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Measurement of the Ratio of the Vector to Pseudoscalar Charm Semileptonic Decay Rate $\frac{\Gamma(D^+ \to \overline{K}^{*0}\mu^+\nu_\mu)}{\Gamma(D^+ \to \overline{K}^{0}\mu^+\nu_\mu)}$

The FOCUS Collaboration ⋆

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Abstract

Using a high statistics sample of photo-produced charm particles from the FOCUS experiment at Fermilab, we report on the measurement of the ratio of semileptonic rates $\frac{\Gamma(D^+\to K\pi\mu^+\nu_\mu)}{\Gamma(D^+\to\overline{K}^0\mu^+\nu_\mu)} = 0.625 \pm 0.045 \pm 0.034$. Allowing for the $K\pi$ S-wave interference measured in [1], we extract the vector to pseudoscalar ratio $\frac{\Gamma(D^+\to\overline{K}^{*0}\mu^+\nu_\mu)}{\Gamma(D^+\to\overline{K}^0\mu^+\nu_\mu)} = 0.594\pm0.043\pm0.033$ and the ratio $\frac{\Gamma(D^+\to\overline{K}^0\mu^+\nu_\mu)}{\Gamma(D^+\to K^-\pi^+\pi^+)} = 1.019\pm0.076\pm0.065$. Our results show a lower ratio for $\frac{\Gamma(D\to K^*\ell\nu)}{\Gamma(D\to K\ell\nu)}$ than has been reported recently and indicate the current world average branching fractions for the decays $D^+ \to \overline{K}^0(\mu^+, e^+)\nu_{\mu,e}$ are low. Using the world average $\mathrm{B}(D^+ \to K^-\pi^+\pi^+)$ [2] we extract $\mathrm{B}(D^+ \to \overline{K}^0\mu^+\nu) = (9.27 \pm 0.69 \pm 0.59 \pm 0.61)$ %.

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1 Introduction

There is currently some controversy concerning the relative rates of the charm vector semileptonic decays that proceed via a K^* and the charm pseudoscalar semileptonic decays that proceed via a K. Theorists originally expected the rates to be about the same [3], but other theoretical predictions and experimental measurements in the 90's tend to favor a smaller vector semileptonic rate (see Tables 1 and 2).

A more recent experimental result [4] indicates that the ratio of rates is closer to unity than measured previously. Rather than measure the rate for the pseudoscalar and vector semileptonic decays directly, as was done in previous measurements by the same experiment [5,6], the result uses the average from the PDG for the determination of $\mathrm{B}(D^+ \to K^0 e^+ \nu_e)$ [2]. Other experiments measure the semileptonic *rates* for D^+ and D^0 decays and form a ratio of vector to pseudoscalar rates (see Table 1).

Since the D^+ and the D^0 particles only differ by the light quark, exclusive semileptonic rates for the decays of these particles are expected to be equal through SU(3) symmetry. A comparison using the current world averages of the pseudoscalar decay branching fractions along with the D^+ and D^0 lifetimes [2] indicates that the pseudoscalar semi-electronic rates (the error on the D^+ pseudoscalar semi-muonic rate is too large for a meaningful comparison) are different at the 99% confidence level: $\Gamma(D^+ \to \overline{K}^0 e^+ \nu_e) - \Gamma(D^0 \to K^- e^+ \nu_e) = -25 \pm 9.7\ \mathrm{ns}^{-1}$. This result is surprising and merits further investigation. We intend to show in this paper that the difference in rates and the recent CLEO [4] result are in part due to the pseudoscalar semileptonic branching fraction reported in the PDG [2] for the decay $D^+ \to K^0 \ell^+ \nu_\ell$ being too low.

We measure the ratio $\frac{\Gamma(D^+ \to K\pi\mu^+\nu_\mu)}{\Gamma(D^+ \to \overline{K}^0\mu^+\nu_\mu)}$ directly, using decays with a very similar topology. Previous measurements of this ratio used comparisons between the D^+ and the D^0, relied on PDG branching ratios, and/or used decays with different topologies (for instance where one of the modes requires an added pion). By reconstructing the neutral kaon in the microvertex detector of the FOCUS experiment [7,8] through the decay $K_S^0 \to \pi^+\pi^-$, we take advantage of the studies and work performed to produce precise lifetime measurements of the long-lived charm particles. By measuring the D^+ decay, we take advantage of the extensive work performed to understand the decay $D^+ \to K^-\pi^+\mu^+\nu_\mu$ [1,9,10] and our result is the first to incorporate the interference effects described in [9] and measured in [1].

The data for this analysis were collected using the Wideband photoproduction experiment FOCUS during the 1996–1997 fixed-target run at Fermilab. The FOCUS detector is a large aperture, fixed-target spectrometer with excellent

vertexing and particle identification used to measure the interactions of high energy photons on a segmented BeO target. The FOCUS beamline [11] and detector [7–9,12] have been described elsewhere.

Table 1
Previous results compared to the FOCUS result. Notice that some results [13–15] are admixtures of different charm species related through the isospin argument, [14,15] are correlated since the same $D^+ \to \overline{K}^{*0}\mu^+\nu$ result is used, and in the CLEO(02) result [4], the $\Gamma(D^+ \to \overline{K}^0 e^+\nu)/\Gamma_{\rm Total}$ comes from the PDG00 [16] estimate for $\Gamma(D^+ \to \overline{K}^0 \ell^+\nu)/\Gamma_{\rm Total}$.

Experiment	Quantity	Result
CLEO(91)[5]	$\frac{\Gamma(D^0\to K^{*-}e^+\nu)}{\Gamma(D^0\to K^-e^+\nu)}$	$0.51 \pm 0.18 \pm 0.06$
CLEO(93)[6]	$\frac{\Gamma(D^0\to K^{*-}e^+\nu)}{\Gamma(D^0\to K^-e^+\nu)}$	$0.60 \pm 0.09 \pm 0.07$
CLEO(93)[6]	$\frac{\Gamma(D^+\to \overline{K}^{*0}e^+\nu)}{\Gamma(D^+\to \overline{K}^{0}e^+\nu)}$	$0.65 \pm 0.09 \pm 0.10$
E691(89)[13]	$\frac{\Gamma(D^+\to \overline{K}^{*0}e^+\nu)}{\Gamma(D^0\to K^-e^+\nu)}$	0.55 ± 0.14
E687(93)[14]	$\frac{\Gamma(D^+\to \overline{K}^{*0}\mu^+\nu)}{\Gamma(D^0\to K^-\mu^+\nu)}$	$0.59 \pm 0.10 \pm 0.13$
E687(95)[15]	$\frac{\Gamma(D^+\to \overline{K}^{*0}\mu^+\nu)}{\Gamma(D^0\to K^-\mu^+\nu)}$	$0.62 \pm 0.07 \pm 0.09$
CLEO(02)[4]	$\frac{\Gamma(D^+\to \overline{K}^{*0}e^+\nu)}{\Gamma(D^+\to \overline{K}^{0}e^+\nu)}$	$0.99 \pm 0.06 \pm 0.07 \pm 0.06$ (± 0.12) [a]
FOCUS(04)	$\frac{\Gamma(D^+\to \overline{K}^{*0}\mu^+\nu)}{\Gamma(D^+\to \overline{K}^{0}\mu^+\nu)}$	$0.594 \pm 0.043 \pm 0.030$

[a] The PDG00 [16] error for $\Gamma(D^+ \to \overline{K}^0 \ell^+\nu)/\Gamma_{Total}$, omitted [18] in the CLEO [4] result, is shown in parentheses.

2 Event Selection

We identify D^+'s through the 3-body decay $D^+ \to h^-\pi^+\mu^+\nu_\mu$ (where the h represents a pion or a kaon and charge conjugate modes are implied throughout this paper). To search for candidate events, a search strategy common to both modes is employed. This is possible since roughly 10% of all K_S^0's reconstructed in the $\pi^+\pi^-$ channel are reconstructed using hits from the FOCUS silicon microvertex detectors. [1] Where possible, we have chosen cuts similar to those optimized in other FOCUS analyses to enhance charm content and particle identification.

Two opposite sign tracks reconstructed using information from the silicon detectors are required to form a vertex with a confidence level exceeding 1%. To suppress background from short-lived, primarily non-charm particles produced

[1] The technique used in [17] to reconstruct $D^+ \to K_S^0\pi^+$ using the bulk of the K_S^0's is not applicable in this case due to the missing neutrino.

Table 2
Theoretical predictions for the Vector to Pseudoscalar ratio. No distinction is made for electrons, muons or charm species. In some cases, listed with a (*), the ratio was calculated from information given in the publication.

Model	$\frac{\Gamma(D\to K^*\ell^+\nu)}{\Gamma(D\to K\ell^+\nu)}$
WSB(85*) [3]	1.13
LFR(90*) [19]	0.7, 0.67
SUMR(91)[20]	0.50 ± 0.15
LAT(92*) [21]	0.86 ± 0.27
LAGR(93*) [22]	0.56
LAT(94) [24]	$1.1 \pm 0.6 \pm 0.3$
MITBAG(94) [23]	0.56
ISGW2(95) [25]	0.54
QPT(96) [26]	0.65
RQM(96) [27]	0.57
QM(97) [28]	0.62
LFM(97) [29]	0.68
SR(97*) [30]	$0.47^{+0.19}_{-0.17}$
QM(00),DISP(01) [31,34]	0.63
COVQ(01*) [32]	1.01, 0.72
CVLFD(01) [33]	0.66, 0.64, 0.67
PCL(01) [35]	0.54, 0.63, 0.57, 0.67
EFT(02) [36]	0.5 ± 0.2

in the targets, this 2-track vertex is required to occur at least 1 standard deviation outside of target material. The two tracks are then formed into a single track that is combined with a candidate muon to make a putative D^+ vertex with a confidence level exceeding 1%. This D^+ vertex is required to occur at least 1 standard deviation outside of target material as well.

Due to the relatively long lifetime and Lorentz boost of charm candidates, the primary interaction vertex and secondary D^+ decay vertex can have a significant separation along the beam direction. Tracks not used to form a D^+ are used to construct a set of candidate primary vertices with confidence level greater than 1%. The highest multiplicity primary vertex candidate with the highest separation, ℓ, from the secondary in units of error, σ_ℓ, greater than 3 is retained. Since the significance of the separation between the interaction and

decay vertices is an essential tool used to separate charm from background, we perform a scan in this variable between $\ell/\sigma_\ell = 3$ and $\ell/\sigma_\ell = 23$ (roughly a factor of 3 in yield) to judge the stability of our result. We find that our results become stable after a cut of $\ell/\sigma_\ell > 5$ is reached. Additionally, the fit quality for the determination of the $D^+ \to K^0_S\mu^+\nu_\mu$ yield becomes optimal ($\chi^2/\mathrm{DOF} < 1$) between $\ell/\sigma_\ell = 11$ and $\ell/\sigma_\ell = 19$. For the final sample, a value of $\ell/\sigma_\ell = 13$ is used. The ratios of the yields of individual modes to that expected from simulations exhibit the same stability. This is a strong indication that our cuts are effective at removing any short-lived (non-charm) backgrounds that could mimic signal.

Muon candidates are required to be within the acceptance of the inner muon system in FOCUS [9]. Since the rate of muon misidentification increases at low momentum, we require that the momentum of muon candidate tracks be greater than 13 GeV/c. Muon candidates are required to have associated hits in the muon system sufficient to meet a minimum confidence level of 5% for the muon hypothesis where at least 5 of 6 planes of the detector must record hits consistent with the candidate track.

To separate decays proceeding through the $K^-\pi^+\mu^+\nu_\mu$ channel, from those decaying through the $K^0_S\mu^+\nu_\mu$ channel, we use additional vertex, particle ID and invariant mass requirements.

The vertex representing the $K^-\pi^+$ is required to occur within 3 standard deviations of the $(K^-\pi^+)\mu^+$ vertex, and a three track vertex formed from the K^-,π^+ and μ^+ tracks must exceed a confidence level of 5%. Since the K^0_S lifetime is long compared even to the charm lifetime, we find a very effective cut to reduce non-K^0_S contamination is to impose a requirement that the K^0_S vertex have a large separation from the charm decay vertex. We require that the vertex representing the $K^0_S \to \pi^-\pi^+$ decay occur at least 15 standard deviations downstream of the $(\pi^-\pi^+)\mu^+$ vertex.

We use the Čerenkov system [12] to identify pions and kaons. For each track, $W_{\mathrm{obs}} = -2\log(\mathcal{L})$ is computed, where $\mathcal{L}$ is the likelihood that a track is consistent with a given particle hypothesis. For the track in the $K^-\pi^+\mu^+$ vertex with charge opposite to the muon, we require $W_{\mathrm{obs}}(\pi) - W_{\mathrm{obs}}(K)$ (kaonicity) be greater than 2.0, and for the track same charge as the muon in the $K^-\pi^+\mu^+$ vertex, we require $W_{\mathrm{obs}}(K) - W_{\mathrm{obs}}(\pi)$ (pionicity) be greater than 0.0. For pions that form a candidate K^0_S, we require that the pion likelihood be favored over the particle hypothesis that forms the minimum likelihood $W_{min} - W_{\mathrm{obs}}(\pi)$ be greater than -5.0 (a loose cut).

Background from $D^+ \to K^-\pi^+\pi^+$ and $D^+ \to K^0_S\pi^+$, where a pion is misidentified as a muon, is reduced by requiring that the visible mass $M(h^-\pi^+\mu^+) < 1.8\ \mathrm{GeV/c^2}$. In order to suppress background from $D^{*+} \to D^0\pi^+ \to (K^-\mu^+\nu_\mu)\pi^+$

we require $M(K^-\pi^+\mu^+) - M(K^-\mu^+) > 0.2$. In order to enrich the K_S^0 sample we require the K_S^0 invariant mass be within 2 standard deviations of the nominal reconstructed value.

To reduce backgrounds from topologies consistent with extra tracks coming from the secondary vertex (such as expected backgrounds like $D^0 \to \overline{K}^0\pi^-\mu^+\nu_\mu$ or high multiplicity charm and non-charm backgrounds not present in the simulation) we reject candidates where any track not used to create the secondary vertex can be included in the secondary vertex with a vertex fit confidence level exceeding 10%.

3 Analysis

We fit the $K_S^0\mu^+$ invariant mass and the $K^-\pi^+$ invariant mass. Fit components are a combination of Monte Carlo histograms: one generated with the mode of interest and others representing background components. A maximum likelihood is used where the predicted number of events in a bin$_i$ is described by:

$$N(\text{predicted})_i = P_S\text{Signal}_i + P_j\text{Background}_{i,j}$$

where P_S and the P_j's (more than one background shape is used) are fit parameters. The number of predicted signal events in the data is then described by $\sum_i^{\#\text{bins}} P_S\text{Signal}_i$ where Signal$_i$ is the number of reconstucted Monte Carlo events for the mode of interest in a given bin.

We find that the fit to determine the yield of the $D^+ \to K^-\pi^+\mu^+\nu_\mu$ events requires only 2 components: one for signal and one for background. The $D^+ \to K^-\pi^+\mu^+\nu_\mu$ matrix element has only recently been fully measured, and we simulate the signal shape and efficiency with the results from [1]. We represent the background shape in the $K^-\pi^+$ mass by generating a Monte Carlo in which we simulate all known charm decay backgrounds while removing $D^+ \to K^-\pi^+\mu^+\nu_\mu$ from the generated particle mix.

We have tried several approaches to fitting the invariant $K_S^0\mu^+$ mass histogram. To generate $D^+ \to \overline{K}^0\mu^+\nu_\mu$ events, we use the simple pole form for the pseudoscalar to pseudoscalar semileptonic form factor where:

$$f_\pm(q^2) = f_\pm(0)/(1 - q^2/M^2_{\text{pole}}), \quad q^2 = (P_D - P_{\text{kaon}})^2 = M^2_{\text{W-virtual}}.$$

We use $M_{\text{pole}} = 2.11$ GeV/c^2 and $f_-/f_+ = -0.7$. Since we accept almost the

entire $K_S^0\mu^+$ invariant mass range, small changes in the choice of M_{pole} and f_-/f_+ have a negligible effect on our final result. To represent the background in the data, several techniques were investigated.

If the background in the data is primarily due to $D \to \overline{K}^0\pi\mu^+\nu_\mu$ when a π^0 or π^+ is missed and $D^+ \to \overline{K}^0\mu^+\nu_\mu$ when either the K_S^0 or the μ^+ is not from the D^+, it should be sufficient to perform a fit including only these two components. We find that this approach produces consistent results but poor fit quality at low ℓ/σ_ℓ's. Fit quality improves though at the cost of stability if the 2 (or more) lowest mass bins are removed from the fit. We also find a slight improvement in fit quality and stability at high ℓ/σ_ℓ if a Breit-Wigner component centered at 892 MeV/c^2 with a width of 50 MeV/c^2 is added to the fit.

In order to try and improve the fit quality, we added a background shape to the previous 3 component fit by generating a Monte Carlo in which we simulate all known charm decay backgrounds while removing $D^+ \to K_S^0\mu^+\nu_\mu$ from the generated particle mix. We found that the fit quality did not improve, the error on the returned fit increased, and the results were not stable below $\ell/\sigma = 9$. Even though the results agreed quite well at higher ℓ/σ, we decided to investigate this behavior further.

In order to reproduce a background shape which may contain unsimulated backgrounds, we took events which had at least one extra track consistent with the secondary vertex at vertex fit confidence levels between 30% and 90%. In order to gauge the specific effect of non-simulated backgrounds on the final result, invariant mass histograms were formed from both the data and from a Monte Carlo in which we simulate all known charm decays except $D^+ \to \overline{K}^0\mu^+\nu_\mu$ and $D^+ \to \overline{K}^0\pi^0\mu^+\nu_\mu$. Using these shapes in addition to the 3 component fit significantly improves the fit quality. The χ^2/DOF for the fit using the background from the data is acceptable ($\sim$ 1 or less) at all ℓ/σ_ℓ's. There is now a component of the signal in the data background subtraction. As a consequence, the signal and background are correlated in the fit, and the fit errors increase. In the fit using the simulated background, we find that the χ^2/DOF is about 1 unit higher than the data represented background fit until ℓ/σ_ℓ's above 11, where the fit quality becomes equivalent between the two representations. The difference in χ^2/DOF at low ℓ/σ_ℓ's is likely due to non-charm or short-lived charm decays, appearing primarily at low $K_S^0\mu$ mass according to the results of the binning tests, which are not included in the simulation. Even though we see a difference between the data and simulated background in χ^2/DOF at low ℓ/σ_ℓ, the results of the fits are in good agreement and stable above $\ell/\sigma_\ell = 3$.

Our quoted result uses the fit to the $K_S^0\mu^+$ invariant mass with the simulated background from higher multiplicity secondary vertices at $\ell/\sigma_\ell = 13$. It is

important to note that in all four of the fits described, the 3 component fit, the fit with the inclusive simulated background, and the fits with the background from higher multiplicity secondaries produce equivalent values above $\ell/\sigma_\ell = 11$. This is likely due to the background being dominated by $D \to K\pi\mu^+\nu_\mu$ decays at higher ℓ/σ_ℓ.

The fit to the data for both modes is presented in Fig. 1. We find 555 ± 39 $K^0_S\mu^+\nu_\mu$ decays and 9871 ± 127 $K^-\pi^+\mu^+\nu_\mu$ decays.

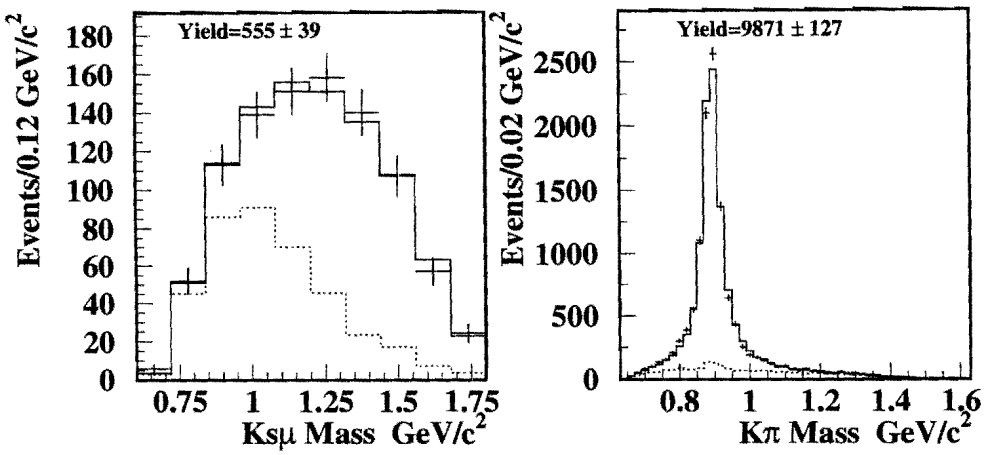

Fig. 1. The fit to the $K^0_S\mu^+$ invariant mass (left) and the $K^-\pi^+$ invariant mass (right). The fit to the data (error bars) is shown as a solid line, and the background described in the text in shown as a dotted line. For both plots, the difference between the solid and dotted lines represents the signal. Note the good fit to the $K^0_S\mu$ invariant mass and the cleanliness of the $K^-\pi^+$ invariant mass.

Thus, our ratio $\frac{\Gamma(D^+\to K\pi\mu^+\nu_\mu)}{\Gamma(D^+\to \overline{K}^0\mu^+\nu_\mu)}$ becomes:

$$\frac{1/2}{2/3}\,\frac{\#K^-\pi^+\mu^+\nu_\mu(\mathrm{FIT})}{\#K^0_S\mu^+\nu_\mu(\mathrm{FIT})}\,\frac{\epsilon(K^0_S\mu^+\nu_\mu)}{\epsilon(K^-\pi^+\mu^+\nu_\mu)}$$

The 2/3 accounts for the probability that $\overline{K}^{*0}$ decays to $K^-\pi^+$ (see above) and the 1/2 accounts for the probability that K^0 decays to K^0_S. The $K^0_S \to \pi^+\pi^-$ branching fraction is accounted for in the Monte Carlo generation. The number of events determined from the fit to the data for each mode is labeled (FIT), and the reconstruction efficiency for each mode determined using the Monte Carlo indicated by an ϵ. In order to quote the K* component as a separate result, we separate the resonant and non-resonant components using the technique outlined in [1]. Fully $95.0\pm0.5\%$ of the $K^-\pi^+$ sample is proceeds through a $\overline{K}^{*0}$. Thus we find:

$$\frac{\Gamma(D^+ \to K\pi\mu^+\nu)}{\Gamma(D^+ \to \overline{K}^0\mu^+\nu)} = 0.625 \pm 0.045$$

and,

$$\frac{\Gamma(D^+ \to \overline{K}^{*0}\mu^+\nu)}{\Gamma(D^+ \to \overline{K}^0\mu^+\nu)} = 0.594 \pm 0.043.$$

Our systematic tests of the result are outlined in the next section.

4 Systematic Checks

Our systematic uncertainty comes from known quantities which are not included in the fit (such as the S-wave contribution estimate), unanticipated variations in the data not accounted for in the simulation, and variations due to the fitting technique.

To determine the amount of $D^+ \to \overline{K}^{*0}\mu^+\nu_\mu$ contained in our $D^+ \to K^-\pi^+\mu^+\nu$ signal, we corrected our estimated yield of $D^+ \to K^-\pi^+\mu^+\nu$ by 0.950 ± 0.005. We compute this fraction by integrating over the $K^-\pi^+\mu^+\nu$ phase space the model intensity and parameters from reference [1] with the S-wave amplitude set to zero and dividing this value by the same with the S-wave amplitude and phase set to the measured values. The uncertainty (± 0.005) is determined by varying the amplitude and phase by the errors indicated in [1] and noting the difference.

In order to assess a systematic uncertainty due to larger variations of M_{pole}, we varied M_{pole} between 1.86 and 2.31 GeV/c^2 in the Monte Carlo generation. The resultant simulated signal histograms are then used to repeat the fit used to obtain the final result. The sample variance from the returned fit results is retained as the systematic uncertainty due to M_{pole}. To estimate the systematic uncertainty due to different values of f_-/f_+, we repeated the fit at $M_{\text{pole}} = 2.11$ GeV/c^2 with $f_-/f_+ = 0.7$ and kept the difference as the error estimate.

In order to assess a systematic error to the measured ratio from unanticipated variations in the data not accounted for in the simulation, we placed a variety of pertinent cuts on the data and computed a sample variance from the returned values. The cuts described below are in addition to those previously applied. Unless a particular meson or mode is mentioned, cuts are applied to both modes used to calculate the ratio simultaneously.

To check for non-K_S^0 $\pi^+\pi^-$ backgrounds, we set the normalized K_S^0 mass cut to values of 3 and 1. To check for backgrounds where a neutral long-lived particle such as a Λ is misidentified as a K_S^0, we made a cut requiring that the difference in the magnitude of the K_S^0 candidate pion momenta be no greater than 70% of the sum. We also increased the Čerenkov requirement on both pions so that $W_{\text{obs}}(K) - W_{\text{obs}}(\pi)$ (pionicity) be greater than 0.0. Since this latter cut primarily removes low momentum K_S^0's, it is an effective tool at examining the K_S^0 acceptance as well.

To check for unanticipated backgrounds and differences between the simulation and the signal mode due to event topology, we investigated how the ratio behaves with a variety of requirements on the detailed location of the reconstructed event in the spectrometer. We required the D^+ vertex be located downstream of the first interaction target, downstream of the second interaction target, upstream of a trigger counter located near the main silicon tracking system, or upstream of a location roughly between the 2 downstream target silicon system planes. We also required the K_S^0 vertex be 2 cm downstream of the D^+ vertex, upstream of the main (last 12 planes) silicon system, or downstream of the target (1st 4 planes) silicon system [8].

To look for short lived or non-charm background, we increased the requirement to 3 standard deviations that the D^+ vertex occur outside of target material, and we required $P_{\text{visible}} > 30$ GeV/c^2. As a check for higher multiplicity decays feeding into the signal, we specified that the maximum allowable confidence level that an additional track be consistent with the secondary be 1%.

In order to specifically look for decays feeding into the signal where a particle is misidentified as a muon, we chose cuts that should reduce the probability of contamination while leaving high efficiency for signal. We placed a cut on the muon requiring that the momentum measured by both magnets in the spectrometer be consistent. We increased the muon momentum cut to > 20 GeV/c^2. We also increased the requirement on the muon identification confidence level to 15%.

To check for additional background in the $K\pi$ mass, we increased the Čerenkov likelihood difference cut on the kaon from > 2.0 to > 4.0, and increased the cut on the mass difference used to cut out D^*'s to 0.25 and 0.30.

We assess a systematic uncertainty from these cut tests by computing a sample variance of the returned values for $\frac{\Gamma(D^+\to K\pi\mu^+\nu)}{\Gamma(D^+\to\overline{K}^0\mu^+\nu)}$. The larger contributions to this systematic error estimate are listed in Table 3.

In order to test the fit, we performed repeated tests where we Poisson fluctuated the bins of both the data and fit histograms and performed repeated fits. Our tests indicate that the $K_S^0\mu^+\nu_\mu$ yield error is underestimated by 6% (due

to finite Monte Carlo statistics), and the statistical error in the final result is boosted to account for this difference. We also looked at the standard deviation between the four different fits tried. Although the fits agree remarkably well at $\ell/\sigma_\ell = 13$, we felt a more conservative approach was to choose a value that was common to four returned ratios below and above the chosen ℓ/σ_ℓ. The systematic uncertainty estimated from the four fits is added in quadrature to the previously described estimates to assess a total systematic uncertainty in the ratio (see Table 3).

Table 3
Larger contributions to the systematic uncertainty in the ratio $\frac{\Gamma(D^+\to K\pi\mu^+\nu)}{\Gamma(D^+\to\overline{K}^0\mu^+\nu)}$. Note that the uncertainty associated with the non-S-wave $K\pi$ contribution applies only to the estimate of the K^* fraction in the $K\pi$ signal.

Systematic Contribution	Value
Normalized K_S^0 Mass Cut	0.008
Secondary Vertex Location	0.017
K_S^0 Vertex Location	0.013
Muon Magnet Consistency	0.012
Muon Momentum Cut	0.008
Total contributions from cut variations	0.028
M_{pole} and f_-/f_+ variation	0.015
Contribution from fit variations	0.013
Total systematic uncertainty	0.034
S-wave Fraction (K^* ratio only)	0.003

5 Summary and Conclusions

Our result represents a substantial improvement over previous results for the ratio of the vector to pseudoscalar decay in the muon channel. We find

$$\frac{\Gamma(D^+ \to K\pi\mu^+\nu)}{\Gamma(D^+ \to \overline{K}^0\mu^+\nu)} = 0.625 \pm 0.045 \pm 0.034$$

and

$$\frac{\Gamma(D^+ \to \overline{K}^{*0}\mu^+\nu)}{\Gamma(D^+ \to \overline{K}^0\mu^+\nu)} = 0.594 \pm 0.043 \pm 0.033.$$

Where the first error is statistical and the second error is the systematic uncertainty. The K^*/K result agrees with older results, but disagrees with a recent result from the CLEO collaboration (with no corrections ($\sim 3\%$ effect) made for phase space and form factors). One possible source of this difference can be due to the vector decay as is discussed in Reference [1]. Another source is detailed below.

By using our vector/pseudoscalar result and our measurement of the branching ratio $\frac{\Gamma(D^+\to\overline{K}^{*0}\mu^+\nu)}{\Gamma(D^+\to K^-\pi^+\pi^+)}$ [10], correcting for the updated values of the S-wave contribution [1], we determine:

$$\frac{\Gamma(D^+ \to \overline{K}^0\mu^+\nu)}{\Gamma(D^+ \to K^-\pi^+\pi^+)} = 1.019 \pm 0.076 \pm 0.065,$$

where we have added the statistical and systematic errors separately in quadrature.

Using the PDG [2] values for the absolute branching fraction of the decay $D^+ \to K^-\pi^+\pi^+$: $(9.1 \pm 06)\%$, we calculate,

$$\mathrm{B}(D^+ \to \overline{K}^0\mu^+\nu) = (9.27 \pm 0.69 \pm 0.59 \pm 0.62)\ \%.$$

The third error is due to uncertainty in the $D^+ \to K^-\pi^+\pi^+$ branching fraction. Our result is a substantial improvement over the present world average [2] of $7.0^{+3.0}_{-2.0}\%$. Besides the difference between the D^+ and D^0 semi-electronic rates mentioned in the introduction, there are other reasons to believe that the PDG value for the D^+ semi-electronic mode $6.5 \pm 0.9\%$ is low.

We can compare the D^0 and D^+ semi-muonic rates as we did for the semi-electronic rates in the introduction. We find good agreement with $\Gamma(D^+ \to \overline{K}^0\mu^+\nu_\mu) - \Gamma(D^0 \to K^-\mu^+\nu_\mu) = 11 \pm 11$ ns^{-1} (where no correction for the difference in phase space between D^+ and D^0 has been made).

It is likely that the semi-muonic rate is lower than the semi-electronic rate by a few percent, and this is consistent with what is measured for the isospin conjugate decay of the D^0 using values from [2]. One sees $\frac{\Gamma(D^0\to K^-e^+\nu_e)/\Gamma(D^0\to K^-\pi^+)}{\Gamma(D^0\to K^-\mu^+\nu_\mu)/\Gamma(D^0\to K^-\pi^+)} = 1.12 \pm 0.07$, for the D^0 but $\frac{\Gamma(D^+\to K^0e^+\nu_e)/\Gamma(D^+\to K^-\pi^+\pi^+)}{\Gamma(D^+\to\overline{K}^0\mu^+\nu_\mu)/\Gamma(D^+\to K^-\pi^+\pi^+)} = 0.72 \pm 0.11$ for the D^+ using the result in this paper. The PDG estimates that this ratio should be around 1.03[2]. This is reasonable since a very large positive ratio for f_-/f_+, which increases the semi-muonic rate, is not likely given the E687 [15] result for f_-/f_+, and radiative corrections for the semi-electronic mode, which can lower the measured semi-electronic rate, are expected to be small. It is also unlikely that any such large, unanticipated corrections apply to the D^+

semi-electronic mode exclusively.

Finally, it is interesting to note that the sum of the rates measured by FOCUS, corrected as suggested in the PDG [2] to estimate the semi-electronic modes, is $B(D^+ \to ((1.05)K\pi + (1.03)K^0)\mu^+\nu_\mu) = 14.9 \pm 1.2\%$. This is closer to the current world average inclusive electronic rate $D^+ \to e^+(\text{anything}) = 17.2 \pm 1.9\%$ than $B(D^+ \to ((3/2)K^-\pi^+ + K^0)\mu^+\nu_\mu) = 12.9^{+1.6}_{-1.4}\%$ (both from [2]), suggesting that a large, previously unseen, semileptonic decay mode for the D^+ is unlikely.

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