $\kappa_f(s,t)$ and $\kappa_{ef}(s,t)$ in the subroutine ROKANC. For INDF = 9, i.e. the $b\bar{b}$ final state, all these form factors were calculated in one-loop approximation with the leading m_t^4 two-loop term. As a result, in ROKANC the initial-state Ze^+e^- form factors for all other final states are generated including next-to-leading two-loop corrections, while for the $b\bar{b}$ final state these corrections were missing. This mismatch also affects the ZFITTER interfaces ZUXSA, ZUTAU and ZUXSA2, which use the language of effective couplings [2], since they are defined to coincide as close as possible with the complete Standard Model prediction in ROKANC if the effective couplings coincide with their Standard Model analogue.

The problem has been alleviated since ZFITTER version 6.41 (15 October 2004). In contrast to the older implementations, $\kappa_e(s,t)$ and $\kappa_f(s,t)$ are not treated symmetrically anymore for INDF = 9, but two-loop electroweak corrections are included in $\kappa_e(s,t)$ for AMT4 $\geq$ 4, not yet in $\kappa_b(s,t)$. The treatment of $\rho_{ef}(s,t)$ and $\kappa_{ef}(s,t)$ has been changed accordingly [3]. Here one can use the fact that the presently known two-loop contributions factorize into initial-state and final-state corrections. For a more detailed discussion see [60].

[3] In ZFITTER version 6.41 an error occured in the treatment of a form factor $\rho_{ef}(s,t)$. It affected the $b\bar{b}$ cross-section only, and was corrected in ZFITTER version 6.42.

6 Input parameters and pseudo observables

For the calculations of pseudo observables (POs), such as partial Z decay widths or effective coupling constants, `ZFITTER` uses the `DIZET` package which calculates such quantities employing the *on-mass-shell (OMS)* renormalization scheme [61, 62] within the Standard Model. The user has to provide a set of values for the so-called input parameter set, which are then used by `ZFITTER` and `DIZET` to calculate the POs. A standard set of imput parameters is given by:

- The electromagnetic coupling constant at the Z pole, or, more precisely, its shift due to the 5-quark flavour hadronic vacuum polarisation, $\Delta\alpha^{(5)}_{\rm had}$ at the Z pole;
- The strong coupling constant at the Z pole, α_S;
- The pole masses of Z boson, top quark and Higgs boson.

These variables are the main physics values to be provided to `ZFITTER` and `DIZET` by the user. The Fermi constant is treated as a constant in the program (see flag `GFER`). In order to use the user-supplied value of the hadronic vacuum polarisation, flag `ALEM=2` must be used. In order to use the latest set of electroweak radiative corrections (see [37, 52] and Section 5), flag `AMT4=6` must be used.

Selected POs calculated by `ZFITTER` and `DIZET` are listed in Tab. 6.1, namely: (i) the W boson mass and the on-shell electroweak mixing angle $s^2_W = 1 - M^2_W/M^2_Z$; (ii) the partial Z decay widths, including the invisible width, simply equal to $3\Gamma_\nu$, and the the total hadronic width, equal to the sum of the five-flavour quarkonic widths, and the total width; (iii) ratios of widths and pole cross-sections as used by, e.g., the LEP EWWG; (iv) the effective electroweak mixing angle $\sin^2\theta^f_{\rm eff}$ according to Eq. (5.6) and the ρ_f parameter $\rho_f = {\rm Re}(\rho^f_Z)$, for leptons and heavy fermions b and c; and (v) the asymmetry parameter $\mathcal{A}_f$ and the forward-backward pole asymmetries $A^{0f}_{\rm FB}$ for leptons and heavy fermions b and c.

The partial decay widths are defined inclusively, i.e., they contain all real and virtual corrections. The ratios for leptons l and quarks q are defined as:

$$R^0_l \;=\; \frac{\Gamma_h}{\Gamma_l} \qquad\qquad R^0_q \;=\; \frac{\Gamma_q}{\Gamma_h}\,, \tag{6.1}$$

while the hadronic and leptonic pole cross-sections are defined as:

$$\sigma^0_h \;=\; 12\pi\,\frac{\Gamma_e\Gamma_h}{M^2_Z\Gamma^2_Z} \qquad\qquad \sigma^0_l \;=\; 12\pi\,\frac{\Gamma_e\Gamma_l}{M^2_Z\Gamma^2_Z}\,. \tag{6.2}$$

The complex variable g_Z^f is defined as the ratio:

$$g_z^f \;=\; \frac{v_f}{a_f} \;=\; 1 - 4|Q_f|\kappa_z^f s_W^2 \tag{6.3}$$

of the complex effective vector and axial couplings of the Z boson to the fermion f:

$$v_f \;=\; \sqrt{\rho_Z^f}\,(T_3^f - 2Q_f\kappa_Z^f s_W^2)\,, \tag{6.4}$$

$$a_f \;=\; \sqrt{\rho_Z^f}\,T_3^f\,. \tag{6.5}$$

Hence one may write:

$$\mathrm{Re}(g_Z^f) \;=\; 1 - 4|Q_f|\sin^2\theta_{\mathrm{eff}}^f. \tag{6.6}$$

The complex form factors ρ_Z^f, κ_Z^f are defined through the amplitude of the Z boson decay into a pair of fermions, as in eq. (5.5).

With these definitions, the asymmetry parameters are then given in terms of the real part of g_Z^f:

$$\mathcal{A}_f \;=\; 2\frac{\mathrm{Re}\,g_z^f}{1 + \left(\mathrm{Re}\,g_Z^f\right)^2}\,. \tag{6.7}$$

The forward-backward pole asymmetries are mere combinations of these asymmetry parameters:

$$A_{\mathrm{FB}}^{0f} \;=\; \frac{3}{4}\,\mathcal{A}_e\,\mathcal{A}_f. \tag{6.8}$$

In terms of the real part of the complex ρ_Z^f parameter, $\rho_f = \mathrm{Re}(\rho_Z^f)$, and the (real) effective electroweak mixing angle $\sin^2\theta_{\mathrm{eff}}^f$ defined earlier, the real effective vector and axial-vector coupling constants as quoted by the LEP EWWG are then defined as:

$$g_{Af} \;=\; \sqrt{\rho_f}\,T_3^f \tag{6.9}$$

$$g_{Vf} \;=\; \sqrt{\rho_f}\,(T_3^f - 2Q_f\sin^2\theta_{\mathrm{eff}}^f)\,. \tag{6.10}$$

With these definitions, it follows that:

$$\frac{g_{Vf}}{g_{Af}} \;=\; \mathrm{Re}(g_Z^f)\,, \tag{6.11}$$

so that the asymmetry parameters are then equivalently given as:

$$\mathcal{A}_f \;=\; 2\frac{g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2}\,. \tag{6.12}$$

Observable	Value
M_W [GeV]	80.3613
s^2_W	0.22335
Γ_ν [MeV]	167.219
Γ_e [MeV]	83.990
Γ_μ [MeV]	83.990
Γ_τ [MeV]	83.800
Γ_c [MeV]	299.966
Γ_b [MeV]	375.729
$\Gamma_{\rm inv}$ [GeV]	0.501657
Γ_h [GeV]	1.741507
Γ_Z [GeV]	2.494944

Observable	Value
R^0_l	20.73462
R^0_b	0.215750
R^0_c	0.172245
σ^0_h [nb]	41.4826
σ^0_l [nb]	2.0006
$\sin^2\theta^{\rm lept}_{\rm eff}$	0.231548
$\sin^2\theta^{b}_{\rm eff}$	0.233032
$\sin^2\theta^{c}_{\rm eff}$	0.231442
ρ_e	1.005165
ρ_b	0.993943
ρ_c	1.005860

Observable	Value
$\mathcal{A}_e$	0.146813
$\mathcal{A}_b$	0.934554
$\mathcal{A}_c$	0.667779
$A^{0,l}_{\rm FB}$	0.016166
$A^{0,b}_{\rm FB}$	0.102904
$A^{0,c}_{\rm FB}$	0.073529

Table 6.1
Predictions for pseudo observables calculated with ZFITTER *and* DIZET *for* $\Delta\alpha^{(5)}_{\rm had} = 0.02758$, $\alpha_S = 0.118$, $M_Z = 91.1875\ GeV$, $M_t = 175\ GeV$ *and* $M_H = 150\ GeV$, *and flags* ALEM=2 *and* AMT4=6.

6.1 Pseudo observables in common blocks of DIZET

The channel dependent quantities are stored in common block COMMON/CDZRKZ/ and in array PARTZ(0:11):

```
 COMMON/CDZRKZ/ARROFZ(0:10),ARKAFZ(0:10),ARVEFZ(0:10),ARSEFZ(0:10)
&             ,AROTFZ(0:10),AIROFZ(0:10),AIKAFZ(0:10),AIVEFZ(0:10)
*---
 DIMENSION NPAR(25),ZPAR(30),PARTZ(0:11),PARTW(3)
*---
 SUBROUTINE DIZET(NPAR,AMW   !  NPAR : FLAGS; AMW : INPUT/OUTPUT
&                     ,AMZ,AMT,AMH,DAL5H,V_TBA,ALSTR   !  INPUT
&                     ,ALQED,ALSTRT,ZPAR,PARTZ,PARTW)  !  OUTPUT
```

The correspondences have not been changed compared to Section 2.5.1 of [2]:

$$\texttt{ARROFZ(0:10)} = (\rho_Z^f)' \,, \tag{6.13}$$
$$\texttt{AROTFZ(0:10)} = \mathrm{Re}\,\rho_Z^f \,, \tag{6.14}$$
$$\texttt{ARKAFZ(0:10)} = \mathrm{Re}\,\kappa_Z^f \,, \tag{6.15}$$
$$\texttt{ARVEFZ(0:10)} = \mathrm{Re}\,g_Z^f \,, \tag{6.16}$$
$$\texttt{AIROFZ(0:10)} = \mathrm{Im}\,\rho_Z^f \,, \tag{6.17}$$
$$\texttt{AIKAFZ(0:10)} = \mathrm{Im}\,\kappa_Z^f \,, \tag{6.18}$$
$$\texttt{AIVEFZ(0:10)} = \mathrm{Im}\,g_Z^f \,, \tag{6.19}$$
$$\texttt{ARSEFZ(0:10)} = \sin^2\theta_{\mathrm{eff}}^f \,, \tag{6.20}$$
$$\texttt{PARTZ(0:11)} = \Gamma_f \,, \tag{6.21}$$

The usual ZFITTER channel assignments as given in Figure 5 of [2] are used, and all quantities but $(\rho_Z^f)'$ have been introduced already. The $(\rho_Z^f)'$ is discussed in Section 2.5.1 of [2]. We mention also here that both options MISC=1,0 (with $(\rho_Z^f)'$ or with $|\rho_Z^f|$) are used in the *Model Independent interfaces* of ZFITTER, see Appendix C.

Note that since the convention of the LEP EWWG is to use the real parameter $\rho_f = \mathrm{Re}(\rho_Z^f)$, not $(\rho_Z^f)'$, the array AROTFZ is generally used, while ARROFZ, RENFAC and SRENFC are usually ignored.

7 Subroutine ZFTEST

The ZFITTER distribution includes subroutine ZFTEST. With ZFTEST the user may test whether ZFITTER has been properly installed. The subroutine calculates cross-sections and asymmetries as functions of $\sqrt{s}$ near the Z peak, below, and above. A sample file zfmai6_42.f runs ZFTEST:

```
*
* MAIN to call ZFTEST with version 6.42
*
      CALL ZFTEST(0)
      END
```

The numerical output should reproduce the Tables given in Subsection 7.1.

7.1 ZFTEST *results*

This Section contains the standard test-outputs, produced by a call to ZFTEST(0). The argument of ZFTEST sets the flag MISC. The default value is MISC=0. The

user's result should be identical to the sample output, apart from a possible flip in the last digit.

```
**************************************************
**************************************************
**           This is ZFITTER version 6.42       **
**                    05/05/18                  **
**************************************************
** http://www.ifh.de/theory/publist.html        **
**************************************************

ZUINIT> ZFITTER defaults:

ZFITTER flag values:
AFBC: 1 SCAL: 0 SCRE: 0 AMT4: 4 BORN: 0
BOXD: 1 CONV: 1 FINR: 1 FOT2: 3 GAMS: 1
DIAG: 1 INTF: 1 BARB: 2 PART: 0 POWR: 1
PRNT: 0 ALEM: 3 QCDC: 3 VPOL: 1 WEAK: 1
FTJR: 1 EXPR: 0 EXPF: 0 HIGS: 0 AFMT: 3
CZAK: 1 PREC:10 HIG2: 0 ALE2: 3 GFER: 2
ISPP: 2 FSRS: 1 MISC: 0 MISD: 1 IPFC: 5
IPSC: 0 IPTO: 3 FBHO: 0 FSPP: 0 FUNA: 0
ASCR: 1 SFSR: 1 ENUE: 1 TUPV: 1 DMWW: 0
DSWW: 0


ZFITTER cut values:
  INDF  ICUT    ACOL    EMIN    S_PR    ANGO    ANG1     SPP
     0    -1    0.00    0.00    0.00    0.00  180.00    0.00
     1    -1    0.00    0.00    0.00    0.00  180.00    0.00
     2    -1    0.00    0.00    0.00    0.00  180.00    0.00
     3    -1    0.00    0.00    0.00    0.00  180.00    0.00
     4    -1    0.00    0.00    0.00    0.00  180.00    0.00
     5    -1    0.00    0.00    0.00    0.00  180.00    0.00
     6    -1    0.00    0.00    0.00    0.00  180.00    0.00
     7    -1    0.00    0.00    0.00    0.00  180.00    0.00
     8    -1    0.00    0.00    0.00    0.00  180.00    0.00
     9    -1    0.00    0.00    0.00    0.00  180.00    0.00
    10    -1    0.00    0.00    0.00    0.00  180.00    0.00
    11    -1    0.00    0.00    0.00    0.00  180.00    0.00



ZFITTER input parameters:
DAL5H =  0.0280398093
ALQED5=  128.886183
 ZMASS =   91.18760;  TMASS =  178.00000
 HMASS =  100.00000
 DAL5H =    0.02804; ALQED5 =  128.88618
 ALFAS =    0.11700;  ALFAT =    0.10637

ZFITTER intermediate results:
 WMASS =   80.39522; SIN2TW =    0.22270

ALPHST =    0.11700;
QCDCOR =  1.00000 1.03938 1.04639 1.03877 1.03177 1.03941 1.04611 1.03877 1.03177 1.03938 1.04639 1.03937 1.02506-0.00002 0.19863

CHANNEL          WIDTH         RHO_F_R        RHO_F_T        SIN2_EFF
-------         -------       --------       --------       --------
nu,nubar        167.299       1.008546       1.008546       0.231036
e+,e-            84.037       1.005790       1.005643       0.231417
mu+,mu-          84.037       1.005790       1.005643       0.231417
tau+,tau-        83.847       1.005790       1.005643       0.231417
u,ubar          300.128       1.006390       1.006339       0.231310
d,dbar          383.033       1.007315       1.007306       0.231183
c,cbar          300.059       1.006390       1.006339       0.231310
s,sbar          383.033       1.007315       1.007306       0.231183
t,tbar            0.000       0.000000       0.000000       0.000000
b,bbar          375.593       0.993885       0.993885       0.233004
hadron         1741.846
total          2495.664
```

SQRT(S) = 89.1875992

INDF	ZUTHSM	ZUXSEC	ZUXSA	ZUXSA2	ZUXAFB	ZUTHSM	ZUXSA	ZUXSA2	ZUXAFB	ZUTPSM	ZUTAU	ZULRSM	ZUALR
0	0.79464	0.79464											
1	0.41291	0.41291	0.41291	0.41291	0.41291	-0.1885	-0.1885	-0.1885	-0.1885				
2	0.41309	0.41309	0.41309	0.41309	0.41309	-0.1885	-0.1885	-0.1885	-0.1885				
3	0.41223	0.41223	0.41223	0.41223	0.41223	-0.1888	-0.1888	-0.1888	-0.1888	-0.1206	-0.1206		
4	1.44131	1.44131	1.44131			-0.0473	-0.0473					0.0841	0.0000
5	1.81653	1.81653	1.81653			0.0544	0.0544					0.1031	2.1932
6	1.44105	1.44105	1.44105			-0.0474	-0.0474					0.0841	2.1932
7	1.81654	1.81654	1.81653			0.0544	0.0544					0.1031	2.0196
8	0.00000	0.00000	0.00000			0.0000	0.0000					0.0000	2.1951
9	1.78022	1.78022	1.78022			0.0542	0.0542					0.1025	0.0000
10	8.29565	8.29565										0.0963	0.0000
11	0.70082	0.70101	0.70090	0.70090		0.4313	0.4312	0.4312					

SQRT(S) = 91.1875992

INDF	ZUTHSM	ZUXSEC	ZUXSA	ZUXSA2	ZUXAFB	ZUTHSM	ZUXSA	ZUXSA2	ZUXAFB	ZUTPSM	ZUTAU	ZULRSM	ZUALR
0	2.91319	2.91319											
1	1.47692	1.47692	1.47692	1.47692	1.47692	0.0003	0.0003	0.0003	0.0003				
2	1.47745	1.47745	1.47745	1.47745	1.47745	0.0003	0.0003	0.0003	0.0003				
3	1.47438	1.47438	1.47438	1.47438	1.47438	0.0001	0.0001	0.0001	0.0001	-0.1431	-0.1431		
4	5.24201	5.24201	5.24201			0.0615	0.0615					0.1418	0.0000
5	6.67109	6.67109	6.67109			0.0971	0.0971					0.1441	2.1932
6	5.24108	5.24108	5.24109			0.0616	0.0616					0.1418	2.1932
7	6.67112	6.67112	6.67113			0.0971	0.0971					0.1441	2.0196
8	0.00000	0.00000	0.00000			0.0000	0.0000					0.0000	2.1951
9	6.54153	6.54153	6.54153			0.0982	0.0982					0.1444	0.0000
10	30.36683	30.36683										0.1434	0.0000
11	1.37174	1.37182	1.37174	1.37174		0.1869	0.1869	0.1869					

SQRT(S) = 93.1875992

INDF	ZUTHSM	ZUXSEC	ZUXSA	ZUXSA2	ZUXAFB	ZUTHSM	ZUXSA	ZUXSA2	ZUXAFB	ZUTPSM	ZUTAU	ZULRSM	ZUALR
0	1.22102	1.22102											
1	0.62880	0.62880	0.62880	0.62880	0.62880	0.1209	0.1209	0.1209	0.1209				
2	0.62905	0.62905	0.62905	0.62905	0.62905	0.1209	0.1209	0.1209	0.1209				
3	0.62781	0.62781	0.62781	0.62781	0.62781	0.1208	0.1208	0.1208	0.1208	-0.1530	-0.1530		
4	2.21270	2.21270	2.21270			0.1293	0.1293					0.1763	0.0000
5	2.80592	2.80592	2.80592			0.1235	0.1235					0.1695	2.1932
6	2.21234	2.21234	2.21235			0.1295	0.1295					0.1763	2.1932
7	2.80593	2.80593	2.80594			0.1235	0.1235					0.1695	2.0196
8	0.00000	0.00000	0.00000			0.0000	0.0000					0.0000	2.1951
9	2.75285	2.75285	2.75285			0.1253	0.1253					0.1704	0.0000
10	12.78974	12.78953										0.1720	0.0000
11	0.52345	0.52335	0.52336	0.52336		0.2174	0.2174	0.2174					

SQRT(S) = 100.

INDF	ZUTHSM	ZUXSEC	ZUXSA	ZUXSA2	ZUXAFB	ZUTHSM	ZUXSA	ZUXSA2	ZUXAFB	ZUTPSM	ZUTAU	ZULRSM	ZUALR
0	0.20718	0.20718											
1	0.11707	0.11707	0.11707	0.11707	0.11707	0.2404	0.2404	0.2404	0.2404				
2	0.11714	0.11714	0.11715	0.11715	0.11714	0.2403	0.2403	0.2403	0.2403				
3	0.11696	0.11696	0.11696	0.11696	0.11696	0.2405	0.2405	0.2405	0.2405	-0.1520	-0.1520		
4	0.39006	0.39006	0.39006			0.2003	0.2003					0.2078	0.0000
5	0.48082	0.48082	0.48082			0.1518	0.1518					0.1962	2.1932
6	0.39002	0.39002	0.39002			0.2006	0.2006					0.2078	2.1932
7	0.48083	0.48083	0.48083			0.1518	0.1518					0.1962	2.0196
8	0.00000	0.00000	0.00000			0.0000	0.0000					0.0000	2.1951
9	0.47229	0.47229	0.47229			0.1546	0.1546					0.1981	0.0000
10	2.21402	2.21402										0.2007	0.0000
11	0.19403	0.19401	0.19400	0.19400		0.6379	0.6381	0.6381					

SQRT(S) = 140.

INDF	ZUTHSM	ZUXSEC	ZUXSA	ZUXSA2	ZUXAFB	ZUTHSM	ZUXSA	ZUXSA2	ZUXAFB	ZUTPSM	ZUTAU	ZULRSM	ZUALR
0	0.02252	0.02252											
1	0.01755	0.01755	0.01755	0.01755	0.01755	0.2920	0.2920	0.2920	0.2920				
2	0.01758	0.01758	0.01758	0.01758	0.01758	0.2917	0.2917	0.2917	0.2917				
3	0.01757	0.01757	0.01757	0.01757	0.01757	0.2917	0.2917	0.2917	0.2917	-0.1171	-0.1171		
4	0.04846	0.04846	0.04846			0.2478	0.2478					0.2115	0.0000
5	0.05371	0.05371	0.05371			0.1693	0.1694					0.2045	2.1932
6	0.04847	0.04847	0.04847			0.2480	0.2480					0.2115	2.1932
7	0.05371	0.05371	0.05371			0.1693	0.1694					0.2045	2.0196
8	0.00000	0.00000	0.00000			0.0000	0.0000					0.0000	2.1951
9	0.05302	0.05302	0.05302			0.1747	0.1747					0.2107	0.0000
10	0.25737	0.25691										0.2083	0.0000
11	0.08494	0.08497	0.08498	0.08498		0.8995	0.8996	0.8996					

SQRT(S) = 175.

INDF	ZUTHSM	ZUXSEC	ZUXSA	ZUXSA2	ZUXAFB	ZUTHSM	ZUXSA	ZUXSA2	ZUXAFB	ZUTPSM	ZUTAU	ZULRSM	ZUALR
0	0.01078	0.01078											
1	0.00939	0.00939	0.00939	0.00939	0.00939	0.2815	0.2815	0.2815	0.2815				
2	0.00941	0.00941	0.00941	0.00941	0.00941	0.2810	0.2810	0.2810	0.2810				
3	0.00940	0.00940	0.00941	0.00941	0.00940	0.2810	0.2810	0.2810	0.2810	-0.1006	-0.1006		
4	0.02428	0.02428	0.02428			0.2444	0.2444					0.2016	0.0000
5	0.02587	0.02587	0.02587			0.1617	0.1618					0.1953	2.1932
6	0.02428	0.02428	0.02428			0.2445	0.2445					0.2016	2.1932
7	0.02587	0.02587	0.02587			0.1617	0.1618					0.1953	2.0196
8	0.00000	0.00000	0.00000			0.0000	0.0000					0.0000	2.1951
9	0.02561	0.02561	0.02561			0.1693	0.1693					0.2063	0.0000
10	0.12590	0.12545										0.1998	0.0000
11	0.05490	0.05491	0.05491	0.05491		0.9019	0.9021	0.9021					

8 Summary

The description of the ZFITTER package has been updated to version 6.42. Besides a short introduction to the physics contents of the program additions, we reproduce, in Appendices, also the technical details like user flags and interface subroutines as complete as necessary for a convenient use of the program.

ZFITTER covers most of the radiative corrections of a practical relevance in the foreseeable future. Yet, several corrections are still needed to close the two loop program of data analysis with ZFITTER:

- cross section (or decay rate) asymmetries, with the Z boson coupling to light quarks or b quarks;
- electroweak corrections to the Z boson width;
- bosonic electroweak corrections to all asymmetries and the effective weak mixing angle.

The inclusion of the two loop electroweak corrections to the $Zb\bar{b}$ vertex will additionally require to account for the mass of the b quark in the one loop electroweak corrections. Nonetheless, it was checked that the one loop corrections with a massive b quark would give negligible effects for the accuracy reached at LEP.

Some bug reports concerning ZFITTER versions between version 6.21 and version 6.42 will be found at the webpage [3].

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For many years, the support of the user community of ZFITTER by the authors of the program was substantial for the numerous applications of the program by the LEP collaborations, the LEP EWWG (LEP Electroweak Working Group), and many other user groups.

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We would like to thank Georg Weiglein for discussions.

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A `DIZET` user guide

This Appendix describes technical details of the `DIZET` package. Not all of them have been influenced by the updates over the years. For the convenience of the user, we nevertheless decided to give a complete overview and will repeat a substantial part of the material, which was already presented in Section 4.1 of [2].

A.1 Structure of `DIZET`

A first call of subroutine `DIZET` returns various pseudo-observables, the W-boson mass, weak mixing angles, the Z-boson width, the W-boson width and other quantities. After the first call to `DIZET`, several subroutines of `DIZET` might be used for the calculation of form factors and couplings. This is described in Section 4.1.1 of [2].

A.2 Input and output of `DIZET`

The `DIZET` argument list contains Input, Output and Mixed (I/O) types of arguments:

```
CALL DIZET(NPAR,AMW,AMZ,AMT,AMH,DAL5H,ALQED,ALSTR,ALSTRT,ZPAR,PARTZ,PARTW)
```

A.2.1 Input and I/O parameters to be set by the user

Input:

`NPAR(1:25), INTEGER*4` vector of flags
`AMT` $= m_t$ – t-quark mass

AMH = M_H – Higgs boson mass
ALSTR = $\alpha_S(M_Z^2)$ – strong coupling at $s = M_Z^2$

I/O:

AMW = M_W, W boson mass, input if NPAR(4) = 2,3, but is being calculated for NPAR(4) = 1
AMZ = M_Z, Z boson mass, input if NPAR(4) = 1,3, but is being calculated for NPAR(4) = 2
DAL5H = $\Delta\alpha_h^{(5)}(M_Z^2)$, hadronic vacuum polarization

The M_Z^2, M_W^2, and $\Delta\alpha_h^{(5)}(M_Z^2)$ cannot be assigned by a parameter statement (input/output variables).

A.2.2 Output of the DIZET *package*

ALQED = $\alpha(M_Z^2)$, calculated from $\Delta\alpha_h^{(5)}(M_Z^2)$, see description of flag ALEM in Subsection A.3
ALSTRT = $\alpha_S(m_t^2)$
ZPAR(1) = DR=Δr, the loop corrections to the muon decay constant
ZPAR(2) = DRREM = $\Delta r_{\rm rem}$, the remainder contribution $\mathcal{O}(\alpha)$
ZPAR(3) = SW2 = s_W^2, squared of sine of the weak mixing angle defined by weak boson masses
ZPAR(4) = GMUC = G_μ, muon decay constant, if NPAR(4) = 1,2, is set in CONST1 depending on flag NPAR(20), see Subsection A.3. (It should be calculated if NPAR(4)=3 from M_Z, M_W, but then it will deviate from the experimental value.)
ZPAR(5-14) – stores effective sines for all partial Z-decay channels:
- 5 – neutrino
- 6 – electron
- 7 – muon
- 8 – τ lepton
- 9 – up quark
- 10 – down quark
- 11 – charm quark
- 12 – strange quark
- 13 – top quark (presently equal to up quark)
- 14 – bottom quark

ZPAR(15) = ALPHST $\equiv \alpha_S(M_Z^2)$
ZPAR(16-30) = QCDCOR(0-14), QCDCOR(I) – array of QCD correction factors for quark production processes and/or Z boson partial width (channel i) into quarks. Enumeration as follows:

$$
\begin{aligned}
\texttt{QCDCOR(0)} &= 1 \\
\texttt{QCDCOR(1)} &= R_V^u(M_Z^2) \\
\texttt{QCDCOR(2)} &= R_A^u(M_Z^2) \\
\texttt{QCDCOR(3)} &= R_V^d(M_Z^2) \\
\texttt{QCDCOR(4)} &= R_A^d(M_Z^2) \\
\texttt{QCDCOR(5)} &= R_V^c(M_Z^2) \\
\texttt{QCDCOR(6)} &= R_A^c(M_Z^2) \\
\texttt{QCDCOR(7)} &= R_V^s(M_Z^2) \\
\texttt{QCDCOR(8)} &= R_A^s(M_Z^2) \\
\texttt{QCDCOR(9)} &= R_V^u(M_Z^2) \quad \text{foreseen for} \quad t\bar{t}\text{-channel} \\
\texttt{QCDCOR(10)} &= R_A^u(M_Z^2) \quad \text{foreseen for} \quad t\bar{t}\text{-channel} \\
\texttt{QCDCOR(11)} &= R_V^b(M_Z^2) \\
\texttt{QCDCOR(12)} &= R_A^b(M_Z^2) \\
\texttt{QCDCOR(13)} &= R_V^S\text{–singlet vector correction} \\
\texttt{QCDCOR(14)} &= f_1, \quad \text{corrections to} \quad A_{FB}\ [63]
\end{aligned}
\tag{A.1}
$$

`PARTZ(I)` – array of partial decay widths of the Z-boson:

- `I` = 0 neutrino
- `I` = 1 electron
- `I` = 2 muon
- `I` = 3 tau
- `I` = 4 up
- `I` = 5 down
- `I` = 6 charm
- `I` = 7 strange
- `I` = 8 top (foreseen, not realized)
- `I` = 9 bottom
- `I` = 10 hadrons
- `I` = 11 total

`PARTW(I)` – array of partial decay widths of the W-boson [4] for the channels:

- `I` = 1 one leptonic
- `I` = 2 one quarkonic
- `I` = 3 totoal

A.3 The flags used by `DIZET`

Since the `DIZET` package may be used as stand-alone in order to compute POs we present here a short description of all flags in `DIZET`. The flag values

[4] The calculation of the W width [64] follows the same principles as that of the Z width and is realized in subroutine `ZWRATE` of `DIZET`. Since the W width is not that important for the description of fermion pair production, we do not go into details.

must be filled in vector NPAR(1:25) by the the user. Most of these flags overlap with the flags set in user subroutine ZUFLAG called by ZFITTER, however, in the stand-alone mode ZUFLAG need not be called. We will show the correspondence between the flag names CHFLAG and the flag values IVALUE used inside DIZET, called with

```
CALL ZUFLAG('CHFLAG', IVALUE)
```

The description is given in the order of the vector NPAR(1:25). Flag values marked as **presently not supported** are not recommended. For instance, they may be chosen for backward compatibility with respect to earlier versions of the code.

NPAR(1) = IHVP → ZUFLAG('VPOL',IHVP) – Handling of hadronic vacuum polarization:
IHVP = **1** (default) by the parameterization of [65]
IHVP = **2** by effective quark masses of [66, 67] **presently not**
IHVP = **3** by the parameterization of [68] **supported**

NPAR(2) = IAMT4 → ZUFLAG('AMT4',IAMT4) – Re-summation of the leading $\mathcal{O}(G_\mu m_t^2)$ electroweak corrections, see Section 5:
IAMT4 = **0** no re-summation
IAMT4 = **1** with re-summation recipe of [69] **presently**
IAMT4 = **2** with re-summation recipe of [43] **not**
IAMT4 = **3** with re-summation recipe of [70] **supported**
IAMT4 = **4** (default) with two-loop sub-leading corrections and re-summation recipe of [23–28]
IAMT4 = **5** with fermionic two-loop corrections to M_W according to [29, 30, 32]
IAMT4 = **6** with complete two-loop corrections to M_W [37] and fermionic two-loop corrections to $\sin^2\theta_{\rm eff}^{\rm lept}$ [52]

NPAR(3) = IQCD → ZUFLAG('QCDC',IQCD) – Handling of internal QCD corrections of order $\mathcal{O}(\alpha\alpha_s)$:
IQCD = **0** no internal QCD corrections
IQCD = **1** by Taylor expansions (fast option) of [71]
IQCD = **2** by exact formulae of [71]
IQCD = **3** (default) by exact formulae of [42]

NPAR(4) = IMOMS – Choice of two input/output parameters from the three parameters $\{G_\mu,\ M_z,\ M_W\}$:
IMOMS = **1** (default) input G_μ, M_z; output M_W), see Eq. (5.1)
IMOMS = **2** input G_μ, M_W; output M_z,
IMOMS = **3** input M_z, M_W; output G_μ, **foreseen, not realized**

NPAR(5) = IMASS – Handling of hadronic vacuum polarization in Δr; for tests only:

IMASS = **0** (default) uses a fit to data
IMASS = **1** uses effective quark masses

NPAR(6) = ISCRE → ZUFLAG('SCRE',ISCRE) – Choice of the scale of the two-loop remainder terms of Δr with the aid of a conversion factor f, for details see [2].
ISCRE = **0** (default) scale of the remainder terms is $K_{\rm scale} = 1$
ISCRE = **1** scale of the remainder terms is $K_{\rm scale} = f^2$
ISCRE = **2** scale of the remainder terms is $K_{\rm scale} = \dfrac{1}{f^2}$

NPAR(7) = IALEM → ZUFLAG('ALEM',IALEM) – Controls the usage of $\alpha(M_Z^2)$, see flowchart in [2]. Inside DIZET, however, its meaning is limited:
IALEM = **0 or 2** $\Delta\alpha_h^{(5)}(M_Z^2)$ must be supplied by the user as input to the DIZET package
IALEM = **1 or 3** $\Delta\alpha_h^{(5)}(M_Z^2)$ is calculated by the program using a parameterization IHVP (default: IALEM=3)

For details see the complete discussion about this flag in Sections 2.8 and 4.2.2 of [2].

NPAR(8) = IMASK – Historical relict of earlier versions. **Presently unused.**

NPAR(9) = ISCAL → ZUFLAG('SCAL',ISCAL) – Choice of the scale of $\alpha_s(\xi m_t)$:
ISCAL = **0** (default) exact AFMT correction [47]
ISCAL = **1,2,3** options used in [72], **presently not supported**
ISCAL = **4** Sirlin's scale $\xi = 0.248$ [73]

NPAR(10) = IBARB → ZUFLAG('BARB',IBARB) – Handling of leading $\mathcal{O}\left(G_\mu^2 m_t^4\right)$ corrections:
IBARB = **0** corrections are not included
IBARB = **1** corrections are applied in the limiting case: Higgs mass negligible with respect to the top mass, [74]
IBARB = **2** (default) analytic results of [75] approximated by a polynomial [76]
These options are inactive for AMT4 = 4.

NPAR(11) = IFTJR → ZUFLAG('FTJR',IFTJR) – Treatment of $\mathcal{O}\left(G_\mu \alpha_s m_t^2\right)$ FTJR corrections [77],
IFTJR = **0** without FTJR corrections
IFTJR = **1,2** with FTJR corrections (default IFTJR=1)

Inside DIZET its meaning is limited. See complete discussion about this flag in [2].

NPAR(12) = IFACR → ZUFLAG('EXPR',IFACR) – Realizes different expan-

sions of Δr [26, 59]:
IFACR = **0** (default) realizes the so-called OMS-I renormalization scheme
IFACR = **1** intermediate step from OMS-I to OMS-II renormalization scheme
IFACR = **2** approaches the spirit of the OMS-II renormalization scheme, a fully expanded option

NPAR(13) = IFACT → ZUFLAG('EXPF',IFACT) – To simulate theoretical uncertainties different expansions of the formfactors ρ and κ are realized in complete analogy to the flag IFACR:
IFACT = **0** (default) OMS-I renormalization scheme
IFACT = **1** intermediate step from OMS-I to OMS-II renormalization scheme
IFACT = **2** approaches the OMS-II renormalization scheme

NPAR(14) = IHIGS → ZUFLAG('HIGS',IHIGS) – Switch on/off resummation of the leading Higgs contribution:
IHIGS = **0** (default) leading Higgs contribution is not re-summed
IHIGS = **1** leading Higgs contribution is re-summed

NPAR(15) = IAFMT → ZUFLAG('AFMT',IAFMT) – Includes the three-loop corrections AFMT$\sim \delta^{\alpha\alpha_s}$ [47] (see also description of flag SCAL):
IAFMT = **0** without AFMT correction
IAFMT = **1** correction $\mathcal{O}(G_f m_t^2 \alpha_s^2)$ is included
IAFMT = **2** corrections $\mathcal{O}(G_f m_t^2 \alpha_s^2)$ and $\mathcal{O}(G_f M_Z^2 \alpha_s^2 + \log(m_t^2))$ are included
IAFMT = **3** (default) corrections $\mathcal{O}(G_f m_t^2 \alpha_s^2)$, $\mathcal{O}(G_f M_Z^2 \alpha_s^2 + \log(m_t^2))$ and $\mathcal{O}(G_f M_Z^2/m_t^2 \alpha_s^2)$ are included

NPAR(16) = IEWLC – Treatment of the remainder terms of ρ and κ (used in ROKAPP together with obsolete option AMT4 = 1-3):
IEWLC = **0** all remainders are set equal to zero
IEWLC = **1** (default) standard treatment

NPAR(17) = ICZAK → ZUFLAG('CZAK',ICZAK) – Treatment of the CKHSS non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections $\Delta_{EW/QCD}$ to the quarkonic width, Γ_q [78, 79]:
ICZAK = **0** without CKHSS corrections
ICZAK = **1** (default) with CKHSS corrections
ICZAK = **2** with CKHSS for pseudoobservables and without CKHSS in case of realistic observables

Inside the weak library DIZET the meaning of ICZAK is limited. See also discussion about this flag in Subsection B.2.

NPAR(18) = IHIG2 → ZUFLAG('HIG2',IHIG2) – Handling of the quadratically enhanced two-loop Higgs contributions to Δr [80, 81]:
IHIG2 = **0** without Higgs corrections

IHIG2 = **1** with Higgs corrections

NPAR(19) = IALE2 → ZUFLAG('ALE2',IALE2) – Treatment of leptonic corrections to $\Delta\alpha$:
IALE2 = **0** for backward compatibility with versions up to v.5.12
IALE2 = **1** with one-loop corrections
IALE2 = **2** with two-loop corrections [82]
IALE2 = **3** (default) with three-loop corrections [83]

NPAR(20) = IGFER → ZUFLAG('GFER',IGFER) – Handling of QED corrections to the Fermi constant:
IGFER = **0** for backward compatibility with versions up to v.5.12
IGFER = **1** one-loop QED corrections for Fermi constant [84–86]
IGFER = **2** two-loop QED corrections for Fermi constant [87, 88]

NPAR(21) = IDDZZ – Used in ZWRATE for internal tests:
IDDZZ = **0** RQCDV(A) are set to 0
IDDZZ = **1** (default) standard treatment of FSR QCD corrections

NPAR(22) = IAMW2 → ZUFLAG('AMW2',IAMW2) – incorporates the fermionic two-loop contributions to the prediction for the W boson mass [29, 30, 32]:
IAMW2 = **0** (default) no two-loop corrections to M_W
IAMW2 = **1** with two-loop corrections to M_W

NPAR(23) = ISFSR → ZUFLAG('SFSR',ISFSR) – allows to switch the final state radiation
ISFSR =**-1** both, QED and QED⊗QCD final state radiation are excluded
ISFSR = **0** final state QED radiation is excluded, QED⊗QCDis included
ISFSR = **1** (default) both, QED and QED⊗QCD final state radiation are included

NPAR(24) = IDMWW → ZUFLAG('DMWW',IDMWW) – Simulation of the theoretical error on M_W for AMT4 = 5, 6:
IDMWW = **-1** minimal value for M_W
IDMWW = **0** (default) no shift on M_W applied
IDMWW = **1** maximal value for M_W

NPAR(25) = IDSWW → ZUFLAG('DSWW',IDSWW) – Simulation of the theoretical error on $\sin^2\theta_{\rm eff}^{\rm lept}$ for AMT4 = 6:
IDSWW = **-1** minimal value for $\sin^2\theta_{\rm eff}^{\rm lept}$
IDSWW = **0** (default) no shift on $\sin^2\theta_{\rm eff}^{\rm lept}$ applied
IDSWW = **1** maximal value for $\sin^2\theta_{\rm eff}^{\rm lept}$

the running QED coupling at scale s is calculated with function XFOTF3 as follows:

$$\alpha(s) = \frac{\alpha}{1 - \frac{\alpha}{4\pi}\texttt{DREAL}(\texttt{XFOTF3}(\texttt{IALEM}, \texttt{IALE2}, \texttt{IHVP}, \texttt{IQCD}, 1, \texttt{DAL5H}, -\texttt{S}))} \,. \tag{A.2}$$

B ZFITTER user guide

This Appendix describes technical details of the ZFITTER package. Not all of them have been influenced by the updates over the years. For the convenience of the user, we nevertheless decided to give a complete overview and will repeat a substantial part of the material, which was already presented in Section 4.2 of [2].

ZFITTER is coded in FORTRAN 77. Double-precision variables have been used throughout the program. The package consists of the following FORTRAN files:

zf630_aux.f

zfbib6_40.f

zfmai6_42.f

zfusr6_42.f

acol6_1p.f

bcqcdl5_14.f

bhang4_640.f

bkqcdl5_14.f

dizet6_42.f

expifi6_30.f

funang6_30.f

m2tcor5_11.f

pairho6_40.f

APV_lib.f

The following routines are normally called in the initialization phase of programs using the ZFITTER package in the order listed below: ZUINIT, ZUFLAG, ZUWEAK, ZUCUTS, ZUINFO. An example of different use is described in Section 2.5 of [2].

B.1 Subroutine ZUINIT

Subroutine ZUINIT is used to initialize variables with their default values. This routine *must* be called before any other ZFITTER routine.

```
CALL ZUINIT
```

B.2 Subroutine ZUFLAG

Subroutine ZUFLAG is used to modify the default values of flags which control various ZFITTER options.

```
CALL ZUFLAG(CHFLAG,IVALUE)
```

Input Arguments:

CHFLAG is the character identifier of a ZFITTER flag.
IVALUE is the value of the flag. See Tab. C.2 for a list of the defaults.

Possible combinations of CHFLAG and IVALUE are listed below: [5]

In Tab. C.2 an overview over all flags used in DIZET and ZFITTER is given. The DIZET flags (vector NPAR(1:25) in DIZET corresponds to NPARD(1:25) in ZFITTER) are described in Subsection A.

AFBC – Controls the calculation of the forward backward asymmetry for interfaces ZUTHSM, ZUXSA, ZUXSA2, and ZUXAFB:
IVALUE = **0** asymmetry calculation is inhibited (can speed up the program if asymmetries are not desired)

[5] It is worth noting that not for all flags the default value is necessarily the preferred value. A typical example is flag FINR, distinguishing two different treatments of FSR, which are relevant in different experimental setups.

IVALUE = **1** (default) both cross-section and asymmetry calculations are done

AFMT – see NPAR(15) in subsection A.3

ALEM – Controls the treatment of the running QED coupling $\alpha(s)$:
IVALUE = **0 or 2** $\Delta\alpha_h^{(5)}(M_Z^2)$ must be supplied by the user as input to the DIZET package; using this input DIZET calculates ALQED = $\alpha(M_Z^2)$
IVALUE = **1 or 3** $\Delta\alpha_h^{(5)}(M_Z^2)$ and $\alpha(M_Z^2)$ are calculated by the program using a parameterization IHVP.

The scale of α(scale) is governed in addition by the flag CONV, see description below and the flowchart, Figure 6 of [2]. Values ALEM = 0,1 are accessible only at CONV = 0. Then for ALEM = 0,1 $\alpha(M_Z^2)$ and for ALEM = 2,3 $\alpha(s)$ are calculated. Values ALEM = 2,3 are accessible for CONV = 0,1,2. Then for CONV = 0 $\alpha(s)$ and for CONV = 1,2 $\alpha(s')$ are calculated. Recommended values: ALEM = 2,3.

ALE2 – see NPAR(19) in subsection A.3

AMT4 – see NPAR(2) in subsection A.3

ASCR – is a hidden flag that handles the treatment of contributions to A_{FB}:
IVALUE = 0 treatment as in versions 5 up to version 6.23
IVALUE = 1 (default) new (and very old) treatment

BARB – see NPAR(10) in subsection A.3

BORN – Controls calculation of QED and Born observables:
IVALUE = **0** (default) QED convoluted observables
IVALUE = **1** electroweak observables corrected by Improved Born Approximation

BOXD – Determines calculation of ZZ and WW box contributions, (see Section 3.3.2 of [2]):
IVALUE = **0** no box contributions are calculated
IVALUE = **1** (default) the boxes are calculated as additive separate contribution to the cross-section
IVALUE = **2** box contributions are added to all four form factors

CONV – Controls the energy scale of running α and EWRC, see Figure 6 of [2]:
IVALUE = **0** $\alpha(s)$
IVALUE = **1** (default) $\alpha(s')$ convoluted
IVALUE = **2** both electroweak radiative correction and α_s are convoluted

CZAK – Treatment of CKHSS non-factorized corrections, [78], [79], see Figure

6 of [2]:
IVALUE = **0** without CKHSS corrections
IVALUE = **1** (default) with CKHSS corrections everywhere
IVALUE = **2** CKHSS corrections are taken into account only in POs, this option is used for tests only

DIAG – Selects type of diagrams taken into account:
IVALUE = **−1** only Z- exchange diagrams are taken into account
IVALUE = **0** Z and γ - exchange diagrams are taken into account
IVALUE = **1** (default) Z and γ exchange and $Z\gamma$ interference are included

EXPF – see NPAR(13) in subsection A.3

EXPR – see NPAR(12) in subsection A.3

ENUE – Treatment of the improved Born approximation for the process $e^+e^- \to \nu_e\bar{\nu}_e$. This option is only available via subroutine COSCUT in zfbib6_34.f and using zfEENN_34.f. This flag is *presently not supported*, see also Section 3.
IVALUE = **−1** s-channel only
IVALUE = **0** s- and t-channels
IVALUE = **1** (default) s+t complete with s-t interference

DMWW – see NPAR(24) in subsection A.3

DSWW – see NPAR(25) in subsection A.3

FBHO – treatment of second order corrections to angular distributions and A_{FB}
IVALUE = **0** (default) treatment as in version 6.21 and before
IVALUE = **1** modified treatment; photonic radiative corrections are improved and $\mathcal{O}(\alpha^2)$ contributions from pairs are included in leading-log approximation, see Section 2.1

FINR – Controls the calculation of final-state radiation,
IVALUE = **−1** final-state QED and QCD correction are not applied;
IVALUE = **0** by s' cut, final-state QED correction is described with the factor $1 + 3\alpha(s)/(4\pi)Q_f^2$
IVALUE = **1** (default) M_{ff}^2 cut, includes complete treatment of final-state radiation with common soft-photon exponentiation

FOT2 – Controls second-order leading log and next-to-leading log QED corrections:
IVALUE = **−1** no initial state radiation QED convolution at all
IVALUE = **0** complete α additive radiator
IVALUE = **1** with logarithmic hard corrections
IVALUE = **2** complete α^2 additive radiator

IVALUE = **3** (default) complete α^3 additive radiator
IVALUE = **4** optional α^3 additive radiator for estimation of theoretical errors [89]
IVALUE = **5** "pragmatic" LLA third order corrections in a factorized form [8]

FSPP – correction due to final state radiation into pairs, see Section 2.4:
IVALUE = **0** (default) no final state state pair corrections
IVALUE = **1** final state state pair contributions are implemented as additive corrections
IVALUE = **2** final state state pair contributions are implemented as multiplicative corrections

FSRS – Final state radiation scale:
IVALUE = **0** $\alpha(0)$, preferred for tight cuts
IVALUE = **1** (default) $\alpha(s)$, preferred for loose cuts

FTJR – Treatment of FTJR corrections [77]:
IVALUE = **0** without FTJR corrections
IVALUE = **1** (default) with FTJR corrections everywhere
IVALUE = **2** FTJR corrections are taken into account only in POs, the option is used for tests only

FUNA – implementation of higher order photonic corrections to the angular distribution, see Section 2.2:
IVALUE = **0** (default) no higher order corrections
IVALUE = **1** higher order photonic LLA corrections are included

GAMS – Controls the s dependence of $\mathcal{G}_Z$, the Z-width function, see Section 3.2 of [2]:
IVALUE = **0** forces $\mathcal{G}_Z$ to be constant
IVALUE = **1** (default) allows $\mathcal{G}_Z$ to vary as a function of s [90].

GFER – see NPAR(20) in subsection A.3

HIGS – see NPAR(14) in subsection A.3

HIG2 – see NPAR(18) in subsection A.3

INTF – Determines if the $\mathcal{O}(\alpha)$ initial-final state QED interference (IFI) is calculated; see Section 2.3:
IVALUE = **0** the interference term is ignored
IVALUE = **1** (default) with IFI in the $\mathcal{O}(\alpha)$
IVALUE = **2** with one-loop IFI corrections and corresponding higher order effects from the exponentiation

IPFC – Pair flavour content for the pair production corrections:

IVALUE = **1** only electron pairs
IVALUE = **2** only muon pairs
IVALUE = **3** only tau-lepton pairs
IVALUE = **4** only hadron pairs
IVALUE = **5** (default) all channels summed
IVALUE = **6** leptonic pairs (without hadrons)

IPSC – Pair production singlet-channel contributions (works with ISPP = 2):
IVALUE = **0** (default) only non-singlet pairs
IVALUE = **1** LLA singlet pairs according to [91]
IVALUE = **2** complete $O(\alpha^2)$ singlet pairs, *ibid*
IVALUE = **3** singlet pairs up to order $(\alpha L)^3$, *ibid*

IPTO – Third (and higher) order pair production contributions [12] (works with ISPP = 2):
IVALUE = **−1** allows to calculate the pure virtual pair contributions separately
IVALUE = **0** only $O(\alpha^2)$ contributions
IVALUE = **1** $O(\alpha^3)$ pairs
IVALUE = **2** some "non-standard" $O(\alpha^3)$ LLA pairs added
IVALUE = **3** (default) $O(\alpha^4)$ LLA electron pairs added

ISPP – Treatment of ISR pairs:
IVALUE = **−1** pairs are treated with a "fudge" factor as in versions up to v.5.14
IVALUE = **0** without ISR pairs
IVALUE = **1** with ISR pairs, [92] with a re-weighting
IVALUE = **2** (default) with ISR pairs according to [12]
IVALUE = **3** with ISR pairs according to [93]
IVALUE = **4** with ISR pairs [93] with extended treatment of hadron pair production

MISC – Controls the treatment of scaling of ρ in the Model Independent approach, see discussion in Subsection 6.1:
IVALUE = **0** (default) non-scaled ρ's are used, AROTFZ-array
IVALUE = **1** scaled ρ's, absorbing imaginary parts, are used, ARROFZ-array

MISD – Controls the s dependence of the Model Independent approach
IVALUE = **0** fixed $s = M_Z^2$ in EWRC, old treatment
IVALUE = **1** (default) ensures equal numbers from all interfaces and for all partial channels but INDF=10 for a large range of $\sqrt{s}$ and for INDF=10 up to 100 GeV

PART – Controls the calculation of various parts of Bhabha scattering:
IVALUE = **0** (default) calculation of full Bhabha cross-section and asymmetry

IVALUE = **1** only s channel
IVALUE = **2** only t channel
IVALUE = **3** only $s - t$ interference

POWR – Controls inclusion of final-state fermion masses in kinematical factors, see Figure 6 of [2]. It acts differently for quarks and leptons. For leptons:
IVALUE = **0** final state lepton masses are set equal to zero
IVALUE = **1** (default) final state lepton masses are retained in all kinematical factors
For quarks it is active only for FINR = –1 and then:
IVALUE = **0** final state quark masses are set equal to zero
IVALUE = **1** (default) final state quark masses are set to their running values (that is, for $c\overline{c}$ and $b\overline{b}$ channels) and retained in all kinematical factors

PREC – is an integer number which any precision governing any numerical integration is divided by, increasing thereby the numerical precision of computation:
IVALUE = **10** (default)
IVALUE = **1 – 99** in some cases when some numerical instability while running v.5.10 was registered, it was sufficient to use PREC = 3, in some other cases (e.g. with P_τ) only PREC = 30 solved the instability

PRNT – Controls ZUWEAK printing:
IVALUE = **0** (default) printing by subroutine ZUWEAK is suppressed
IVALUE = **1** each call to ZUWEAK produces some output

QCDC – see NPAR(3) in subsection A.3

SCAL – see NPAR(9) in subsection A.3

SCRE – see NPAR(6) in subsection A.3

SFSR – see NPAR(23) in subsection A.3

TUPV – simulates theoretical uncertainties in APV, see Sect. 4:
IVALUE = 1 (default)
IVALUE = 2,3 for variation, see Sect. 4

VPOL – see NPAR(1) in subsection A.3

WEAK – Determines if the weak loop calculations are to be performed
IVALUE = **–1** is only valid for INDF=–1 and represents the the options WEAK=1,2 without electroweak corrections for W boson exchange in the t-channel
IVALUE = **0** no weak loop corrections to the cross-sections are calculated and weak parameters are forced to their Born values, i.e. $\rho_{ef} = \kappa_{e,f,ef} = 1$

IVALUE = **1** (default) weak loop corrections to the cross-sections are calculated

IVALUE = **2** weak loop corrections are calculated but some higher order corrections that do not propagate via DIZET are swiched off (ADDIME, ADDIMF; these corrections are small at LEP2 and they are not included by using DIZET with other codes

B.3 Subroutine ZUWEAK

Subroutine ZUWEAK is used to perform the weak sector calculations. These are done internally with DIZET, see Section A. The routine calculates a number of important electroweak parameters which are stored in common blocks for later use (see Section 6.1). If any ZFITTER flag has to be modified this must be done before calling ZUWEAK.

```
CALL ZUWEAK(ZMASS,TMASS,HMASS,DAL5H,ALFAS)
```

Input Arguments:

ZMASS is the Z mass M_Z in GeV.
TMASS is the top quark mass m_t in GeV, [10-400].
HMASS is the Higgs mass M_H in GeV, [10-1000].
DAL5H is the value of $\Delta\alpha_h^{(5)}(M_Z^2)$.
ALFAS is the value of the strong coupling constant α_s at $q^2 = M_Z^2$ (see factors QCDCOR in Tab. A.1).

Computing time may be saved by performing weak sector calculations only once during the initialization of the ZFITTER package. This is possible since weak parameters are nearly independent of s near the Z peak, e.g. $\sim \ln s/M_Z^2$. However, the incredible precision of LEP1 data forced us to give up this option, see description of flag MISD.

B.4 Subroutine ZUCUTS

Subroutine ZUCUTS is used to define kinematic and geometric cuts for each fermion channel: it selects the appropriate QED calculational *chain*.

```
CALL ZUCUTS(INDF,ICUT,ACOL,EMIN,S_PR,ANG0,ANG1,SIPP)
```

Input Arguments:

INDF is the fermion index (see Tab. C.1 and Figure 5 of [2]).

ICUT controls the kinds of cuts (*chain*) to be used.

- ICUT = **−1:** (default) allows for an s' cut (a cut on $M^2_{f\bar{f}}$, the fermion and antifermion invariant mass); the fastest branch based on [94]
- ICUT = **0: not recommended**; branch is known to contain bugs. It allows for a cut on the acollinearity ACOL of the $f\bar{f}$ pair, on the minimum energy EMIN of both fermion and antifermion, and for a geometrical acceptance cut [95] [6]
- ICUT = **1:** s' or $M^2_{f\bar{f}}$ cuts and geometrical acceptance cut, based on [99]
- ICUT = **2:** new branch, replaces ICUT = 0 for realistic cuts ACOL and EMIN, based on [96]
- ICUT = **3:** the same branch, using ACOL cut and EMIN cut but also with possibility to impose an additional acceptance cut [98]

ACOL is the maximum acollinearity angle $\xi^{\max}$ of the $f\bar{f}$ pair in degrees (ICUT = 0,2,3).

EMIN is the minimum energy $E_f^{\min}$ of the fermion and antifermion in GeV (ICUT = 0,2,3).

S_PR is the minimum allowed invariant $f\bar{f}$ mass $M^2_{f\bar{f}}$ in GeV (ICUT = −1,1) or, with some approximations, the minimum allowed invariant mass of the propagator after ISR [7]

ANG0 (default = 0°) is the minimum polar angle ϑ in degrees of the final-state antifermion.

ANG1 (default = 180°) is the maximum polar angle ϑ in degrees of the final-state antifermion.

SIPP is a parameter for cuts on the invariant mass of secondary pairs for FSPP=1,2 (see subsection 2.4). In older versions before v.6.30 the parameter SIPP governed the calulation of corrections due to initial state pairs and was recommended to be chosen equal to s'.

B.5 Subroutine ZUINFO

Subroutine ZUINFO prints the values of ZFITTER flags and cuts.

```
CALL ZUINFO(MODE)
```

[6] As was shown recently [96–98], the old results of [95] contained bugs which occasionally didn't show up in comparisons as e.g. in [39]. The option is retained for back-compatibility with older versions only.

[7] The invariant mass of the propagator is not an observable quantity unless specific assumptions on ISR and FSR are made.

Input Argument:

MODE controls the printing of ZFITTER flag and cut values.
MODE = **0:** Prints all flag values.
MODE = **1:** Prints all cut values.

C Interface routines of ZFITTER

This Appendix describes technical details of the interface routines of ZFITTER. Not all of them have been influenced by the updates over the years. For the convenience of the user, we nevertheless decided to give a complete overview and will repeat a substantial part of the material, which was already presented in Section 4.3 of [2].

Note that subroutine ZUWEAK must be called prior to the interfaces. As a consequence, the flags used in ZUWEAK may influence the calculation of cross-sections and asymmetries in the interfaces described now.

All subroutines need the following input arguments:

INDF is the fermion index (see Tab. C.1).
SQRS is the centre-of-mass energy $\sqrt{s}$ in GeV.
ZMASS is the Z mass M_Z in GeV.

We just mention that the interface using an S-matrix inspired language is realized with the SMATASY package [100–103].

C.1 Subroutine ZUTHSM

Subroutine ZUTHSM is used to calculate Standard Model cross-sections and forward-backward asymmetries.

```
CALL ZUTHSM(INDF,SQRS,ZMASS,TMASS,HMASS,DAL5H,ALFAS,XS*,AFB*)
```

Input Arguments:

TMASS is the top quark mass m_t in GeV, [10-400].
HMASS is the Higgs mass M_H in GeV, [10-1000].

DAL5H is the value of $\Delta\alpha_h^{(5)}(M_Z^2)$.
ALFAS is the value of the strong coupling constant α_s at $q^2 = M_Z^2$ (see also flag QCDC and factors QCDCOR).

Output Arguments[8]:

XS is the total cross-section σ_T in nb.
AFB is the forward-backward asymmetry A_{FB}.

Output Internal Flag:

INTRF=1

C.2 Subroutine ZUATSM

Subroutine ZUATSM is used to calculate differential cross-sections, $d\sigma/d\cos\theta$, in the Standard Model.

```
CALL ZUATSM(INDF,SQRS,ZMASS,TMASS,HMASS,DAL5H,ALFAS,CSA*,DXS*)
```

Input Arguments:

TMASS is the top quark mass m_t in GeV, [10-400].
HMASS is the Higgs mass M_H in GeV, [10-1000].
ALQED is the value of the running electromagnetic coupling constant.
ALFAS is the value of the strong coupling constant α_s at $q^2 = M_Z^2$ (see factors QCDCOR).
CSA is the cosine of the scattering angle.

Output Arguments:

DXS is the theoretical differential cross-section.

Output Internal Flag:

INTRF=1

[8] An asterisk (*) following an argument in a calling sequence is used to denote an output argument.

C.3 Subroutine ZUTPSM

Subroutine ZUTPSM is used to calculate the tau polarization and tau polarization asymmetry in the Standard Model.

```
CALL ZUTPSM(SQRS,ZMASS,TMASS,HMASS,DAL5H,ALFAS,TAUPOL*,TAUAFB*)
```

Input Arguments:

HMASS is the Higgs mass M_H in GeV, [10-1000].
DAL5H is the value of $\Delta\alpha_h^{(5)}(M_Z^2)$.
ALFAS is the value of the strong coupling constant α_s at $q^2 = M_Z^2$ (see factors QCDCOR).

Output Arguments:

TAUPOL is the tau polarization $A_{\rm pol}$ of Equation 3.312 of [2].
TAUAFB is the tau polarization forward-backward asymmetry $A_{FB}^{\rm pol}$ as defined in Equation 3.313 of [2].

Output Internal Flag:

INTRF=1

C.4 Subroutine ZULRSM

Subroutine ZULRSM is used to calculate the left-right asymmetry in the Standard Model.

```
CALL ZULRSM(INDF,SQRS,ZMASS,TMASS,HMASS,DAL5H,ALFAS,POL,XSPL*,XSMI*)
```

Input Arguments:

TMASS is the top quark mass m_t in GeV, [40-300].
HMASS is the Higgs mass M_H in GeV, [10-1000].
DAL5H is the value of $\Delta\alpha_h^{(5)}(M_Z^2)$.

ALFAS is the value of the strong coupling constant α_s at $q^2 = M_Z^2$ (see also flag ALST).
POL is the degree of longitudinal polarization of electrons.

Output Arguments:

XSPL is the cross-section for POL > 0
XSMI is the cross-section for POL < 0

Output Internal Flag:

INTRF=1

C.5 Subroutine ZUXSA

Subroutine ZUXSA is used to calculate cross-section and forward-backward asymmetry as described in Section 3.7.5 of [2] as functions of $\sqrt{s}$, M_Z, Γ_Z.

```
CALL ZUXSA(INDF,SQRS,ZMASS,GAMZ0,MODE,GVE,XE,GVF,XF,XS*,AFB*)
```

Input Arguments:

GAMZ0 is the total Z width Γ_Z in GeV.
MODE determines which weak couplings are used:
- MODE = **0:** XE (XF) are effective axial-vector couplings $\bar{a}_{e,f}$ for electrons (final-state fermions).
- MODE = **1:** XE (XF) are the effective weak neutral-current amplitude normalizations $\bar{\rho}_{e,f}$ for electrons (final-state fermions).

GVE is the effective vector coupling for electrons $\bar{g}_e$.
XE is the effective axial-vector coupling $\bar{a}_e$ or weak neutral-current amplitude normalization $\bar{\rho}_e$ for electrons (see MODE).
GVF is the effective vector coupling for the final-state fermions $\bar{g}_f$.
XF is the effective axial-vector coupling $\bar{a}_f$ or the weak neutral-current amplitude normalization $\bar{\rho}_f$ for the final-state fermions (see MODE).

Output Arguments:

XS is the cross-section σ_T in nb.
AFB is the forward-backward asymmetry A_{FB}.

Output Internal Flag:

`INTRF=3`

C.6 Subroutine ZUXSA2

Subroutine `ZUXSA2` is used to calculate lepton cross-section and forward-backward asymmetry as functions of $\sqrt{s}$, M_Z, Γ_Z, and of the weak couplings *assuming lepton universality*. This routine is similar to `ZUXSA` except that the couplings are squared.

```
CALL ZUXSA2(INDF,SQRS,ZMASS,GAMZ0,MODE,GV2,X2,XS*,AFB*)
```

Input Arguments:

`GAMZ0` is the total Z width Γ_Z in GeV.
`MODE` determines which weak couplings are used:
- `MODE` = **0:** `X2` is the square of the effective axial-vector coupling $\bar{a}_l$ for leptons.
- `MODE` = **1:** `X2` is the square of the effective neutral-current amplitude normalization $\bar{\rho}_l$ for leptons.

`GV2` is the square of the effective vector coupling $\bar{g}_l$ for leptons.
`X2` is the square of the effective axial-vector coupling $\bar{a}_l$ or neutral-current amplitude normalization $\bar{\rho}_l$ for leptons (see `MODE`).

Output Arguments:

`XS` is the cross-section σ_T in nb.
`AFB` is the forward-backward asymmetry A_{FB}.

Output Internal Flag:

`INTRF=4`

C.7 Subroutine ZUTAU

Subroutine `ZUTAU` is used to calculate the τ^+ polarization as a function of $\sqrt{s}$, M_Z, Γ_Z, and the weak couplings.

```
CALL ZUTAU(SQRS,ZMASS,GAMZ0,MODE,GVE,XE,GVF,XF,TAUPOL*,TAUAFB*)
```

Input Arguments:

GAMZ0 is the total Z width Γ_z in GeV.
MODE determines which weak couplings are used:
- MODE = **0:** XE (XF) is the effective axial-vector coupling $\bar{a}_{e,f}$ for electrons (final-state fermions).
- MODE = **1:** XE (XF) is the effective weak neutral-current amplitude normalization $\bar{\rho}_{e,f}$ for electrons (final-state fermions).

GVE is the effective vector coupling for electrons $\bar{g}_e$.
XE is the effective axial-vector coupling $\bar{a}_e$ or weak neutral-current amplitude normalization $\bar{\rho}_e$ for electrons (see MODE).
GVF is the effective vector coupling for the final-state fermions $\bar{g}_f$.
XF is the effective axial-vector coupling $\bar{a}_f$ or weak neutral-current amplitude normalization $\bar{\rho}_f$ for the final-state fermions (see MODE).

Output Arguments:

TAUPOL is the τ-lepton polarization λ_τ
TAUAFB is the forward-backward asymmetry for polarized τ-leptons $A_{FB}^{\rm pol}$

Output Internal Flag:

INTRF=3

C.8 Subroutine ZUXSEC

Subroutine ZUXSEC is used to calculate the cross section as a function of $\sqrt{s}$, M_Z, Γ_z, Γ_e and Γ_f.

```
CALL ZUXSEC(INDF,SQRS,ZMASS,GAMZ0,GAMEE,GAMFF,XS*)
```

Input Arguments:

GAMZ0 is the total Z width Γ_z in GeV.
GAMEE is the partial Z decay width Γ_e in GeV.
GAMFF is the partial Z decay width Γ_f in GeV; if INDF=10, GAMFF=Γ_h.

Output Internal Flag:

`INTRF=2`

C.9 Subroutine `ZUXAFB`

Subroutine `ZUXAFB` is used to calculate the cross section as a function of $\sqrt{s}$, M_z, Γ_z, Γ_e and Γ_f.

```
CALL ZUXAFB(INDF,SQRS,ZMASS,GAMZ0,PFOUR,PVAE2,PVAF2,XS*,AFB*)
```

Input Arguments:

`GAMZ0` is the total Z width Γ_z in GeV.
`PFOUR` is the product of vector and axial-vector couplings $\bar{g}_e\bar{a}_e\bar{g}_f\bar{a}_f$.
`PVAE2` is $\bar{g}_e^2 + \bar{a}_e^2$.
`PVAF2` is $\bar{g}_f^2 + \bar{a}_f^2$.

Output Argument:

`XS` is the cross-section σ_T in nb.
`AFB` is the forward-backward asymmetry A_{FB}.

Output Internal Flag:

`INTRF=5`

C.10 Subroutine `ZUALR`

Subroutine `ZUALR` is reserved for the fit of A_{LR}.

```
CALL ZUALR(SQRS,ZMASS,GAMZ0,MODE,GVE,XE,GVF,XF,TAUPOL*,TAUAFB*)
```

Output Internal Flag:

`INTRF=6`

C.11 Subroutine ZVWEAK

The strength of the Wtb vertex, $|V_{tb}|$, can be changed by using the subroutine ZVWEAK instead of ZUWEAK; see Section 1.

```
CALL ZVWEAK(ZMASS,TMASS,HMASS,DAL5H,V_TB,ALFAS)
```

Input Arguments:

V_TB is the value of $|V_{tb}|$; the Standard Model calculation of ZUWEAK corresponds to $|V_{tb}| = 1$.

ZVWEAK has to be complemented with new interface routines, all now with the the additional input argument V_TB:

```
ZVTHSM(INDF,SQRS,ZMASS,TMASS,HMASS,DAL5H,V_TB,ALFAS,XS*,AFB*)
ZVTPSM(SQRS,ZMASS,TMASS,HMASS,DAL5H,V_TB,ALFAS,TAUPOL*,TAUAFB*)
ZVLRSM(INDF,SQRS,ZMASS,TMASS,HMASS,DAL5H,V_TB,ALFAS,POL,XSPL*,XSMI*)
ZVATSM(INDF,SQRS,ZMASS,TMASS,HMASS,DAL5H,V_TB,ALFAS,CSA,DXS*)
```

These subroutines replace ZUTHSM, ZUTPSM, ZULRSM and ZUATSM.

C.12 Subroutine ZU_APV

With subroutine ZU_APV measurements of the weak charge, Q_W, (see Section 4) can be included into the global tests of the Standard Model.

```
CALL ZU_APV(ZMASS,TMASS,HMASS,SIN2TW,UMASS,DMASS,C1U*,C1D*,C2U*,C2D*)
```

Input Arguments:

SIN2TW is the sin of the weak mixing angle.
UMASS is the u-quark mass (constituent).
DMASS is the d-quark mass (constituent).

Output Argument:

C1U – coupling parameter, $C_{1u} = 2a_e v_u$
C1D – coupling parameter, $C_{1d} = 2a_e v_d$

`C2U` – coupling parameter, $C_{2u} = 2v_e a_u$
`C2D` – coupling parameter, $C_{2d} = 2v_e a_d$

Final-state fermions	$\nu\bar{\nu}$	e^+e^-	$\mu^+\mu^-$	$\tau^+\tau^-$	$u\bar{u}$	$d\bar{d}$	$c\bar{c}$	$s\bar{s}$	$t\bar{t}$	$b\bar{b}$	hadrons	Bhabha
`INDF`	0	1	2	3	4	5	6	7	8	9	10	11

Table C.1
Indices for the selection of final states. Note that INDF=0 returns values for one neutrino species, while INDF=10 returns values for the five-flavour inclusive (udscb) hadronic channel. Also note that `INDF` *= 0,1 includes only s-channel calculations while* `INDF` *= 8 always returns zero.*

I	'FLAG'	name in programs	Position in DIZET NPAR(1:25)	Position in ZFITTER NPAR(1:30)	default value
1	AFBC	IAFB		13	1
2	SCAL	ISCAL	9	15	0
3	SCRE	ISCRE	6		0
4	AMT4	IAMT4	2	16	4
5	BORN	IBORN		14	0
6	BOXD	IBOX		4	1
7	CONV				1
8	FINR	IFINAL		9	1
9	FOT2	IPHOT2		10	3
10	GAMS			5	1
11	DIAG			7	1
12	INTF	INTERF		8	1
13	BARB	IBARB	10		2
14	PART	IPART		17	0
15	POWR				1
16	PRNT				0
17	ALEM	IALEM	7	20	3
18	QCDC	IQCD	3	3	3
19	VPOL	IHVP	1	2	1
20	WEAK	IWEAK		1	1
21	FTJR	IFTJR	11		1
22	EXPR	IFACR	12		0
23	EXPF	IFACT	13	19	0
24	HIGS	IHIGS	14		0
25	AFMT	IAFMT	15		3
26	CZAK	ICZAK	17		1
27	PREC	NPREC			10
28	HIG2	IHIG2	18		0
29	ALE2	IALE2	19	21	3
30	GFER	IGFER	20		2
31	ISPP	ISRPPR			2
32	FSRS				1
33	MISC	IMISC			0
34	MISD	IMISD			1
35	IPFC				5
36	IPSC				0
37	IPTO				3
38	FBHO				0
39	FSPP	IFSPPR			0
40	FUNA	IFUNAN			0
41	ASCR				1
42	SFSR	ISFSR	23		1
43	ENUE				1
44	TUPV	ITUPV			1
45	DMWW	IDMWW	24		0
46	DSWW	IDSWW	25		0
		IMOMS	4		1
		IMASS	5		0
		IMASK	8		0
		IEWLC	16		1
		IDDZZ	21		1
		IAMW2	22		0

Table C.2

Flag settings for ZFITTER *and* DIZET*; the flags are listed in the order of vector* IFLAGS*. The corresponding names used internally in the programs, the position of the flags in vector* NPAR(1:25) *of* DIZET *(called* NPARD *in* ZFITTER*) and* NPAR(1:30) *of* ZFITTER *and the default values are given*

References

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