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What Fraction of Boron-8 Solar Neutrinos arrive at the Earth as a ν_2 mass eigenstate?*

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Abstract

We calculate the fraction of ^{8}B solar neutrinos that arrive at the Earth as a ν_2 mass eigenstate as a function of the neutrino energy. Weighting this fraction with the ^{8}B neutrino energy spectrum and the energy dependence of the cross section for the charged current interaction on deuteron with a threshold on the kinetic energy of the recoil electrons of 5.5 MeV, we find that the integrated weighted fraction of ν_2's to be 91±2% at the 95% CL. This energy weighting procedure corresponds to the charged current response of the Sudbury Neutrino Observatory (SNO). We have used SNO's current best fit values for the solar mass squared difference and the mixing angle, obtained by combining the data from all solar neutrino experiments and the reactor data from KamLAND. The uncertainty on the ν_2 fraction comes primarily from the uncertainty on the solar δm^2 rather than from the uncertainty on the solar mixing angle or the Standard Solar Model. Similar results for the Super-Kamiokande experiment are also given. We extend this analysis to three neutrinos and discuss how to extract the modulus of the Maki-Nakagawa-Sakata mixing matrix element U_{e2} as well as place a lower bound on the electron number density in the solar ^{8}B neutrino production region.

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* Dedicated to the memory of John Bahcall who championed solar neutrinos for many lonely years.

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I. INTRODUCTION

Recently the KamLAND [1] and Sudbury Neutrino Observatory (SNO) [2] experiments have given a precise determination of the neutrino solar mass squared difference and mixing angle responsible for the solar neutrino deficit first observed in the Davis [3] experiment when compared to the theoretical calculations by Bahcall [4]. Subsequently this deficit has been observed by many other experiments [5, 6], while the theoretical calculations of the neutrino flux based on the Standard Solar Model (SSM) has been significantly improved[7]. When all of these results are combined in a two neutrino fit as reported by SNO [2], the allowed values for the solar mass squared difference, $\delta m^2_\odot$, and the mixing angle, $\theta_\odot$, are individually (for 1 degree of freedom) restricted to the following range[1],

$$\begin{aligned} \delta m^2_\odot &= 8.0^{+0.4}_{-0.3} \times 10^{-5}\mathrm{eV}^2, \\ \sin^2\theta_\odot &= 0.310 \pm 0.026, \end{aligned} \tag{1}$$

at the 68 % confidence level. Maximal mixing, $\sin^2\theta_\odot = 0.5$, has been ruled out at greater than 5 σ. The solar neutrino data is consistent with $\nu_e \to \nu_\mu$ and/or ν_τ conversion. The precision on $\delta m^2_\odot$ comes primarily from the KamLAND experiment [1] whereas the precision on $\sin^2\theta_\odot$ comes primarily from the SNO experiment [2].

The physics responsible for the reduction in the solar ^{8}B electron neutrino flux is the Wolfenstein matter effect [9] with the electron neutrinos produced above the Mikeyev-Smirnov (MS) resonance [10]. The combination of these two effects in the large mixing angle (LMA) region, given by Eq. (1), implies that the ^{8}B solar neutrinos are produced and propagate adiabatically to the solar surface, and hence to the earth, as almost a pure ν_2 mass eigenstate.[2] Since, approximately one third of the ν_2 mass eigenstate is ν_e, this explains the solar neutrino deficit first reported by Davis. If the ^{8}B solar neutrinos arriving at the Earth were 100% ν_2, then the *day-time* Charged Current (CC) to Neutral Current (NC) ratio, CC/NC, measured by SNO would be exactly $\sin^2\theta_\odot$, the fraction of ν_e in ν_2 in the two neutrino analysis.

Of course, the ν_2 mass eigenstate purity of the solar ^{8}B neutrinos is not 100%, as we will

[1] We use the notation of [8] with the subscript "$\odot$" reserved for the two neutrino analysis whereas the subscript "12" is reserved for the three neutrino analysis.

[2] Without the matter effect, the fraction of ν_2's would be simply $\sin^2\theta_\odot$, i.e. about 31%, and energy independent.

see later, some fraction arrive as ν_1's and if the electron neutrino has a non-zero component in ν_3 (i.e. non-zero $\sin^2\theta_{13}$) then there will be a small fraction arriving as ν_3's. For all practical solar neutrino experiments, these mass eigenstates can be considered to be incoherent, see [11]. The mass eigenstate purity of the ^{8}B solar neutrinos is the main subject of this paper. In the next section we will summarize the important physics of the MSW-LMA solar neutrino solution outlined above and calculate the mass eigenstate purity of ^{8}B neutrinos as a function of the neutrino energy in a two neutrino analysis for both the SNO and Super-Kamiokande (SK) experiments. In section 3 we will discuss what happens in a full three neutrino analysis. In section 4, as an application of the previous sections, we will discuss the possibility of extracting information about the solar interior independently from the standard solar model. Finally, in section 5, we present our summary and conclusions.

II. TWO NEUTRINO ANALYSIS:

A. ^{8}B ν_2 Fraction

In the two neutrino analysis, let $f_1(E_\nu)$ and $f_2(E_\nu)$ be the fraction of ^{8}B solar neutrinos of energy E_ν which exit the Sun and thus arrive at the Earth's surface as either a ν_1 or a ν_2 mass eigenstate, respectively. Following the analytical studies of Ref. [12], these fractions are given by

$$f_1(E_\nu) = \langle \cos^2\theta_\odot^N - P_x \cos 2\theta_\odot^N \rangle_{^8\mathrm{B}}, \tag{2}$$

$$f_2(E_\nu) = \langle \sin^2\theta_\odot^N + P_x \cos 2\theta_\odot^N \rangle_{^8\mathrm{B}}, \tag{3}$$

where $\theta_\odot^N$ is the mixing angle defined at the ν_e production point, P_x is the probability of the neutrino to jump from one mass eigenstate to the other during the MS-resonance crossing, and the sum is constrained to be 1, $f_1 + f_2 = 1$. The average $\langle \cdots \rangle_{^8\mathrm{B}}$ is over the electron density of the ^{8}B ν_e production region in the center of the Sun predicted by the Standard Solar Model [13]. The mixing angle, $\theta_\odot^N$, and the mass difference squared, δm_N^2, at the production point are

$$\sin^2\theta_\odot^N = \frac{1}{2}\left\{1 + \frac{(A - \delta m_\odot^2 \cos 2\theta_\odot)}{\sqrt{(\delta m_\odot^2 \cos 2\theta_\odot - A)^2 + (\delta m_\odot^2 \sin 2\theta_\odot)^2}}\right\}, \tag{4}$$

$$\delta m_N^2 = \sqrt{(\delta m_\odot^2 \cos 2\theta_\odot - A)^2 + (\delta m_\odot^2 \sin 2\theta_\odot)^2} \tag{5}$$

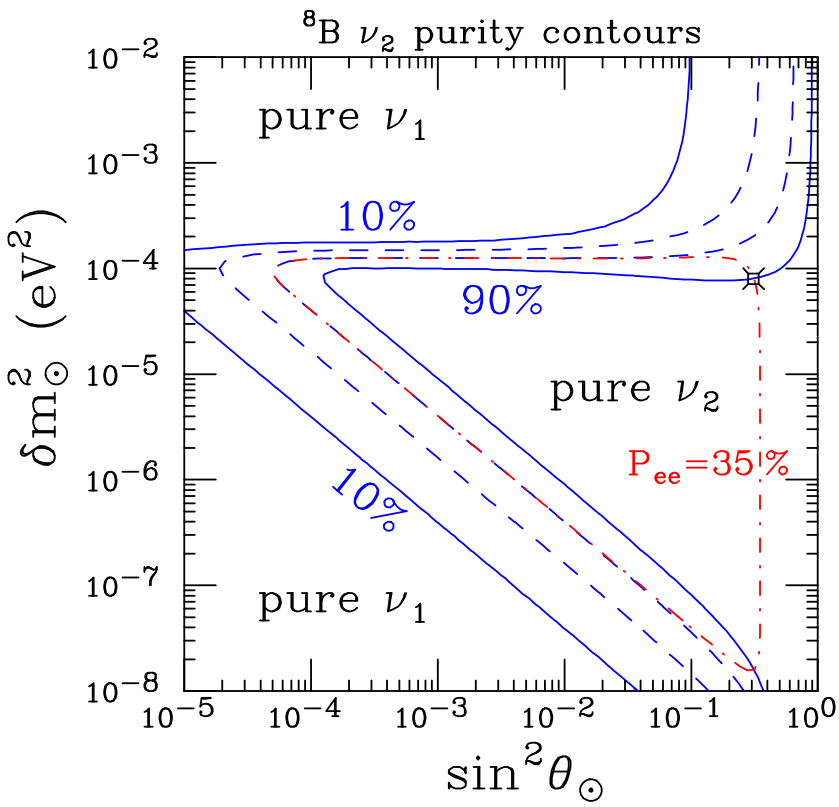

FIG. 1: The solid and dashed (blue) lines are the 90, 65, 35 and 10% iso-contours of the fraction of the solar ^{8}B neutrinos that are ν_2's in the $\delta m^2_\odot$ and $\sin^2\theta_\odot$ plane. The current best fit value, indicated by the open circle with the cross, is close to the 90% contour. The iso-contour for an electron neutrino survival probability, P_{ee}, equal to 35% is the dot-dashed (red) "triangle" formed by the 65% ν_2 purity contour for small $\sin^2\theta_\odot$ and a vertical line in the pure ν_2 region at $\sin^2\theta_\odot = 0.35$. Except at the top and bottom right hand corners of this triangle the ν_2 purity is either 65% or 100%.

where

$$A \equiv 2\sqrt{2}G_F(Y_e\rho/M_n)E_\nu = 1.53\times 10^{-4}\mathrm{eV}^2\left(\frac{Y_e\rho\ E_\nu}{\mathrm{kg.cm}^{-3}\mathrm{MeV}}\right), \tag{6}$$

is the matter potential, E_ν is the neutrino energy, G_F is the Fermi constant, Y_e is the electron fraction (the number of electron per nucleon), M_n is the nucleon mass and ρ is the matter density. The combination $Y_e\rho/M_n$ is just the number density of electrons.

Fig. 1 shows, for a wide range of $\delta m^2_\odot$ and $\sin^2\theta_\odot$, the iso-contours of

$$f_2 \equiv \langle f_2(E_\nu)\rangle_E, \tag{7}$$

where $\langle\cdots\rangle_E$ is the average over the ^{8}B neutrino energy spectrum [14] convoluted with the energy dependence of the CC interaction $\nu_e + d \to p + p + e^-$ cross section [15] at

SNO with the threshold on the recoil electron's kinetic energy of 5.5 MeV. Here we use $\sin^2\theta_\odot$ as the metric for the mixing angle as it is the fraction of ν_e's in the vacuum ν_2 mass eigenstate. In this work, we mainly focus on SNO rather than SK since the former is the unique solar neutrino experiment which can measure the total active ^{8}B neutrino flux as well as ^{8}B electron neutrino flux, independently from the SSM prediction and other experiments. However, we give a brief discussion on SK later in this section.

In the LMA region the propagation of the neutrino inside the Sun is highly adiabatic [10, 12, 16], i.e. $P_x \approx 0$, therefore,

$$f_2(E_\nu) \equiv 1 - f_1(E_\nu) \;=\; \langle \sin^2\theta_\odot^N \rangle_{^8\mathrm{B}}. \tag{8}$$

Due to the fact that ^{8}B neutrinos are produced in a region where the density is significantly higher (about a factor of four) than that of the MS-resonance value, the average $\langle f_2(E_\nu)\rangle_E$ is close to 90% for the current solar best fit values of the mixing parameters from the recent KamLAND plus SNO analysis [2]. Since $\sin^2\theta_\odot^N \to 1$ when $A/\delta m^2_\odot \to \infty$ (see Eq. (4)), we can see that at the high energy end of the ^{8}B neutrinos $\langle \sin^2\theta_\odot^N \rangle_{^8\mathrm{B}}$ must be close to 1.

We can check our result using the analysis of SNO with a simple back of the envelope calculation. In terms of the fraction of ν_1 and ν_2 the *day-time* CC/NC of SNO, which is equal to the day-time average ν_e survival probability, $\langle P_{ee}\rangle$, is given by

$$\left.\frac{\mathrm{CC}}{\mathrm{NC}}\right|_{\mathrm{day}} \;=\; \langle P_{ee}\rangle = f_1\cos^2\theta_\odot + f_2\sin^2\theta_\odot, \tag{9}$$

where f_1 and f_2 are understood to be the ν_1 and ν_2 fractions, respectively, averaged over the ^{8}B neutrino energy weighted with the CC cross section, as mentioned before. Using the central values reported by SNO [3],

$$\left.\frac{\mathrm{CC}}{\mathrm{NC}}\right|_{\mathrm{day}} = 0.347 \pm 0.038, \tag{10}$$

which was obtained from Table XXVI of Ref. [2], and the current best fit value of the mixing angle, we find $f_2 = (1 - f_1) \approx 90\%$, as expected. Due to the correlations in the uncertainties

[3] For the sake of simplicity and transparentness of the discussion, we have avoided the Earth matter effect which causes the so called regeneration of ν_e during night, by simply restricting our analysis to the day time neutrino flux throughout this paper. We note that due to the large error, the observed night-day asymmetry at SNO is consistent with any value from -8 to 5% [2] whereas the expected night-day asymmetry, 2(N-D)/(N+D), is about 2.2-3.5% for the current allowed solar mixing parameters [21]. Thus the difference between the day and the day plus night average CC/NC is less than 2% and much smaller than SNO's 10% measurement uncertainty on CC/NC.

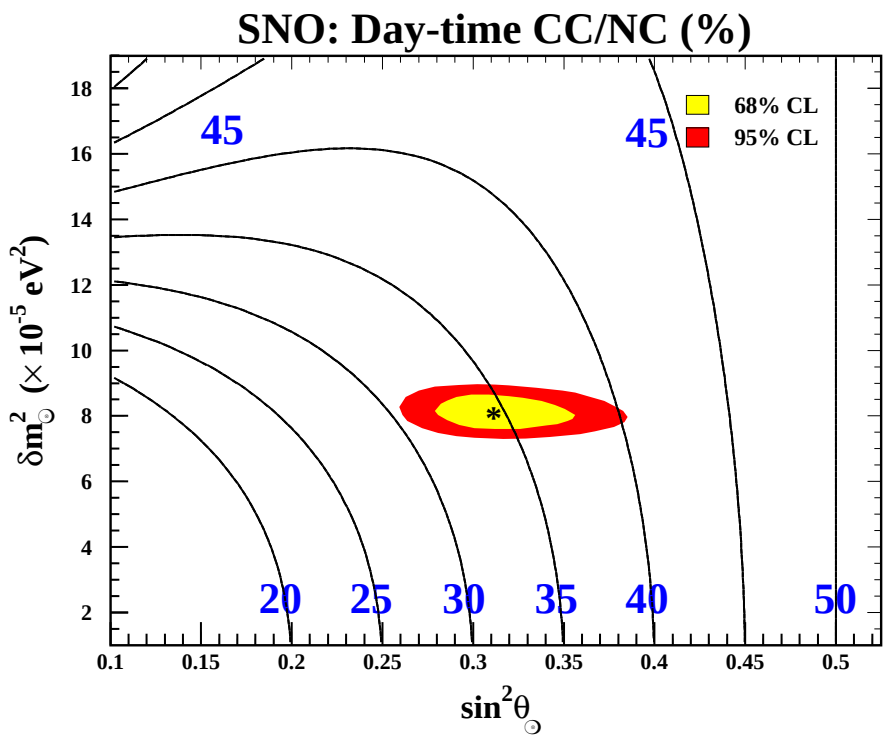

FIG. 2: SNO's Day-time CC/NC ratio in the $\delta m^2_\odot$ versus $\sin^2\theta_\odot$ plane. At small values of $\delta m^2_\odot$, the Day-time CC/NC ratio equals $\sin^2\theta_\odot$. The current allowed region at 68 and 95% CL from the combined fit of KamLAND and solar neutrino data [2] are also shown by the shaded areas with the best fit indicated by the star.

between the CC/NC ratio and $\sin^2\theta_\odot$ we are unable to estimate the uncertainty on f_2 here. Note, that if the fraction of ν_2 were 100%, then $\frac{\rm CC}{\rm NC} = \sin^2\theta_\odot$.

Alternatively, we can rewrite Eq. (9) as[4]

$$\sin^2\theta_\odot \;=\; \frac{1}{1-2f_1}\left(\frac{\rm CC}{\rm NC} - f_1\right). \tag{11}$$

Thus how much CC/NC differs from $\sin^2\theta_\odot$ is determined by how much f_2 differs from 100%, i.e. the size of f_1. In Fig. 2 we have plotted the contours of the day-time CC/NC ratio in the $\sin^2\theta_\odot$ versus $\delta m^2_\odot$ plane for the LMA region. Clearly, at smaller values of $\delta m^2_\odot$ the day time CC/NC tracks $\sin^2\theta_\odot$ whereas at larger values an appreciable difference appears. This difference is caused by a decrease (increase) in the fraction that is ν_2 (ν_1) as $\delta m^2_\odot$ gets larger. Hence if we know the ν_1 or ν_2 fraction we can easily calculate $\sin^2\theta_\odot$ from Eq. (11) using a

[4] The relationship between day-time $\frac{\rm CC}{\rm NC}$ and $\theta_\odot$ $\left(= \arcsin\sqrt{(\frac{\rm CC}{\rm NC} - f_1)/(1-2f_1)}\right)$ or $\tan^2\theta_\odot$ $\left(= (\frac{\rm CC}{\rm NC} - f_1)/(1 - f_1 - \frac{\rm CC}{\rm NC})\right)$ is not as transparent as $\sin^2\theta_\odot$.

measured value of the day-time CC/NC ratio.

A similar analysis can also be performed using the event rate of the elastic scattering (ES) at SK and/or at SNO. In fact, ES is related to the ν_1 and ν_2 fractions, as follows,

$$\frac{\mathrm{ES}}{\mathrm{NC}} = f_1(\cos^2\theta_\odot + r\sin^2\theta_\odot) + f_2(\sin^2\theta_\odot + r\cos^2\theta_\odot) \tag{12}$$

where $r \equiv \langle\sigma_{\nu_{\mu,\tau}e}\rangle/\langle\sigma_{\nu_e e}\rangle \approx 0.155$ is the ratio of the ES cross sections for $\nu_{\mu,\tau}$ and ν_e [18], averaged over the observed neutrino spectrum. Note that we are normalizing the ES event rate to that of SNO NC such that Eq. (12) is valid independent of the SSM prediction of the ^{8}B neutrino flux.

In general, in the presence of neutrino flavor transitions, the fraction of ν_1 and ν_2 are not the same for ES and CC because the energy dependence of the cross sections are different. However, in Ref. [17], it was suggested that if we set analysis threshold energies for SK and SNO appropriately as $T_{\mathrm{SNO}} = 0.995\ T_{\mathrm{SK}} - 1.71$ (MeV), where T_{SNO} and T_{SK} are the kinetic energy threshold of the resulting electron, the energy response of these detectors become practically identical [17]. Thus, using such a set of thresholds, even if there is a spectral distortion in the recoil electron energy spectrum, to a good approximation, SK/SNO ES and SNO CC are related as follows,

$$\frac{\mathrm{ES}}{\mathrm{NC}} = \frac{\mathrm{CC}}{\mathrm{NC}} + r\left(1 - \frac{\mathrm{CC}}{\mathrm{NC}}\right), \tag{13}$$

and all the results we obtained for SNO in this paper are equally valid for ES at SK and/or at SNO provided the energy thresholds are set appropriately[5].

In Fig. 3(a) we show the ν_2 fraction, $f_2(E_\nu)$, versus E_ν. The rapid decrease in the ν_2 fraction below $E_\nu \sim 8$ MeV is responsible for the expected spectral distortion at energies near threshold in both SNO (see Fig. 36 of Ref. [2]) and SK (see Fig. 51 of the last Ref. in [5]). For a neutrino energy near 10 MeV, the SNO sweet spot, the 90% CL variation in $\delta m^2_\odot$ changes $f_2(E_\nu)$ more than the 90% CL variation in $\sin^2\theta_\odot$. Whereas in Fig. 3(b) we give the fraction of ν_2's above a given energy both unweighted and weighted by the energy dependence of the CC interaction and ES cross sections. Note, that above a neutrino energy of 7.5 MeV there is little difference between the weighted and unweighted integrated ν_2 fraction. Furthermore, in Fig. 3(c), we show the fraction of ν_2's above a given kinetic energy

[5] In fact this suggest an alternative to looking for a spectral distortion to test MSW, compare ES to (1-r) CC + rNC for a variety of kinetic energy thresholds.

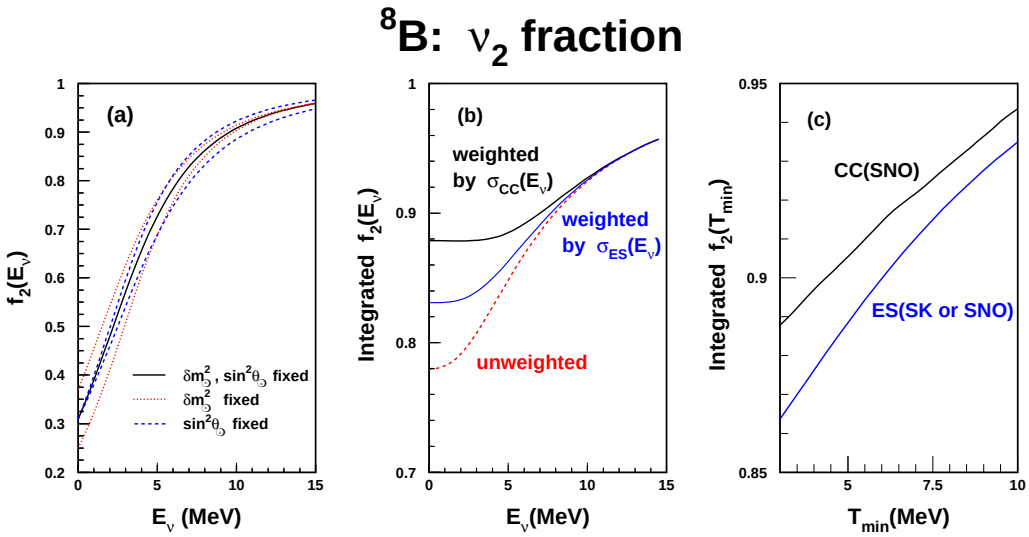

FIG. 3: (a) The fraction of ν_2, $f_2(E_\nu)$, as a function of the neutrino energy. The solid (black) curve is obtained using the central values for $\delta m^2_\odot = 8.0 \times 10^{-5}$ eV^2 and $\sin^2\theta_\odot = 0.31$ whereas the blue dashed (red dotted) lines are the 90% CL range varying $\delta m^2_\odot$ ($\sin^2\theta_\odot$) but holding $\sin^2\theta_\odot$ ($\delta m^2_\odot$) fixed at the central value, Eq. (1). (b) The integrated fraction of ^{8}B neutrinos which are ν_2's above an energy, E_ν, dashed (red) curve. Whereas, the solid black and blue curves are weighted by the energy dependence of the charge current (CC) cross section [15] and the elastic scattering (ES) cross section [18], respectively. (c) The integrated fraction of ^{8}B neutrinos as a function of the threshold kinetic energy of the recoil electrons for CC (SNO) and ES (SK or SNO) reactions.

for the recoil electron for both CC (SNO) and ES (SK or SNO) reactions. We observe that for the same threshold, f_2 for ES is always smaller than that for CC. This is expected since unweighted f_2 is a increasing function of E_ν and CC cross section increase more rapidly with energy than that of ES cross section. Hereafter, unless otherwise stated, we focus on the SNO CC reaction, as the results for ES reaction are qualitatively similar and the thresholds can be adjusted to give identical results for all practical purposes.

In Fig. 4 we give the breakdown into ν_1 and ν_2 for the raw ^{8}B spectrum as well as the spectrum weighted by the energy dependence of the CC interaction using a threshold of 5.5 MeV for the kinetic energy of the recoil electrons. Here we have used the current best fit values for $\delta m^2_\odot$ and $\sin^2\theta_\odot$.

How does the fraction of ν_2 vary if we allow $\delta m^2_\odot$ and $\sin^2\theta_\odot$ to deviate from their best fit values? In Fig. 5(a) we show the contours of the fraction of ν_2 in the $\delta m^2_\odot$ versus $\sin^2\theta_\odot$ plane where we have weighted the spectrum by the energy dependence of the CC interaction cross-section, and we have used a threshold on the kinetic energy of the recoil electrons of 5.5

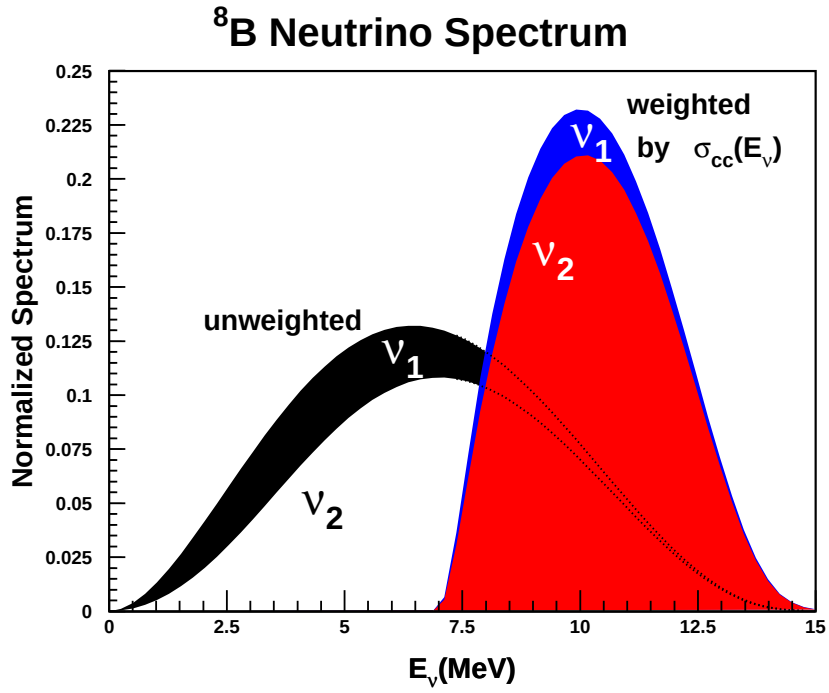

FIG. 4: The normalized ^{8}B energy spectrum broken into the ν_1 and ν_2 components. The left hand curves (black and white) are unweighted whereas the right hand curves (blue and red) are weighted by the energy dependence of the CC cross section [15] with a threshold of 5.5 MeV for the recoil electron's kinetic energy.

MeV. This energy dependence mimics the energy dependence of the SNO detector. Because of the strong correlation between $\sin^2\theta_\odot$ and the day-time CC/NC ratio we also give the contours of the fraction of ν_2 in the $\delta m^2_\odot$ versus day-time CC/NC plane in Fig. 5(b). Thus the ^{8}B energy weighted average fraction of ν_2's observed by SNO is

$$f_2 = 91 \pm 2\% \quad \text{at the 95\% CL.} \tag{14}$$

This is the two neutrino answer to the question posed in the title of this paper. We note, however, that as we showed in Fig. 3(c) the value of f_2 is a function of the threshold energy and also depends on the experiment. We estimate that for SK with the current 4.5 MeV threshold for the kinetic energy of the recoil electrons, that

$$f_2 = 88 \pm 2\% \quad \text{at the 95\% CL.} \tag{15}$$

The uncertainty is dominated by the uncertainty in $\delta m^2_\odot/A$. However, the uncertainty on $\delta m^2_\odot$ is approximately 5% from the KamLAND data whereas the uncertainty on the matter potential, A, in the region of ^{8}B production of the Standard Solar Model is 1-2%, see [19]. Hence, the uncertainty on $\delta m^2_\odot$ dominates.

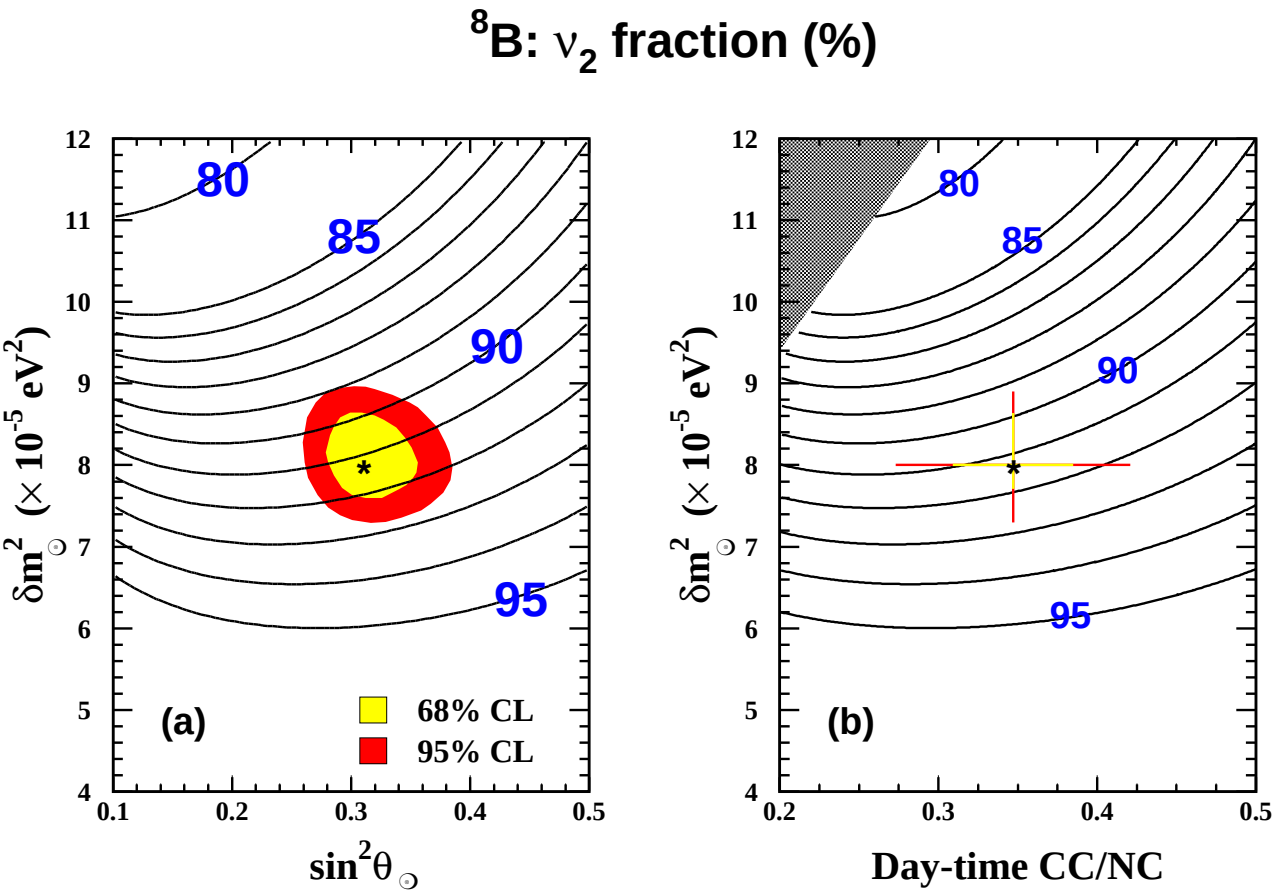

FIG. 5: (a) The ν_2 fraction (%) in the $\delta m^2_\odot$ versus $\sin^2\theta_\odot$ plane. As in Fig. 2, the current allowed region is also shown. (b) The ν_2 fraction (%) in the $\delta m^2_\odot$ versus the Day-time CC/NC ratio of SNO plane. We have excluded a region in the top left hand corner of this plot which corresponds to $\sin^2\theta_\odot < 0.1$. The current allowed range is indicated by the cross.

For the current allowed values for $\delta m^2_\odot$ and $\sin^2\theta_\odot$, the ratio

$$\frac{\delta m^2_\odot \sin 2\theta_\odot}{A(^8\mathrm{B}) - \delta m^2_\odot \cos 2\theta_\odot} \approx \frac{3}{4}, \tag{16}$$

where $A(^8\mathrm{B})$ is obtained using a typical number density of electrons at ^{8}B neutrino production ($Y_e\rho \approx 90$ g.cm^{-3}) and the typical energy of the observed ^{8}B neutrinos (≈ 10 MeV).

For the best fit central values of $\delta m^2_\odot$ and $\sin^2\theta_\odot$, given by Eq.(1), let us define an effective matter potential for the ^{8}B neutrinos, $A^{^8\mathrm{B}}_{eff}$, such that the left hand side of Eq.(4) equals our best fit value for the fraction that is ν_2. Thus,

$$\begin{aligned} A^{^8\mathrm{B}}_{eff} &\equiv \delta m^2_\odot \sin 2\theta_\odot \left[\cot 2\theta_\odot + \frac{2f_2 - 1}{2\sqrt{f_2(1-f_2)}}\right] \\ &= 1.36 \times 10^{-4}\ \mathrm{eV}^2, \end{aligned} \tag{17}$$

for $f_2 = 0.910$. This $A^{^8\mathrm{B}}_{eff}$ corresponds to a $Y_e\rho E_\nu = 0.892$ kg cm^{-3} MeV, the effective mixing angle, $\theta^N_\odot|_{eff} = 73^\circ$ and the effective $\delta m^2_N|_{eff} = 13.6 \times 10^{-5}$ eV^2.

We can then use this $A^{^8\mathrm{B}}_{eff}$ to perform a Taylor series expansion about the best fit point as follows

$$f_2 = \langle \sin^2\theta^N_\odot \rangle_{^8\mathrm{B}} \approx \frac{9}{10} + \frac{24}{125}\xi + \mathcal{O}(\xi^2) \qquad \text{with } \xi \equiv \frac{3}{4} - \frac{\delta m^2_\odot \sin 2\theta_\odot}{(A^{^8\mathrm{B}}_{eff} - \delta m^2_\odot \cos 2\theta_\odot)}. \tag{18}$$

This simple expression reproduces the values of f_2 to high precision throughout the 95% allowed region of the KamLAND and the solar neutrino experiments given in Fig. 5(a). In this sense our $A^{^8\mathrm{B}}_{eff}$ is the effective matter potential for the ^{8}B neutrinos. An expansion in $\delta m^2_\odot/A$ around its typical value of 0.6 could also be used but the coefficients are ever more complex trigonometric functions of $\theta_\odot$, whereas with our ξ expansion the coefficients are small rational numbers.

B. ^{7}Be and pp neutrinos

For ^{7}Be and pp neutrinos the fractions of ν_1 and ν_2 are much closer to the vacuum values of $\cos^2\theta_\odot$ and $\sin^2\theta_\odot$ respectively, as they are produced well below (more than a factor of two) the MS-resonance in the Sun, and an expansion in $A/\delta m^2_\odot$ is the natural one. In the third Ref. in [16], the electron neutrino survival probability was obtained by a similar expansion around the average of the matter potential. Using this expansion, we find that

$$f_2 = 1 - f_1 = \sin^2\theta^N_\odot = \sin^2\theta_\odot \; + \; \frac{1}{2}\sin^2 2\theta_\odot \left(\frac{A}{\delta m^2_\odot}\right) + \mathcal{O}\left(\frac{A}{\delta m^2_\odot}\right)^2 \tag{19}$$

$$\text{with} \quad A^{^7\mathrm{Be}}_{eff} = 1.1 \times 10^{-5}\mathrm{eV}^2 \qquad \text{and} \quad A^{\mathrm{pp}}_{eff} = 0.31 \times 10^{-5}\mathrm{eV}^2, \tag{20}$$

where the averaged value of the energy (weighted by the cross section) as well as the electron densities used are, respectively, $\langle E_\nu \rangle_{\mathrm{pp}} = 0.33$ MeV and $\langle Y_e\rho \rangle_{\mathrm{pp}} = 62$ g/cm^3 for pp, and $\langle E_\nu \rangle_{^7\mathrm{Be}} = 0.86$ MeV and $\langle Y_e\rho \rangle_{^7\mathrm{Be}} = 81$ g/cm^3 for ^{7}Be. Thus $f_2(^7\mathrm{Be}) = 37 \pm 4(7)\%$ and $f_2(\mathrm{pp}) = 33 \pm 4(7)\%$ at 68 (95) % CL where the uncertainty here is dominated by our knowledge of $\sin^2\theta_\odot$.

C. Two Neutrino Summary

In Fig. 6 we give the neutrino mass spectrum, the value of fraction of ν_2's ($\sin^2\theta^N_\odot$) and the fractional flux as function of the electron number density times neutrino energy, $Y_e\rho E_\nu$,

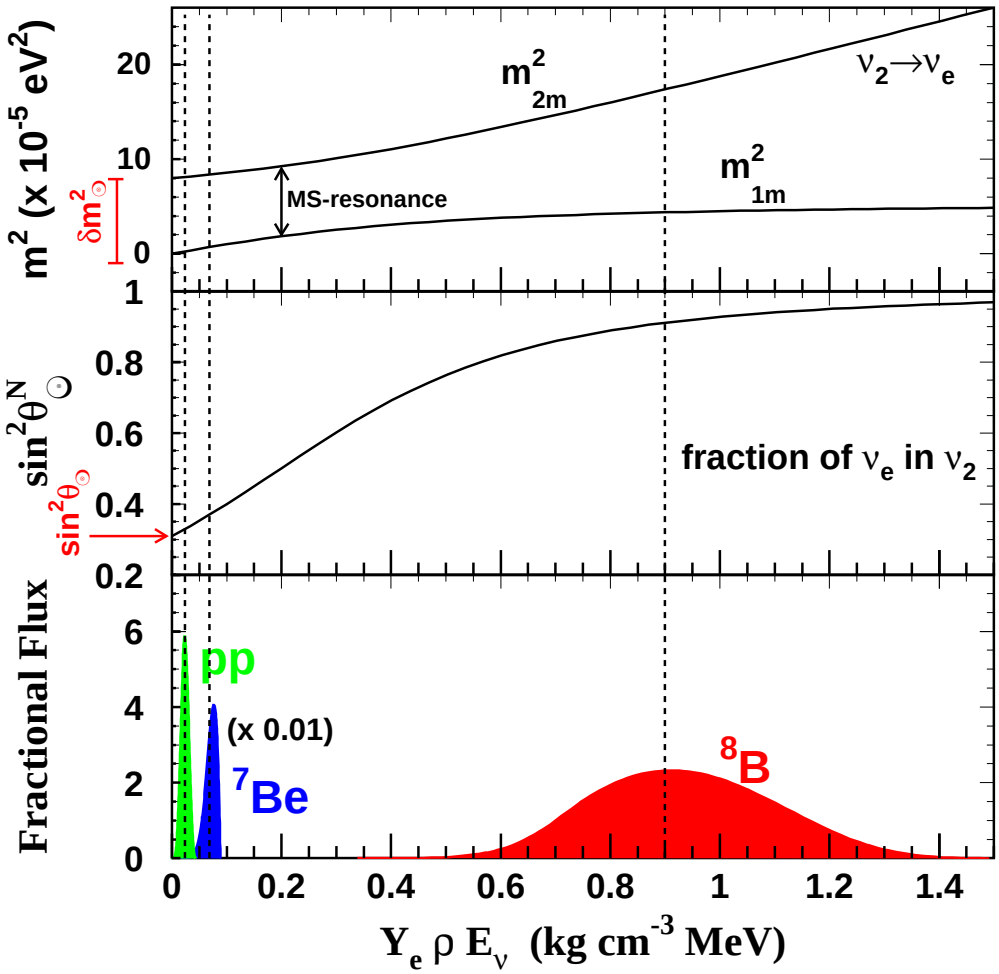

FIG. 6: The Mass spectrum (top panel), the fraction of ν_2's produced, $\sin^2\theta_\odot^N$, (middle panel) and the fractional flux (bottom panel) versus the product of the electron fraction, Y_e, the matter density, ρ, and the neutrino energy, E_ν, for the best fit values $\delta m^2_\odot = 8.0 \times 10^{-5}\text{eV}^2$ and $\sin^2\theta_\odot = 0.310$. The vertical dashed lines give the value of $Y_e\rho E_\nu$ which reproduces the average ν_2 fractions, 91, 37 and 33% for ^{8}B, ^{7}Be and pp respectively. This value of $Y_e\rho E_\nu = 0.89$ kg cm^{-3} MeV, for the ^{8}B neutrinos, gives a production mixing angle equal to 73° and a production $\delta m^2_N = 14 \times 10^{-5}$ eV^2. $Y_e\rho E_\nu = 1$ kg cm^{-3} MeV corresponds, in terms of the matter potential, to 15.3×10^{-5} eV^2, see Eq. (6).

which is proportional to the matter potential, for the ^{8}B, ^{7}Be and pp neutrinos using the best fit values of $\delta m^2_\odot$ and $\sin^2\theta_\odot$ in Eq. (1). The ^{8}B energy spectrum has been weighted by the energy dependence of the CC interaction of SNO with a 5.5 MeV threshold on the kinetic energy of the recoil electrons whereas the pp energy spectrum has been weighted by the energy dependence of the charged current interaction on Gallium with a 0.24 MeV

threshold. The vertical dashed lines gives the value of $Y_e \rho E_\nu$ which reproduces the average ν_2 fraction using the simple expression in Eq. (4) and are useful for the approximations given in Eqs. (18) and (19).

The energy weighted ν_2 fractions for ^{8}B, ^{7}Be and pp neutrinos using a two neutrino analysis, at the 95% CL, are

$$f_2(^8\mathrm{B}) = 91 \pm 2\%, \tag{21}$$

$$f_2(^7\mathrm{Be}) = 37 \pm 4\%, \tag{22}$$

$$f_2(\mathrm{pp}) = 33 \pm 4\%, \tag{23}$$

where the uncertainties for ^{7}Be and pp are dominated by the uncertainty on $\sin^2\theta_\odot$ whereas for ^{8}B the uncertainty is dominated by the uncertainty on $\delta m^2_\odot$. The ν_1 fractions, f_1, are simply $1 - f_2$.

III. THREE NEUTRINO ANALYSIS

For the three neutrino analysis we first must discuss the size of the component of ν_3 which is ν_e, i.e. the size of $\sin^2\theta_{13}$. This mixing angle determines the size of the effects on ν_e associated with the atmospheric mass squared difference. The best constraint on θ_{13} comes from the CHOOZ reactor experiment [20] which gives a limit on $\sin^2\theta_{13}$, as

$$0 \leq \sin^2\theta_{13} < 0.04, \tag{24}$$

at the 90 % CL for $\delta m^2_{31} = 2.5 \times 10^{-3}\mathrm{eV}^2$. This constraint depends on the precise value of δm^2_{31} with a stronger (weaker) constraint at higher (lower) allowed values of δm^2_{31}.

So far the inclusion of genuine three flavor effects has not been important because these effects are controlled by the two small parameters

$$\frac{\delta m^2_{21}}{\delta m^2_{32}} \approx 0.03 \quad \text{and/or} \quad \sin^2\theta_{13} \leq 0.04. \tag{25}$$

However as the accuracy of the neutrino data improves it will become inevitable to take into account genuine three flavor effects. See [21, 22], for recent studies on the impact of θ_{13} on solar neutrinos.

Suppose that Double CHOOZ [23], T2K [24] or NOνA [25] or some other experiment measures a non-zero value for $\sin^2\theta_{13}$. What effect does this have on the previous analysis?

How does this change our knowledge of the solar parameters and the relationship between solar mixing angle and the fraction of ν_2?

Our knowledge of the solar δm^2 comes primarily from the KamLAND experiment where the effects of the atmospheric δm^2 are averaged over many oscillations, thus to high accuracy

$$\delta m^2_{21} \;=\; \delta m^2_{\odot}, \tag{26}$$

i.e. the solar δm^2 remains unaffected. Remember, we are using the notation δm^2_{21} and $\sin^2\theta_{12}$ for the three neutrino analysis to distinguish it from $\delta m^2_{\odot}$ and $\sin^2\theta_{\odot}$ used in the two neutrino analysis.

A. ^{8}B 3 Neutrino Analysis

For the mixing angle $\sin^2\theta_{12}$ the situations is more complicated in the three neutrino analysis. The ^{8}B electron neutrino survival probability measured by SNO's day-time CC/NC ratio can be written as

$$\frac{\mathrm{CC}}{\mathrm{NC}} \;=\; \mathcal{F}_1\cos^2\theta_{13}\cos^2\theta_{12} + \mathcal{F}_2\cos^2\theta_{13}\sin^2\theta_{12} + \mathcal{F}_3\sin^2\theta_{13}, \tag{27}$$

where $\mathcal{F}_1$, $\mathcal{F}_2$ and $\mathcal{F}_3$ are the fraction of ν_1, ν_2 and ν_3 respectively, satisfying $\mathcal{F}_1+\mathcal{F}_2+\mathcal{F}_3 = 1$. The ν_3 fraction is given by

$$\mathcal{F}_3 = \left(1 \pm \frac{2A}{|\delta m^2_{31}|}\right)\sin^2\theta_{13} \approx \sin^2\theta_{13}, \tag{28}$$

where +(-) sign refers to the normal, $\delta m^2_{31} > 0$ (inverted, $\delta m^2_{31} < 0$) mass hierarchy. The small correction factor $\frac{2A}{|\delta m^2_{31}|} \sim 10\%$ comes from matter effects associated with atmospheric δm^2 in the center of the Sun. We will ignore this correction since it is small and currently the sign is unknown. Hence, $\mathcal{F}_1 + \mathcal{F}_2 = 1 - \mathcal{F}_3 = \cos^2\theta_{13}$.

With this approximation the ν_1 and ν_2 fractions can be written as

$$\mathcal{F}_1 = \cos^2\theta_{13}\;\langle\cos^2\theta^N_{12}\rangle_{^8\mathrm{B}} \qquad \text{and} \qquad \mathcal{F}_2 = \cos^2\theta_{13}\;\langle\sin^2\theta^N_{12}\rangle_{^8\mathrm{B}}. \tag{29}$$

where the average $\langle\cdots\rangle_{^8\mathrm{B}}$ is over the solar production region and the energy of the observed neutrinos. $\sin^2\theta^N_{12}$ is given by Eq. (4) with the replacements $\sin^2\theta_{\odot} \to \sin^2\theta_{12}$ and $A \to A\cos^2\theta_{13}$ [26].

In going from the two neutrino analysis to the three neutrino analysis the quantity that must remain unchanged is the value of the electron neutrino survival probability, i.e. the

CC/NC ratio. This implies that we must adjust the value of $\sin^2\theta_{12}$ and hence the fractions of ν_1 and ν_2 so that the CC/NC ratio remains constant. We have performed this procedure numerically and report the result as a Taylor series expansion in the fraction of ν_1's about $\sin^2\theta_{13} = 0$. If we write

$$\mathcal{F}_1(\sin^2\theta_{13}) = \mathcal{F}_1(0) + \alpha\sin^2\theta_{13} + \mathcal{O}(\sin^4\theta_{13}), \tag{30}$$

$$\text{then} \quad \mathcal{F}_1(0) \equiv f_1, \quad \text{and} \quad \alpha \equiv \left.\frac{d\mathcal{F}_1}{d\sin^2\theta_{13}}\right|_{\sin^2\theta_{13}=0}. \tag{31}$$

In Fig.7(a) we have plotted the contours of $\alpha \equiv \left.\frac{d\mathcal{F}_1}{d\sin^2\theta_{13}}\right|_0$ in the $\delta m^2_\odot$ versus $\sin^2\theta_\odot$ plane. Near the best values this total derivative is close to zero, i.e.

$$\left.\frac{d\mathcal{F}_1}{d\sin^2\theta_{13}}\right|_{\sin^2\theta_{13}=0} = 0.00^{+0.02}_{-0.04} \tag{32}$$

at the 68% CL. As $\sin^2\theta_{13}$ grows above zero, the size of $\mathcal{F}_1$ is influenced by a number of effects; the first is the factor of $\cos^2\theta_{13}$ in Eq. (29) which reduces $\mathcal{F}_1$, the second is the matter potential A which is reduced to $A\cos^2\theta_{13}$ raising the fraction $\mathcal{F}_1$ and third is the value of $\sin^2\theta_{12}$ which changes to hold the CC/NC ratio fixed. By coincidence the sum of these effects approximately cancel at the current best fit values and the fraction of ν_1 remains approximately *unchanged* as $\sin^2\theta_{13}$ gets larger. This implies that the fraction of ν_2 is reduced by $\sim \sin^2\theta_{13}$ since the sum of $\mathcal{F}_1 + \mathcal{F}_2$ is simply $\cos^2\theta_{13}$, thus

$$\mathcal{F}_1 \approx f_1 = 0.09 \mp 0.02, \tag{33}$$

$$\mathcal{F}_2 = f_2 - \sin^2\theta_{13} \approx 0.91 \pm 0.02 - \sin^2\theta_{13}, \tag{34}$$

$$\mathcal{F}_3 = \sin^2\theta_{13}. \tag{35}$$

Remember f_i and $\mathcal{F}_i$ are the fractions of the i-th mass eigenstate in the two and three neutrino analysis, respectively. The uncertainty comes primarily from the uncertainty in $\delta m^2_\odot$ measured by KamLAND.

As a use of these fractions one can for example evaluate the MNS matrix element, $|U_{e2}|^2 = \cos^2\theta_{13}\sin^2\theta_{12}$, by rewriting Eq. (27) as

$$|U_{e2}|^2 = \cos^2\theta_{13}\sin^2\theta_{12} = \frac{(\frac{\text{CC}}{\text{NC}} - \cos^2\theta_{13}\mathcal{F}_1)}{(\cos^2\theta_{13} - 2\mathcal{F}_1)}, \tag{36}$$

where terms of $\mathcal{O}(\sin^4\theta_{13})$ have been dropped. Performing a Taylor series expansion about

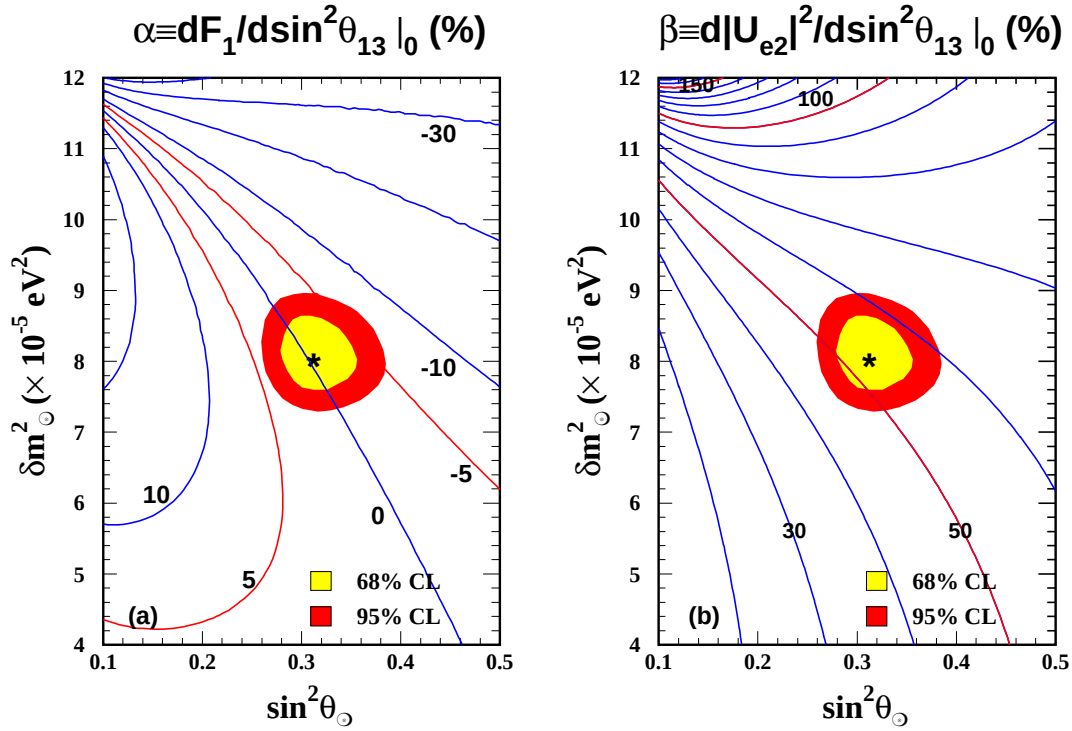

FIG. 7: Iso-contours of the derivatives of $\mathcal{F}_1$ (a) and $|U_{e2}|^2$ (b) with respect $\sin^2\theta_{13}$ evaluated at $\sin^2\theta_{13}=0$ in the $\delta m^2_\odot$ versus $\sin^2\theta_\odot$ plane. The contours are labeled as in per cent. The 68 and 95 % CL allowed regions are also indicated.

$\sin^2\theta_{13}=0$, we find

$$|U_{e2}|^2 \;=\; \sin^2\theta_\odot^{\rm 8B} + \beta\sin^2\theta_{13} + \mathcal{O}(\sin^4\theta_{13}), \tag{37}$$

$$\text{with}\quad \beta \;\equiv\; \left.\frac{d|U_{e2}|^2}{d\sin^2\theta_{13}}\right|_0 = \frac{(f_1-\alpha)+(1+2\alpha)\sin^2\theta_\odot}{(1-2f_1)}. \tag{38}$$

For the current allowed region of the solar parameters, this implies that

$$|U_{e2}|^2 \;\approx\; \sin^2\theta_\odot^{\rm 8B} + (0.53^{+0.06}_{-0.04})\sin^2\theta_{13}, \tag{39}$$

at the 68% CL, i.e. the three neutrino $|U_{e2}|^2$ is approximately equal to the $\sin^2\theta_\odot^{\rm 8B}$ using a two neutrino analysis of only the ^{8}B electron neutrino survival probability using the KamLAND's $\delta m^2_\odot$ constraint plus 53% of $|U_{e3}|^2$ determined, say, by a CHOOZ-like reactor experiment, see Fig. 7(b).

If a similar analysis is performed for the three neutrino sine squared solar mixing angle $\sin^2\theta_{12}$, the total derivative with respect to $\sin^2\theta_{13}$ is simply $(\beta+\sin^2\theta_\odot)$. For $\tan^2\theta_{12}$ the total derivative is $(\beta+\sin^2\theta_\odot)/\cos^4\theta_\odot$. Alternatively we can turn this discussion inside out and write the ^{8}B effective two component $\sin^2\theta_\odot$ in terms of three component quantities as

$$\sin^2\theta_\odot^{\rm 8B} \;=\; \sin^2\theta_{12} - (\beta+\sin^2\theta_{12})\sin^2\theta_{13}. \tag{40}$$

For KamLAND, the equivalent relationship is

$$\sin^2\theta_\odot^{^8\mathrm{Kam}} \;=\; \sin^2\theta_{12} - \left(\frac{\sin^2 2\theta_{12}}{2\cos 2\theta_{12}}\right)\sin^2\theta_{13}. \tag{41}$$

For the current best fit values $(\beta + \sin^2\theta_{12}) \approx 0.90$ is close to $\sin^2 2\theta_{12}/2\cos 2\theta_{12} \approx 1.1$, i.e. in a two component analysis the difference between the solar ^{8}B and KamLAND $\sin^2\theta_\odot$'s is approximately $0.2\sin^2\theta_{13}$.

B. ^{7}Be and pp 3 Neutrino Analysis

Performing a similar 3 neutrino analysis for the pp (or ^{7}Be) neutrinos we find that the fraction of neutrino mass eigenstates is

$$\mathcal{F}_1 \;\approx\; \cos^2\theta_\odot - \frac{1}{2}\sin^2 2\theta_\odot\left(\frac{A}{\delta m^2_\odot}\right) + \frac{\sin^2\theta_\odot}{\cos 2\theta_\odot}\sin^2\theta_{13} = f_1 + 0.82\sin^2\theta_{13}, \tag{42}$$

$$\mathcal{F}_2 \;\approx\; \sin^2\theta_\odot + \frac{1}{2}\sin^2 2\theta_\odot\left(\frac{A}{\delta m^2_\odot}\right) - \frac{\cos^2\theta_\odot}{\cos 2\theta_\odot}\sin^2\theta_{13} = f_2 - 1.8\sin^2\theta_{13}, \tag{43}$$

$$\mathcal{F}_3 \;\approx\; \sin^2\theta_{13}, \tag{44}$$

where the $\sin^2\theta_\odot$ here is determined from the pp (or ^{7}Be) neutrinos. Terms of order $\mathcal{O}\left(A/\delta m^2_\odot\right)^2$, $\mathcal{O}(\sin^4\theta_{13})$ and $\mathcal{O}\left(\sin^2\theta_{13}A/\delta m^2_\odot\right)$ have been dropped here. The two neutrino fractions f_1 and f_2 are given in Eq. (19).

Again we can use these fractions to determine the $|U_{e2}|^2$ element of the MNS matrix

$$|U_{e2}|^2 \;=\; \sin^2\theta_\odot - \left(\frac{\cos^2\theta_\odot}{\cos 2\theta_\odot}\right)\sin^2\theta_{13} \approx \sin^2\theta_\odot - 1.8\;\sin^2\theta_{13}. \tag{45}$$

Comparing this equation with Eq. (39) appears to be in contradiction but this is not so since if $\sin^2\theta_{13} \neq 0$ then the two component analysis of the ^{8}B and pp (or ^{7}Be) neutrinos will lead to different values of $\sin^2\theta_\odot$, in fact

$$\sin^2\theta_\odot^{\mathrm{pp}} - \sin^2\theta_\odot^{^8\mathrm{B}} \approx 2.3\sin^2\theta_{13}. \tag{46}$$

This difference has been extensively exploited in Ref. [22] to determine $\sin^2\theta_{13}$ using only solar neutrino experiments. Their $\sin^2\theta_{12}$ versus $\sin^2\theta_{13}$ figures, e.g. Fig. 6, demonstrates this point in a clear and useful fashion. Also, the numerical values of our derivatives of $|U_{e2}|^2$ are consistent with the inverse of the slopes of their Fig. 6.

Eqs. (39) and (45) also imply that the uncertainty in the determination of $|U_{e2}|^2$ from the current unknown value of $\sin^2\theta_{13}$ is smaller for the analysis of ^{8}B neutrinos than pp or ^{7}Be neutrinos. Of course the current uncertainty on the two neutrino $\sin^2\theta_\odot$ dominates.

IV. PROBING THE SOLAR INTERIOR BY ^{8}B NEUTRINOS

In this section, as an application of our analysis, we will invert the discussions found in Ref. [27] where the validity of the MSW physics has been tested assuming the standard solar model (SSM) prediction of the electron number density as well as ^{8}B neutrino production region. Here, we will discuss what can be said about these quantities, assuming the validity of the MSW effect in the LMA region. While there is no strong reason to doubt the correctness of the SSM, which is in good agreement also with the helioseismological data [28], it is nevertheless interesting if we can test it independently.

Since the propagation of ^{8}B neutrinos, in the Sun, is highly adiabatic in the LMA region, the fraction of ν_2, and consequently, the SNO CC/NC ratio is determined only by the effective value of the matter potential, $A^{^8\mathrm{B}}_{eff}$, defined in Section II(A). This implies that if we can measure $\sin^2\theta_\odot$ using an experiment independent of the ^{8}B solar neutrinos, then from the measured value of SNO's CC/NC ratio we can determine the value of $A^{^8\mathrm{B}}_{eff}$. Note, that we can not extract information on the electron number density distribution or the ^{8}B neutrino production distribution, separately, but only on $A^{^8\mathrm{B}}_{eff}$ which is a single characteristic of the convolution of these two distributions.

For the two flavor neutrino analysis, if we rewrite the definition of the effective matter potential $A^{^8\mathrm{B}}_{eff}$ given by Eq.(17) using the relationship between f_2 and SNO's CC/NC ratio, Eq.(9), we obtain

$$A^{^8\mathrm{B}}_{eff} \;=\; \delta m^2_\odot \sin 2\theta_\odot \left[\cot 2\theta_\odot + \frac{1-2\frac{\mathrm{CC}}{\mathrm{NC}}}{2\sqrt{(\cos^2\theta_\odot - \frac{\mathrm{CC}}{\mathrm{NC}})(\frac{\mathrm{CC}}{\mathrm{NC}} - \sin^2\theta_\odot)}}\right]. \tag{47}$$

This expression allows us to obtain a value of $A^{^8\mathrm{B}}_{eff}$ from $(\sin^2\theta_\odot, \delta m^2_\odot)$ measured independent of ^{8}B neutrinos and SNO's ^{8}B neutrino CC/NC ratio. We can convert this into an effective value of the electron number density, $Y_e\rho\,|^{^8\mathrm{B}}_{eff}$, in the solar ^{8}B production region, as follows

$$Y_e\rho\,|^{^8\mathrm{B}}_{eff} \equiv \frac{M_n}{2\sqrt{2}G_F}\frac{A^{^8\mathrm{B}}_{eff}}{\langle E_\nu\rangle_{^8\mathrm{B}}}, \tag{48}$$

where $\langle E_\nu\rangle_{^8\mathrm{B}} = 10.5$ MeV is the CC cross section weighted average energy of neutrinos observed by SNO. For a given solar model, the value of $Y_e\rho\,|^{^8\mathrm{B}}_{eff}$ can be calculated for any value of $\sin^2\theta_\odot$ and $\delta m^2_\odot$. The SSM prediction is that $Y_e\rho\,|^{^8\mathrm{B}}_{eff} = 85$ g cm^{-3} at the current best fit point[6]. As a comparison the mean value of $Y_e\rho$ over the ^{8}B production region is 90

[6] Because of the way we have defined $A^{^8\mathrm{B}}_{eff}$, our $Y_e\rho\,|^{^8\mathrm{B}}_{eff}$ has a weak dependence on $\sin^2\theta_\odot$ and $\delta m^2_\odot$ but

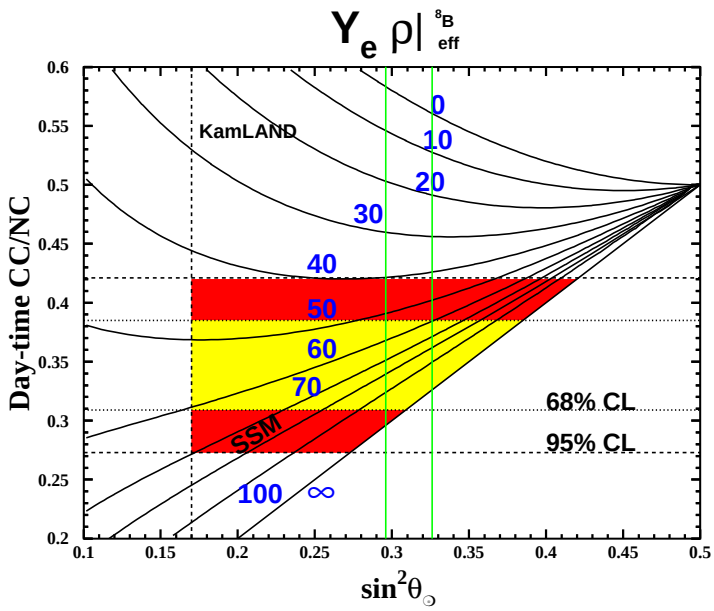

FIG. 8: The iso-contours of $Y_e\rho\ |^{8\mathrm{B}}_{eff}$ in the $\sin^2\theta_\odot - \mathrm{CC/NC}|_{\mathrm{day}}$ plane. The line labeled SSM is the Standard Solar Model prediction for $Y_e\rho\ |^{8\mathrm{B}}_{eff}$ (≈ 85 g/cm^3). The range of observed values of CC/NC are indicated by the shaded horizontal bands. The KamLAND experiment places a lower bound on $\sin^2\theta_\odot$ independent of solar neutrinos at 0.17, see [1]. The vertical band indicate the uncertainty which could be expected by future reactor experiments [29].

g cm^{-3}. The reason that $Y_e\rho\ |^{8\mathrm{B}}_{eff}$ is below the mean value is because values of $Y_e\rho$ below the mean pull down the ν_2 fraction more than values above the mean raise the ν_2 fraction.

We show in Fig. 8, the iso-contours of $Y_e\rho\ |^{8\mathrm{B}}_{eff}$ in the $\sin^2\theta_\odot - \mathrm{CC/NC}|_{\mathrm{day}}$ plane, for the current best fitted value of $\delta m^2_\odot$. The observed range of SNO's CC/NC are shown by the horizontal lines[7]. From this plot, we can derive the *lower bound* on $Y_e\rho\ |^{8\mathrm{B}}_{eff}$ which is 40 g/cm^3 for any value of $\theta_\odot$ at 95 % CL. Future reactor neutrino oscillation experiments [29] can perform a 2-3% measurement of $\sin^2\theta_\odot$. The 68% range of $\sin^2\theta_\odot$ is indicated by vertical lines in this figure. However, such precision on $\sin^2\theta_\odot$ will not reduce the allowed values for $Y_e\rho\ |^{8\mathrm{B}}_{eff}$ unless the error on the measured value of CC/NC is reduced.

A three neutrino analysis is needed if $U_{e3} \neq 0$ and this can be performed using Eq. (47)

this variation is less than 2% over the 95% CL allowed region.

[7] Another horizontal band could be included by combining the Super-Kamiokande Electron Scattering measurement with the SNO Neutral Current measurement. However, since the uncertainty on the NC measurement dominates this would produce a similar sized band.

with the following replacements,

$$\theta_\odot \to \theta_{12}, \quad \delta m^2_\odot \to \delta m^2_{21}/\cos^2\theta_{13} \quad \text{and} \quad \frac{\text{CC}}{\text{NC}} \to \frac{\text{CC}}{\text{NC}}\,\frac{1}{\cos^4\theta_{13}}. \tag{49}$$

A weak upper bound could be derived using a precision measurement of the ^{7}Be and/or pp electron neutrino survival probability in a similar fashion. As A_{eff} gets larger, the fraction of ν_2 gets larger, see Eq.(19), and hence the electron neutrino survival probability gets smaller for fixed values of the mixing parameters. The upper bound arises when this survival probability is below the measured survival probability at some confidence level, assuming that the mixing parameters have been determined independent of these solar neutrinos.

V. SUMMARY AND CONCLUSIONS

We have performed an extensive analysis of the mass eigenstate fractions of ^{8}B solar neutrinos using only two mass eigenstates ($\sin^2\theta_{13} = 0$) and with three mass eigenstates ($\sin^2\theta_{13} \neq 0$). In the two neutrino analysis the ν_2-fraction is $91 \pm 2\%$. The remaining $9 \mp 2\%$ is, of course, in the ν_1 mass eigenstate. With these fractions in hand, which are primarily determined by the solar δm^2 measured by the KamLAND experiment, the sine squared of the solar mixing angle is simply related to CC/NC ratio measured by the SNO experiment. For completeness the mass eigenstate fractions for ^{7}Be and pp are also given.

Allowing for small but non-zero $\sin^2\theta_{13}$, in a full three neutrino analysis, we found very little change in the fraction of ν_1's. This implies, since the ν_3 fraction is $\sin^2\theta_{13}$, that the ν_2 fraction is reduced by $\sin^2\theta_{13}$. That is, the ν_2-fraction is

$$91 \pm 2 - 100\sin^2\theta_{13}\ \% \quad \text{at the 95\% CL.} \tag{50}$$

Since the CHOOZ experiment constrains the value of $\sin^2\theta_{13} < 0.04$ at the 90% CL this places a lower bound on the ν_2 fraction of ^{8}B solar neutrinos in the mid-eighty percent range making the ^{8}B solar neutrinos the purest mass eigenstate neutrino beam known so far, and it is a ν_2 beam!

As an example of the use of these mass eigenstate fractions, we have shown that for the ^{8}B neutrinos observed by the SNO experiment, the U_{e2}-element of the MNS matrix is given by

$$|U_{e2}|^2 \approx \sin^2\theta^{^8B}_\odot + (0.53^{+0.06}_{-0.04})\sin^2\theta_{13}. \tag{51}$$

Where $\sin^2\theta_{\odot}^{^8B}$ is the sine squared of the solar mixing angle determined by using a two neutrino analysis of the ^8B neutrinos plus KamLAND. An analysis for this $\sin^2\theta_{\odot}^{^8B}$ obtained from the SK, SNO and KamLAND data [30] gives $\sin^2\theta_{\odot}^{^8B} = 0.30^{+0.11}_{-0.08}$ at the 95% CL. With the data currently available this is our best estimate of $|U_{e2}|^2$ and is the most accurately known MNS matrix element.

Finally, we have also demonstrated the possibility of probing the solar interior by ^{8}B neutrinos. We have derived a lower bound on the average electron number density over the region where the solar ^{8}B neutrinos are produced which is 50% of the Standard Solar model value.

Acknowledgments

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