
The Double Simplex

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A NEW WAY TO ENVISION PARTICLES AND INTERACTIONS

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1 Felicitações!

It is my great pleasure to join in celebrating Gustavo Branco's research in CP violation and flavor physics, and to look forward to many insightful contributions still to come. I am also very glad to have the opportunity to express my appreciation for the school of theoretical physics that has developed here in Lisbon through the efforts of Gustavo and colleagues. I particularly respect the attention to challenging problems that characterizes the Lisbon school, and I will allude to some of those hard problems in my remarks today.

2 Presenting Particle Physics

For some time, I have been concerned about the constricted portrayal of the aspirations of particle physics in the popular scientific media. Not long ago, when the first superconducting dipole was lowered into the Large Hadron Collider tunnel at CERN, the BBC informed its listeners that the point of the LHC is “to discover the sought-after Higgs boson, or ‘God particle,’ which explains why matter has mass.” An indistinct feeling that particle physics will be over, once we succeed in ticking off this year's Holy Grail, is standard fare in the press. Of course, it is Quantum Chromodynamics (the strong interaction), not the Higgs boson, that generates most of the visible mass in the Universe, so even this straitened view of what the LHC may bring is garbled.

More troubling to me, reliance on shorthands such as “the search for the Higgs boson” (as a token for uncovering the origin of electroweak symmetry breaking through a thorough exploration of the 1-TeV scale) seems to have narrowed discourse within our field. It is not rare for our own colleagues to talk in terms of a very limited menu of opportunities for discovery. That would be cause for concern, even if the shorthands didn't spill over into the media. As we embark upon the LHC adventure, we will need open and prepared minds!

The iconic representations of the standard model of particle physics summarize the state of our knowledge, but conceal the state of our ignorance. I have a lot of affection for the table of particles and interactions shown

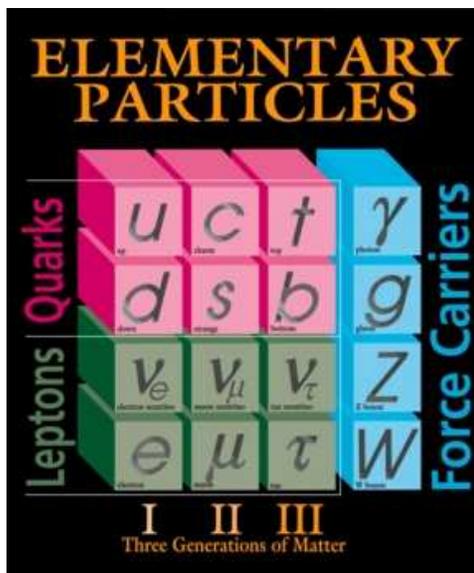


Figure 1: The ubiquitous chart of quarks, leptons, and force carriers.

in Figure 1, which represents the enthusiastic work of many teachers and grew out of a Conference on the Teaching Modern Physics held at Fermilab in 1986. It encodes a lot of information in compact form, and—when used to start a conversation—can be a very attractive visual aid. But the chiseled-in-stone appearance makes it look so finished!

The celebrated chart (see Figure 2) of fundamental particles and interactions created by the Contemporary Physics Education Project (<http://www.cpepweb.org/>), and available as wall chart, poster, and place mat, has had a global reach—more than 200,000 distributed. It, too, has helped move particle physics into the classroom, and it presents many essential notions of the standard model.

Neither of these valuable pedagogical tools does much to suggest the plethora of open questions that animate current research in the field—questions to which we are led, in part, by the success of the standard model. As Presidential Science Advisor John Marburger pointedly observed to me recently, I have been telling him for two decades how important is the search for the agent of electroweak symmetry breaking, and yet the idea of the Higgs boson is nowhere to be found in either of our popular charts.

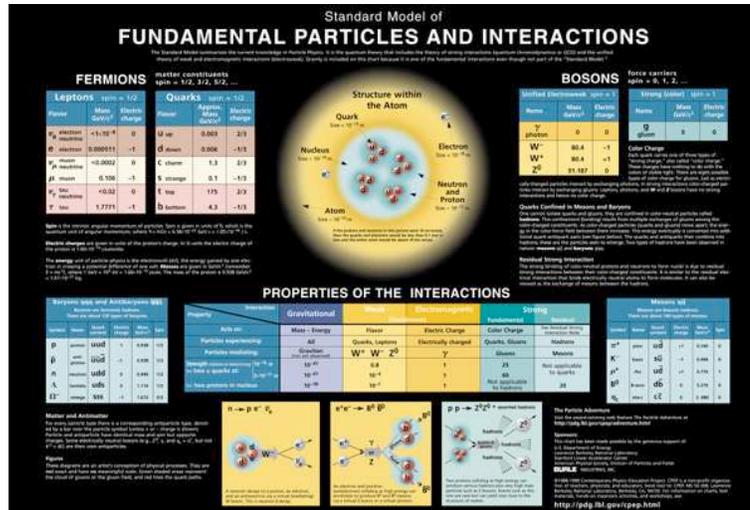


Figure 2: CPEP’s standard-model wall chart.

I want to describe today a work in progress, my attempt to create a three-dimensional object that expresses a new way to envision the particles and interactions. My goal is to represent what we know is true, what we hope might be true, and what we don’t know—in other terms, to show the connections that are firmly established, those we believe must be there, and the open issues.

Galileo asserted that the Book of Nature is written in the language of mathematics. I am comfortable with the gentler claim that mathematics is one of the languages in which we have learned to listen best to Nature. I feel that there is great value in striving to expose our insights and pose our new questions without hiding behind equations. I hope you will agree that the challenge of presenting our science without mathematical formalism is worthy and interesting. I hope, too, that you will be motivated to build on my approach or—even better—to make your own beginning.

Any chart or mnemonic device should be an invitation to narrative and a spur to curiosity, and that is what I intend for the geometrical construction I call the double simplex. I want also to express the spirit of play, of successive approximations, that animates the way scientists work. So instead of dissecting a completed double simplex, I will build it up, step by step. That

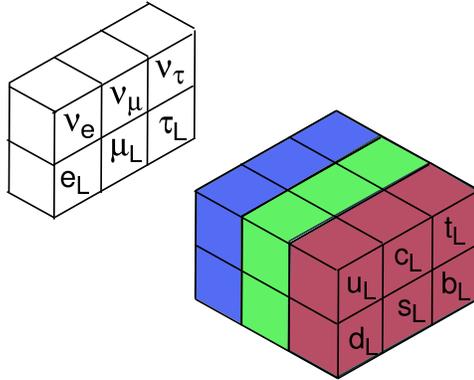


Figure 3: The left-handed doublets of quarks and leptons that inspire the structure of the electroweak theory.

will help us to see where choices are to be made and to encounter some of the fascinating questions we face.

3 Ground Rules

Our picture of matter is based on the discovery of a set of pointlike constituents: the quarks and the leptons, as depicted in Figure 3, and a few fundamental forces derived from gauge symmetries. The quarks are influenced by the strong interaction, and so carry *color*, the strong-interaction charge, whereas the colorless leptons do not feel the strong interaction. The pairs (ν_e, e) , (u, d) , etc., are emblematic of the transitions induced by the charged-current weak interactions. The notion that the quarks and leptons are elementary—structureless and indivisible—is necessarily provisional, limited by our current resolution, $r \lesssim 10^{-18}$ m.

Looking a little more closely at the constituents of matter, we find that our world is not as neat as the simple cartoon vision of Figure 3. The left-handed and right-handed fermions behave very differently under the influence of the weak interactions. A more complete picture is given in Figure 4, which represents the way we looked at the world before the discovery of neutrino oscillations that require neutrino mass and almost surely imply the exis-

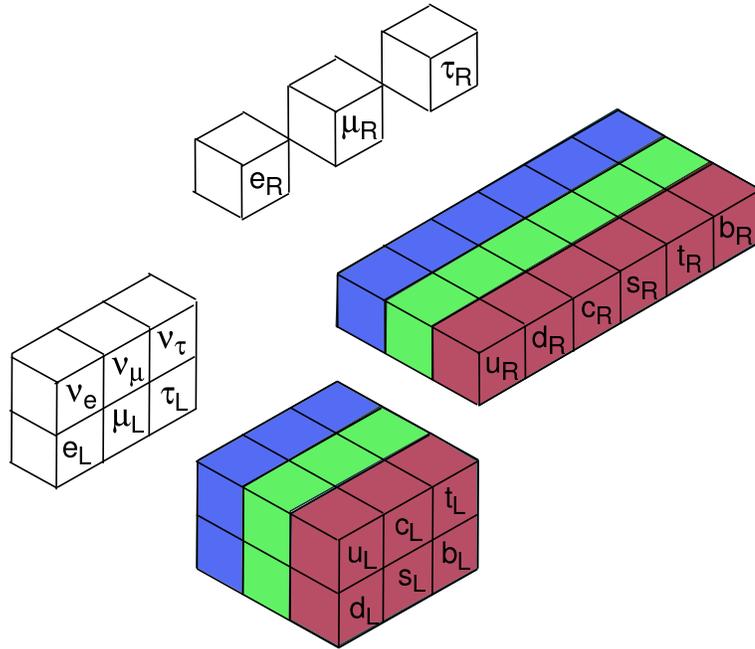


Figure 4: Left-handed doublets, right-handed singlets of quarks and leptons.

tence of right-handed neutrinos.¹ Neutrinos aside, the striking fact is the asymmetry between left-handed fermion doublets and right-handed fermion singlets, which is manifested in *parity violation* in the charged-current weak interactions. What does this distinction mean?

I think that everyone in this amphitheatre learned about parity violation at school, and perhaps familiarity has dulled our senses a bit. [I estimate that I have personally written down the left-handed doublets 10^4 times . . .] It is worth remembering that parity violation came as a stunning surprise to our scientific ancestors. Wolfgang Pauli was moved to send a black-bordered note to Victor Weisskopf, bearing the text shown in Figure 5. It seems to me that nature's broken mirror—the distinction between left-handed and right-handed fermions—qualifies as one of the great mysteries. Even if we will not get to the bottom of this mystery next week or next year, it should be prominent in our consciousness—and among the goals we present to others

¹It is probably time to adopt the quark convention and label the neutrinos by mass eigenstates ν_1, ν_2, ν_3 instead of the flavor eigenstates ν_e, ν_μ, ν_τ .

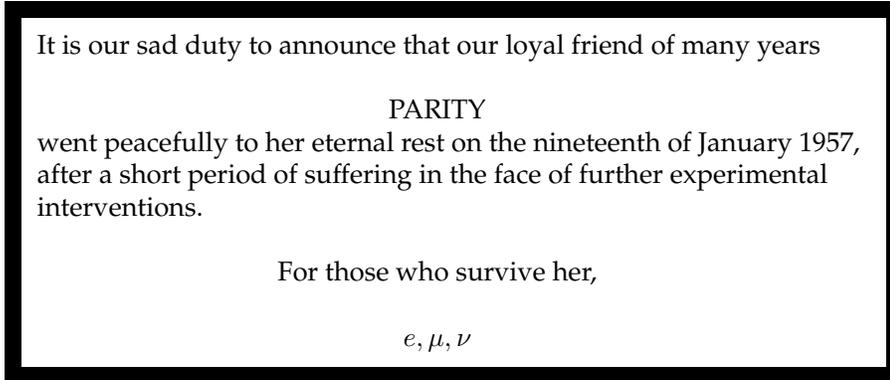


Figure 5: Pauli to Weisskopf, on learning of parity violation in β decay.

as the aspirations of our science.

The insight that local gauge symmetries imply interactions leads us to see the W^\pm bosons as the agents of symmetry transformations that take $u \leftrightarrow d$, and so forth. In similar fashion, the gluons of $SU(3)$ are symmetry operators that transform quark colors, preserving flavor. But there is more to our understanding of the world than is revealed by identifying the symmetries represented in Figure 4. The electroweak gauge symmetry is hidden, $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em}$. If it were not, the world would be very different.

At first sight, \square All the quarks and leptons would be massless and move at the speed of light. \square Electromagnetism as we know it would not exist, but there would be a long-range weak-hypercharge force. \square QCD would confine quarks and generate baryon masses roughly as we know them. \square The Bohr radius of “atoms” consisting of an electron or neutrino attracted by the weak-hypercharge interaction to the nucleons would be infinite. \square Beta decay, inhibited in our world by the great mass of the W boson, would not be weak. \square The unbroken $SU(2)_L$ interaction would confine objects that carry weak isospin.

It is fair to say that electroweak symmetry breaking shapes our world! In fact, when we take into account every aspect of the influence of the strong interactions, the analysis of how the world would be is very subtle and fascinating. Please take time to think about a not-so-simple homework problem: *What would the everyday world be like if there were no mechanism,*

like the Higgs mechanism, to break the electroweak symmetry? Consider the effects of all of the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge interactions.

Because one of my goals is to devise a metaphor that evokes open questions, highlighting what we do not know, my intention is not to make predictions that prejudge Nature's answers, but to create a language nimble enough to relate many stories. A discovery that breaks out of the framework should represent not just a choice among alternatives we already recognize, but a fundamental revision of our thinking. After this prologue, let us begin!

4 Toward the double simplex

Both quarks and leptons are spin- $\frac{1}{2}$, pointlike fermions that occur in $SU(2)_L$ doublets. The obvious difference is that quarks carry $SU(3)_c$ color charge whereas leptons do not, so we could imagine that quarks and leptons are simply distinct and unrelated species. But we have reason to think otherwise. The proton's electric charge very closely balances the electron's, $(Q_p + Q_e)/e < 10^{-21}$, suggesting that there must be a link between protons—hence, quarks—and electrons—hence, leptons. Moreover, quarks and leptons are required, in matched pairs, for the electroweak theory to be anomaly-free, so that quantum corrections respect the symmetries on which the theory is based.

It is fruitful to display the color-triplet red, green, and blue quarks in the equilateral triangle weight diagram for the $\mathbf{3}$ representation of $SU(3)_c$, as shown in the left panel of Figure 6. There I have filled in the plane between them to indicate the transitions mediated by gluons. The equality of proton and (anti)electron charges and the need to cancel anomalies in the electroweak theory suggest that we join the quarks and leptons in an extended family, or multiplet. Pati and Salam provided an apt metaphor when they proposed that we regard lepton number as a fourth color. To explore that possibility, I have placed the lepton in Figure 6 at the apex of a tetrahedron that corresponds to the fundamental $\mathbf{4}$ representation of $SU(4)$.

If $SU(4)$ is not merely a useful *classification symmetry* for the quarks and leptons, but a *gauge symmetry*, then there must be new interactions that transform quarks into leptons, as indicated by the gold lines in the right panel of Figure 6. Are they present in Nature? If leptoquark transitions do

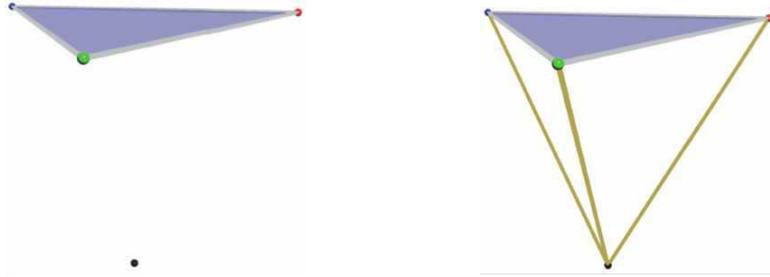


Figure 6: Left panel: A color triplet of quarks and a color singlet lepton, arrayed to explore lepton number as a fourth color. Right panel: Tetrahedron representing the $\mathbf{4}$ representation of $SU(4)$, showing the hypothetical leptoquark transitions.

exist, they can mediate reactions that change baryon and lepton number, such as proton decay. The long proton lifetime tells us that leptoquark transitions must be far weaker than the strong, weak, and electromagnetic interactions of the standard model. What accounts for the febleness of leptoquark transitions?

Our world isn't built of a single quark flavor and a single lepton flavor. The left-handed quark and lepton doublets offer a key clue to the structure of the weak interactions. We can represent the (u_L, d_L) and (ν_L, e_L) doublets by decorating the tetrahedron, as shown in Figure 7. The orange stalks connecting $u_L \leftrightarrow d_L$ and $\nu_L \leftrightarrow e_L$ stand for the W bosons that mediate

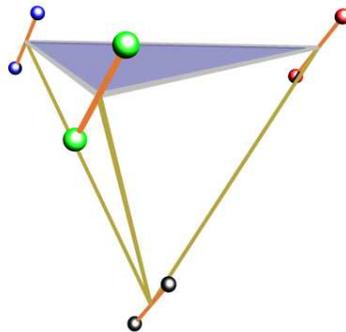


Figure 7: The $SU(4)$ tetrahedron, decorated with left-handed fermions.

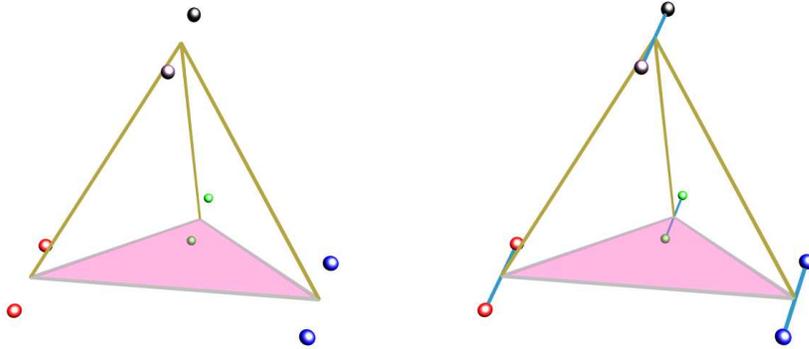


Figure 8: The inverted tetrahedron, decorated with right-handed quark (d_R, u_R) and lepton (e_R, ν_R) pairs. The left panel depicts our current understanding, without right-handed charged currents; the right panel shows how a right-handed W_R -boson could be added.

the charged-current weak interactions. Using the same object for quark and lepton transitions is a token for the universality of the weak charged-current couplings.

What about the right-handed fermions? In quantum field theory, it is equivalent to talk about left-handed *antifermions*. That observation motivates me to display the right-handed quarks and leptons as decorations on an inverted tetrahedron. The right-handed fermions are, by definition, singlets under the usual left-handed weak isospin, $SU(2)_L$, so I give the decorations an orthogonal orientation. Neutrino oscillations make us almost certain that a right-handed neutrino exists,² so I have placed a right-handed neutrino in Figure 8. I have given it a different coloration from the established leptons as a reminder that we have not proved its existence, and we do not know its nature.

We do not know whether the pairs of quarks and leptons carry a right-handed weak isospin, in other words, whether they make up $SU(2)_R$ doublets. We do know that we have—as yet—no experimental evidence for right-handed charged-current weak interactions. Accordingly, I will generally display the right-handed fermions without a connecting W_R -boson, as shown in the left panel of Figure 8. Is there a right-handed charged-current

²A purely left-handed Majorana mass term remains a logical, though not especially likely, possibility.

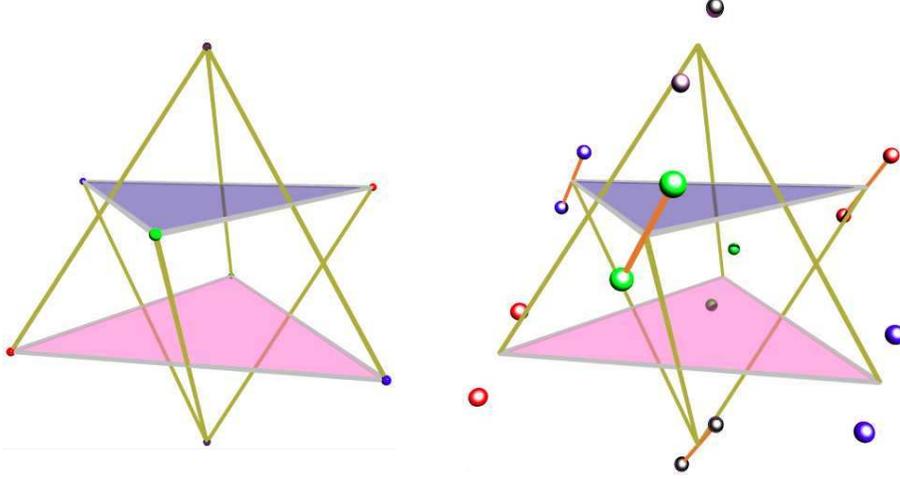


Figure 9: The double simplex, undecorated (left panel) and decorated with one generation of quarks and leptons (right panel).

interaction? If not, we come back to the question that shook our ancestors: what is the meaning of parity violation, and what does it tell us about the world? If we should discover—or wish to conjecture—a right-handed charged current, it can be added to our graphic, as shown in the right-panel of Figure 8. If there is a right-handed charged-current interaction, restoring parity invariance at high energy scales, what makes that interaction so feeble that we haven't yet observed it?

If parity violation in the weak interactions teaches us of an important asymmetry between left-handed and right-handed fermions, the nonvanishing masses of the quarks and leptons inform us that left and right cannot be entirely separate. Coupling the left-handed particle to its right-handed counterpart is what endows fermions with mass. For example, the mass term of the electron in the Lagrangian of quantum electrodynamics is

$$\mathcal{L}_e = -m_e \bar{e} e = -m_e \bar{e} \left[\frac{1}{2}(1 - \gamma_5) + \frac{1}{2}(1 + \gamma_5) \right] e = -m_e (\bar{e}_R e_L + \bar{e}_L e_R). \quad (1)$$

How shall we combine left with right? A suggestive structure is the pair of interpenetrating tetrahedra shown in Figure 9. Mathematicians refer to a tetrahedron as a *simplex* in three-dimensional space, so I call this construction the *double simplex*. We will return to the question of mass momentarily.

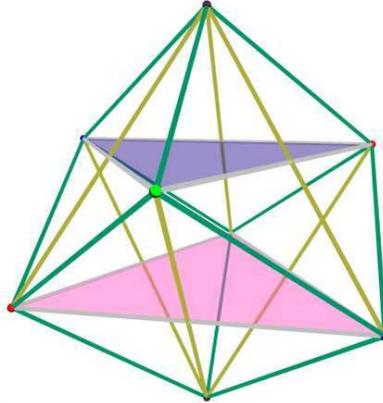


Figure 10: The double simplex, with additional interactions suggested by the shape of the figure indicated as green links.

Now think of holding the double simplex in your hand, turning it over to behold its symmetrical form. Your eyes may be drawn to link the nearest unconnected vertices, as shown in Figure 10. When that happens, you are visualizing one aspect of unification: When we combine the two sets of particles into one representation, we are invited to consider the possibility of new transformations that take any member of the extended family into any other. In this case, the hypothetical new interactions are easy to visualize, because the double simplex can be inscribed in a cube. Do some of these interactions exist? If so, why are they so weak that we have not yet observed them?

If we think of the double simplex as composed of left-handed particles and left-handed antiparticles, the agents of change will be new gauge bosons, since gauge-boson interactions preserve chirality. If we take the two tetrahedra to stand for left-handed and right-handed particles, then the new connections will be spin-zero particles.

Fermion masses tell us that the left-handed and right-handed fermions are linked, but we do not know what agent makes the connection. In the standard $SU(2)_L \otimes U(1)_Y$ electroweak theory, it is the Higgs boson—the avatar of electroweak symmetry breaking—that endows the fermions with mass. But this has not been proved by experiment, and it is certainly

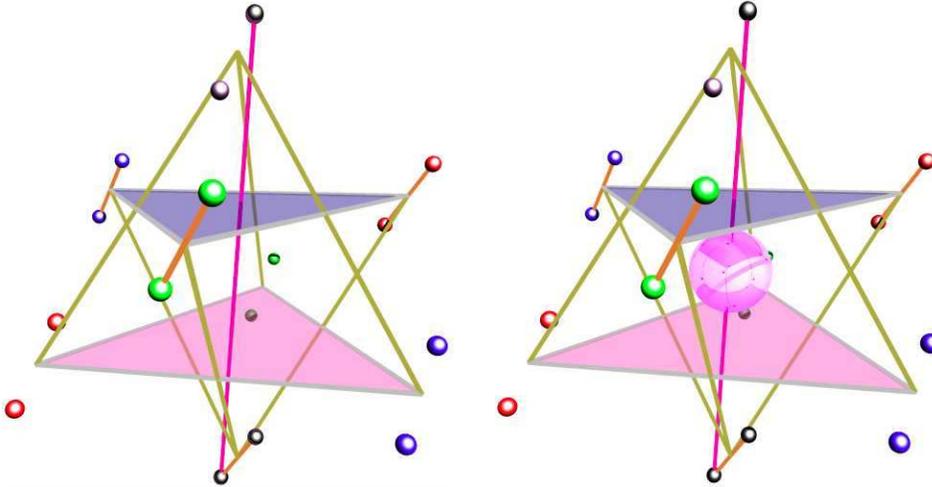


Figure 11: The connection between e_R and e_L implied by the electron's nonzero mass.

conceivable that some entirely different mechanism is the source of fermion mass.

I draw the connection between the left-handed and right-handed electrons in Figure 11. The left-hand panel shows the link between e_L and e_R . In the right-hand panel, I show the connection veiled within an iridescent globe that represents our ignorance of the symmetry-hiding phase transition that links left and right. [Critical opalescence is a marker for a phase transition.] It is excellent to find that the central mystery of the standard model—the nature of electroweak symmetry breaking—appears at the center of the double simplex!

Connecting all the left-handed fermions to their right-handed counterparts³ leads us to the representation given in Figure 12. Does one agent give masses to all the quarks and leptons? (That is the standard-model solution.) If so, what distinguishes one fermion species from another? We do not know the answer, and for that reason I contend that *fermion mass is evidence for physics beyond the standard model*. Let us illustrate the point

³I omit the neutrinos in this brief tour, because there are several possible origins for neutrino mass.

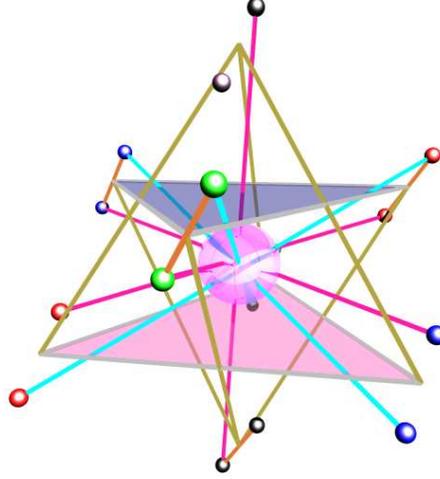


Figure 12: The connections that give rise to mass for the quarks and leptons of the first generation.

in the standard-model context. The mass of fermion f is given by

$$\mathcal{L}_f = -\frac{\zeta_f v}{\sqrt{2}}(\bar{f}_R f_L + \bar{f}_L f_R) = -\frac{\zeta_f v}{\sqrt{2}}\bar{f}f, \quad (2)$$

where $v/\sqrt{2} = (G_F\sqrt{8})^{-1/2} \approx 174$ GeV is the vacuum expectation value of the Higgs field. The *Yukawa coupling* ζ_f is not predicted by the electroweak theory, nor does the standard model relate different Yukawa couplings. In any event, we do not know whether one agent, or two, or three, will give rise to the electron, up-quark, and down-quark masses.

Of course, the world we have discovered until now consists not only of one family of quarks and one family of leptons, but of the three pairs of quarks and three pairs of leptons. We do not know the meaning of the replicated generations, and indeed we have no experimental indication to tell us which pair of quarks is to be associated with which pair of leptons.

Lacking any understanding of the relation of one generation to another, I depict the three generations in the double simplex simply by replicating the decorations to include three pairs of quarks and three pairs of leptons, as shown in the left panel of Figure 13. The connections that generate the fermion masses are indicated in the right panel of Figure 13. Look

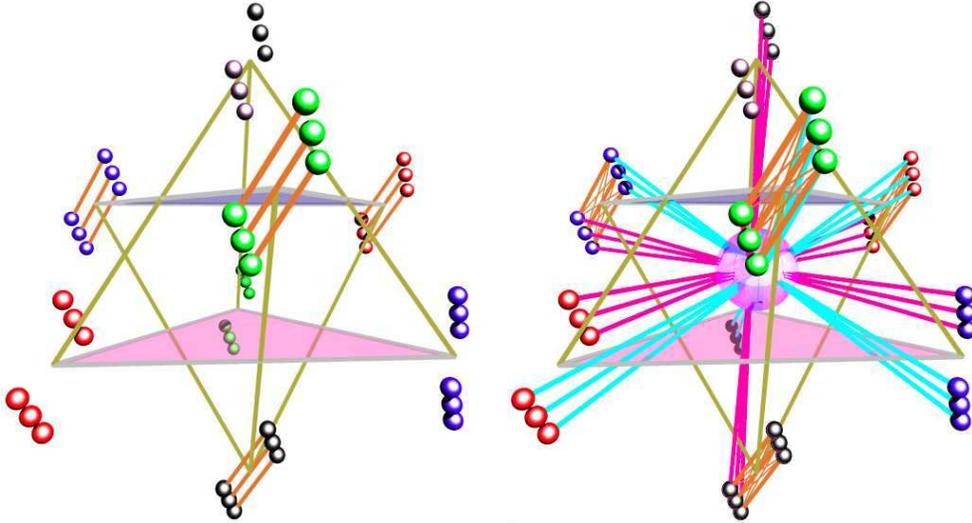


Figure 13: Left panel: Three generations of quarks and leptons. Right panel: The connections that give rise to mass and mixing for three generations of quarks and leptons.

closely at the lines representing weak charged-current transitions, before and after left is joined with right. In the case of more than one generation, the connections that endow the fermions with mass also determine the mixing among generations, the suppressed transitions such as $u \leftrightarrow s$ and $u \leftrightarrow b$.

We may not yet understand the character of the connections between left-handed and right-handed fermions, but this representation emphasizes that the connections are there. If, for example, you are a BaBar graduate student measuring a rare B decay in order to determine the quark-mixing matrix parameter V_{td} , the double simplex shows that you are not measuring an isolated quantity, but one tied to many other aspects of particle physics.

With three generations, the Yukawa couplings may have complex phases that give rise to \mathcal{CP} -violating transitions. Although it is correct to say that the standard model describes the observed examples of \mathcal{CP} violation, I would like to insist that because the standard model does not prescribe the Yukawa couplings, *\mathcal{CP} violation—like fermion mass—is evidence for physics beyond the standard model.*

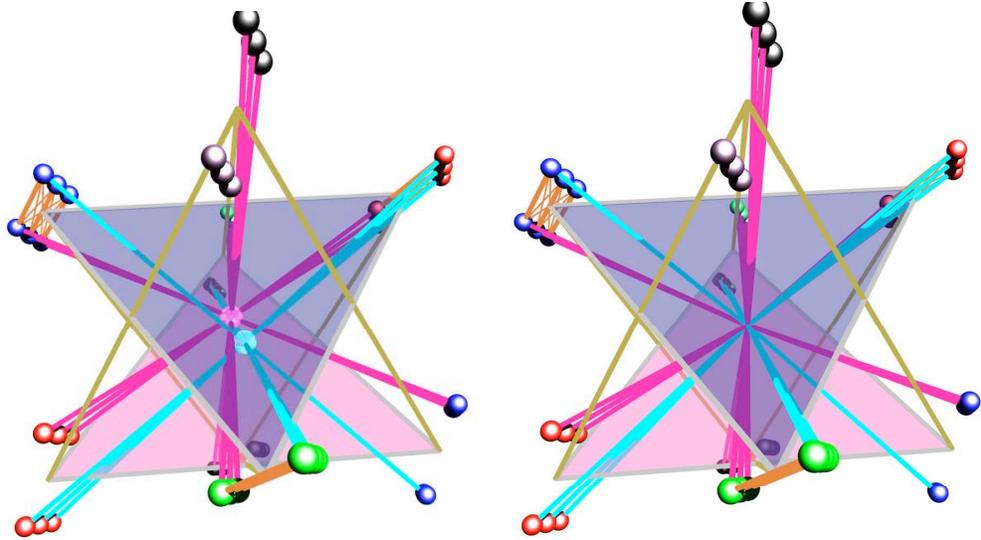


Figure 14: Left panel: Connections between left-handed fermions and their right-handed counterparts, with separate agents for the $I_3 = +\frac{1}{2}$ and $I_3 = -\frac{1}{2}$ particles. Right panel: Standard-model depiction of the connections.

I have said that I intend the double simplex as a device for eliciting questions. Might it also lead us to inventions? Here we have the first example. If we remove the iridescent globe to see what lies within, the left panel of Figure 14 shows that all the u, c, t connections pass through a common point, whereas the d, s, b , and charged-lepton connections pass through a distinct point. One agent gives mass to the weak-isospin $+\frac{1}{2}$ fermions, another agent sets the masses of the weak-isospin $-\frac{1}{2}$ fermions.

Now, this situation is not astounding to a roomful of theoretical physicists—we recognize it as characteristic of supersymmetric models or two-Higgs-doublet models. The fun comes because we did not (consciously!) build it in; we labeled the fermions in an obvious way and connected the dots. Had we never encountered the two-Higgs-doublet possibility before, we could have noticed it through the double simplex!

I hasten to add that this solution is not forced on us. The right panel of Figure 14 represents the standard model, and it is easy to represent many other possibilities as well. As we hoped, the double simplex offers both a

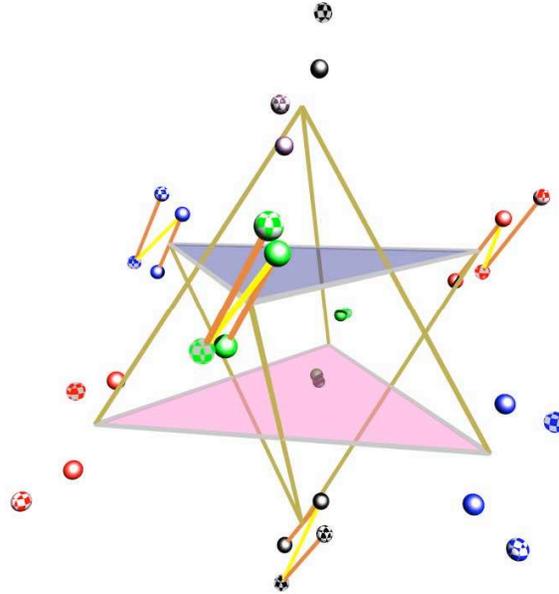


Figure 15: Squarks and sleptons (checked balls) added to the one-generation double simplex.

flexible language and a path to discovery.

Once the double simplex had taken shape, my colleagues and I amused ourselves by extending the basic object to display various sorts of physics beyond the standard model. I show a simple example in Figure 15. The superpartners of quarks and leptons are displayed as checked balls outboard of their standard-model partners. Charged-current weak interactions between squarks or sleptons are mediated by the same W -boson as those between quarks or leptons. Depicting the transitions by the same orange stalks in both cases is a metaphor for the equal couplings of particles and their superpartners. In addition, supersymmetry implies new interactions that change quarks into squarks, or leptons into sleptons. These are mediated by winos, depicted by yellow lines. It is obvious how to represent strong interactions among squarks, mediated by gluons, and quark–squark transitions, induced by gluinos.

We have found it easy to construct visual metaphors for all manner of new physics, including technicolor, Kaluza–Klein excitations, and Majorana

neutrinos. We have thus achieved the goal of devising an object that can be used not only to represent what we know, but also to illustrate new possibilities.

5 Closing remarks

The object of our double-simplex construction project has been to identify important topical questions for particle physics without plunging into formalism. As a theoretical physicist, I have deep respect for the power of mathematics to serve as a refiner's fire for our ideas. But I hope this exercise has helped you to see the power and scope of physical reasoning and the insights that can come from building and looking at a physical object with an inquiring spirit—even if the physical object inhabits an abstract space!

The mathematical underpinnings of the double simplex do bring discipline to the questions it elicits. The structure of the double simplex is based on the $SU(4) \otimes SU(2) \otimes SU(2)$ decomposition of $SO(10)$. A three-dimensional solid (tetrahedron) represents the fundamental $\mathbf{4}$ representation of $SU(4)$. It is decorated at the vertices with dumbbells representing the $SU(2)_L$ and $SU(2)_R$ quantum numbers. The vertical coordinate of $SU(4)$ can be read as $B - L$, the difference of baryon number and lepton number. The group $SO(10)$ is a useful classification symmetry, because its 16-dimensional fundamental representation contains an entire generation of the known quarks and leptons. Using $SO(10)$ as a coordinate system, if you like, carries no implication that it is the symmetry of the world, or that it is the basis of a unified theory of the strong, weak, and electromagnetic interactions.

We have succeeded in mapping current knowledge in a manner that evokes important questions, but we shouldn't be lulled into thinking that we have found all the important questions. We don't know whether we possess all the important clues. When Mendele'ev created his periodic table of the elements, he knew nothing of the noble gases. His seven-column table (in modern notation) was enormously informative and stimulating, but an attempted to create a theory of Mendele'ev's chart and nothing else would presumably have failed, because an important puzzle piece was missing. Might we be missing a crucial piece of our puzzle?

Let us summarize some questions we have encountered in the course of

building up the double simplex. □ Are quarks and leptons elementary? □ What is the relationship of quarks to leptons? □ Are there right-handed weak interactions? □ Are there new quarks and leptons? □ Are there new gauge interactions linking quarks and leptons? □ What is the relationship of left-handed & right-handed particles? □ What is the nature of the right-handed neutrino? □ What is the nature of the mysterious new force that hides electroweak symmetry? □ Are there different kinds of matter? □ Are there new forces of a novel kind? □ What do generations mean? How many? □ Which quarks go with which leptons □ Is there a family symmetry? □ What makes a top quark a top quark, and an electron an electron? □ What is the (grand) unifying symmetry? What hides it?

What can we say about the “homework problem” I posed at the end of §3? First, it is clear that quarks and leptons would remain massless, because mass terms are not permitted in our left-handed world if the electroweak symmetry remains manifest.⁴ We have done nothing to QCD, so that would still confine the (massless) color-triplet quarks into color-singlet hadrons, with very little change in the masses of those stable structures. In particular, the nucleon mass would be essentially unchanged, but the proton would outweigh the neutron because the down quark now does not outweigh the up quark, and that change will have its own consequences.

An interesting and slightly subtle point is that, even in the absence of a Higgs mechanism, the electroweak symmetry is broken by QCD. As we approach low energy in QCD, confinement occurs and the chiral symmetry that treated the massless left-handed and right-handed quarks as separate objects is broken. The resulting communication between the left-handed and right-handed worlds engenders a breaking of the electroweak symmetry. The trouble is that the scale of electroweak symmetry breaking is measured by the pseudoscalar decay constant of the pion, so the amount of mass acquired by the W and Z is set by f_π , not by what we know to be the electroweak scale: it is off by a factor of 2500.

But the fact is that the electroweak symmetry is broken, so the world without a Higgs mechanism—but with strong-coupling QCD—is a world in which the $SU(2)_L \otimes U(1)_Y$ becomes $U(1)_{em}$. Because the W and Z have masses, the weak-isospin force, which we might have taken to be a confining force in the absence of symmetry breaking, is not confining. Beta decay is

⁴I assume for this discussion that all the trappings of the Higgs mechanism, including Yukawa couplings for the fermions, are absent.

very rapid, because the gauge bosons are very light. The lightest nucleus is therefore one neutron; there is no hydrogen atom. Analyses of what would happen to big-bang nucleosynthesis in this world suggest that some light elements such as helium would be created. Because the electron is massless, the Bohr radius of the atom is infinite, so there is nothing we would recognize as an atom, there is no chemistry as we know it, there are no stable composite structures like the solids and liquids we know.

I invite you to explore this scenario in even greater detail. To do so is at least as challenging as trying to understand the world we do live in! The point is to see how very different the world would be, if it were not for the mechanism of electroweak symmetry breaking whose inner workings we intend to explore and understand in the next decade. What we are really trying to get at, when we look for the source of electroweak symmetry breaking, is why we don't live in a world so different, why we live in the world we do. I think that's a glorious question, one of the deepest questions that human beings have ever tried to engage, and we will find the answer!

In closing, I wish Gustavo well in his new and venerable estate, and I count on the Lisbon school to be in the thick of pursuing the questions we have discussed today. Fermi National Accelerator Laboratory is operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the U.S. Department of Energy. I am grateful to Gui Rebelo and the organizing committee for the kind invitation to participate in this celebration, and for warm hospitality. I thank many colleagues, notably Carl Albright, Uli Baur, Bogdan Dobrescu, Chris Hill, Andreas Kronfeld, Joe Lykken, Jack Marburger, Uli Nierste, Yasonuri Nomura, Dave Rainwater, and Maria Spiropulu, for their valuable contributions to the development of double simplex. My primitive sketchbook, containing interactive graphics and photographs of ball-and-stick models, with a minimal explanatory text, is available for browsing at <http://lutece.fnal.gov/DoubleSimplex>.