

Resonant leptogenesis in a predictive $SO(10)$ grand unified model

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Abstract

An $SO(10)$ grand unified model considered previously by the authors featuring lopsided down quark and charged lepton mass matrices is successfully predictive and requires that the lightest two right-handed Majorana neutrinos be nearly degenerate in order to obtain the LMA solar neutrino solution. Here we use this model to test its predictions for baryogenesis through resonant-enhanced leptogenesis. With the conventional type I seesaw mechanism, the best predictions for baryogenesis appear to fall a factor of three short of the observed value. However, with a proposed type III seesaw mechanism leading to three pairs of massive pseudo-Dirac neutrinos, resonant leptogenesis is decoupled from the neutrino mass and mixing issues with successful baryogenesis easily obtained.

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I. INTRODUCTION

The accumulation of refined data on both neutrino oscillation and quark flavor physics has posed increasingly severe tests for quark and lepton models which attempt to explain the mass and mixing data. This is especially true for Grand Unified models which must relate both the quark and lepton sectors. In fact, at present several such models [1] do survive these tests given the present levels of experimental precision for the CKM quark and MNS lepton mixing matrix elements. Two other hurdles for these Grand Unified models involve the issues of flavor-changing neutral currents, for example in $\mu \rightarrow e + \gamma$ and $\tau \rightarrow \mu + \gamma$ decays, and the survival of baryogenesis in the early Universe. It is the latter hurdle which we wish to address in this paper.

It appeared early on that Grand Unified Theories (GUTs) provided the necessary conditions for successful baryogenesis as postulated by Sakharov [2]: baryon-violating interactions which involve both C - and CP -violation and which occur out-of-equilibrium in the early Universe. Since then we have learned that the baryon-violating interactions which occur near the GUT scale conserve $B - L$, so that any net baryon number generated by them can be washed out by sphaleron $B + L$ interactions which occur in thermal equilibrium with the expanding universe. Only if the net baryon number is generated with $\Delta(B - L) \neq 0$ interactions will it not be erased by the sphaleron interactions [3].

For this reason, Fukugita and Yanagida suggested that leptogenesis may play a necessary primary role for baryogenesis [4]. An excess in lepton number generated by the lepton-violating decays of the heavy right-handed neutrinos can be converted into a baryon excess by sphaleron interactions in thermal equilibrium above the critical electroweak symmetry-breaking temperature. In this decay or direct “ ϵ' ” CP -violating scenario, the CP violation is generated through an interference between the decay tree graph and the absorptive part of the one-loop decay vertex. This mechanism requires very heavy N_i , $i = 1, 2, 3$ right-handed neutrinos, with the lightest being heavier than roughly 10^9 GeV. Thus with a moderate hierarchy of heavy right-handed neutrinos and with the lightest satisfying the above bound, satisfactory leptogenesis can be generated.

However, it has been observed that successful leptogenesis may also arise through indirect “ ϵ ” CP -violating mass-mixing effects in the decays of two quasi-degenerate heavy right-handed neutrinos [5]. In other words, resonant enhancement of the lepton asymmetry can be generated if the two neutrinos are sufficiently close in mass that level crossing can occur. Ellis, Raidal, and Yanagida as well as Akhmedov, Frigerio and Smirnov have performed phenomenological studies [6] of neutrino mass matrices and shown that sufficient lepton asymmetry can be generated in the case that the level crossing involves the two lighter right-handed neutrinos, N_1 and N_2 ; moreover, the quasi-degenerate masses can lie as low as 10^8 GeV, at least a factor of 10 below the bound obtained in the generic hierarchical case. Recently Pilaftsis and Underwood have proposed a model of the lepton sector in which the nearly degenerate neutrino pair is as light as 1 TeV [7].

It is important to test this suggested scenario in a realistic model of quark and lepton masses in as quantitative a manner as possible. At issue is whether or not the desired results for both leptogenesis and neutrino oscillations can be obtained simultaneously for the model in question. In this paper we explore leptogenesis with such resonant enhancement in an $SO(10)$ GUT model proposed several years ago by us in collaboration with Babu [8] with

a series of refinements by us since then [9]. This model is numerically very predictive and is found to explain accurately the present data on quark masses and mixings as well as the large mixing angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) [10] solar neutrino solution with near maximal atmospheric neutrino mixing. The lopsided texture of the 2-3 submatrices of the down quark and charged lepton mass matrices neatly explains the small mixing in that sector of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, i.e., small V_{cb} and V_{ts} , as well as the near maximal mixing in that sector of the Maki-Nakagawa-Sakata (MNS) matrix [11], i.e., large $U_{\mu 3}$. The LMA solar neutrino solution arises from the type I seesaw mechanism as an interplay between the Dirac neutrino matrix and the right-handed Majorana mass matrix, M_R . In fact, the 2-3 submatrix of M_R has a zero determinant which nearly replicates the corresponding sector for the right-handed Dirac neutrino matrix product, $M_N^\dagger M_N$. With the full structure of M_R spelled out to give the LMA solution, one finds the lighter two masses N_1 and N_2 are nearly degenerate, so at least the possibility of resonant enhancement of leptogenesis exists in this model. We shall examine how successful the model can be in achieving leptogenesis sufficient to generate the desired amount of baryogenesis observed in Nature.

As an alternative approach, we can reformulate the model with the type III seesaw mechanism developed in [12,13]. Whereas in the usual type I see-saw mechanism there are three superheavy Majorana neutrinos, in the type III see-saw, there are six superheavy neutrinos that typically form three pseudo-Dirac pairs. The desired resonant enhancement of leptogenesis can result from the mixing of the two neutrinos of the lightest pseudo-Dirac pair. In this scheme, the LMA solar neutrino mixing solution emerges rather naturally without fine-tuning of the Dirac and Majorana neutrino matrices, and the heavy lepton decay asymmetry is effectively decoupled from the light neutrino neutrino mass and mixing issues. As a result we shall see that satisfactory leptogenesis is easily obtained in this scenario.

In Sect. II we summarize briefly the features of the $SO(10)$ model and take the opportunity to update the input parameters so as to give even better agreement with the quark and lepton mass and mixing data. The formulas relevant for leptogenesis involving two quasi-degenerate right-handed neutrinos are summarized in Sect. III. We then apply these formulas to the model, modify the model slightly and compare the leptogenesis and oscillation results with experimental information. In Sect. IV the necessary formulas are presented for the type III seesaw and numerically satisfactory examples are given. Conclusions follow in Sect. V.

II. $SO(10)$ MODEL WITH $U(1) \times Z_2 \times Z_2$ FLAVOR SYMMETRY

The GUT model in question [8,9] is based on the grand unified group $SO(10)$ with a $U(1) \times Z_2 \times Z_2$ flavor symmetry. The model involves a minimum set of Higgs fields which solves the doublet-triplet splitting problem. This requires just one $\mathbf{45}_H$ whose VEV points in the $B-L$ direction, two pairs of $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$'s which stabilize the solution, along with several Higgs fields in the $\mathbf{10}_H$ representations and Higgs singlets [14]. The Higgs superpotential exhibits the $U(1) \times Z_2 \times Z_2$ symmetry which is used for the flavor symmetry of the GUT model. The combination of VEVs, $\langle \mathbf{45}_H \rangle_{B-L}$, $\langle 1(\mathbf{16}_H) \rangle$ and $\langle 1(\overline{\mathbf{16}}_H) \rangle$ break

$SO(10)$ to the Standard Model. The electroweak VEVs arise from the combinations $v_u = \langle 5(\mathbf{10}_H) \rangle$ and $v_d = \langle \bar{5}(\mathbf{10}_H) \rangle \cos \gamma + \langle \bar{5}(\mathbf{16}'_H) \rangle \sin \gamma$, while the combination orthogonal to v_d gets massive at the GUT scale. As such, Yukawa coupling unification can be achieved at the GUT scale with $\tan \beta \sim 2 - 55$, depending upon the $\bar{5}(\mathbf{10}_H) - \bar{5}(\mathbf{16}'_H)$ mixing present for the v_d VEV. In addition, matter superfields appear in the following representations: $\mathbf{16}_1, \mathbf{16}_2, \mathbf{16}_3; \mathbf{16}, \bar{\mathbf{16}}, \mathbf{16}', \bar{\mathbf{16}}', \mathbf{10}_1, \mathbf{10}_2$, and $\mathbf{1}$'s, where all but the $\mathbf{16}_i$ ($i = 1, 2, 3$) get superheavy.

The mass matrices then follow from Froggatt-Nielsen diagrams [15] in which the superheavy fields $\mathbf{16}, \bar{\mathbf{16}}, \mathbf{16}', \bar{\mathbf{16}}', \mathbf{10}_1, \mathbf{10}_2$, and $\mathbf{1}$ are integrated out. These diagrams, cf. [16], correspond to the following effective Yukawa operators for the indicated Dirac mass matrix elements:

$$\begin{aligned}
33 &: \mathbf{16}_3 \cdot \mathbf{10}_H \cdot \mathbf{16}_3 \\
23 &: [\mathbf{16}_2 \cdot \mathbf{10}_H]_{\bar{\mathbf{16}}} [\mathbf{45}_H \cdot \mathbf{16}_3]_{\mathbf{16}} / M_G \\
23 &: [\mathbf{16}_2 \cdot \mathbf{16}_H]_{\mathbf{10}} [\mathbf{16}'_H \cdot \mathbf{16}_3]_{\mathbf{10}} / M_G \\
13 &: [\mathbf{16}_1 \cdot \mathbf{16}_3]_{\mathbf{10}} [\mathbf{16}_H \cdot \mathbf{16}'_H]_{\mathbf{10}} / M_G \\
12 &: [\mathbf{16}_1 \cdot \mathbf{16}_2]_{\mathbf{10}} [\mathbf{16}_H \cdot \mathbf{16}'_H]_{\mathbf{10}} / M_G \\
11 &: \mathbf{16}_1 \cdot \mathbf{10}_H \cdot \mathbf{16}_1 \cdot (\mathbf{1}_H)^2 / M_G^2
\end{aligned} \tag{1}$$

where the subscripts to the brackets indicate how the fields inside the brackets are contracted. The Dirac mass matrices for the up quarks, down quarks, neutrinos and charged leptons are then found to be

$$\begin{aligned}
M_U &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon/3 \\ 0 & \epsilon/3 & 1 \end{pmatrix} m_U, & M_D &= \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon/3 \\ \delta' e^{i\phi} & \sigma + \epsilon/3 & 1 \end{pmatrix} m_D, \\
M_N &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & -\epsilon & 1 \end{pmatrix} m_U, & M_L &= \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon \\ \delta' e^{i\phi} & -\epsilon & 1 \end{pmatrix} m_D.
\end{aligned} \tag{2}$$

The above textures give the Georgi-Jarlskog relations [17] between the quark and lepton GUT scale masses, $m_s^0 \simeq m_\mu^0/3$, $m_d^0 \simeq 3m_e^0$ with Yukawa coupling unification holding for $\tan \beta \sim 5$. (Here the Dirac matrices are written with the convention that the left-handed fields label the rows and the left-handed conjugate fields label the columns. The opposite convention was used in some earlier references for this model. Hence the matrices here are the transpose of those given in those earlier papers.)

All nine quark and charged lepton masses, plus the three CKM angles and CP phase, are well-fitted with the eight input parameters (the older choice is indicated in parentheses)

$$\begin{aligned}
m_U &\simeq 113 \text{ GeV}, & m_D &\simeq 1 \text{ GeV}, \\
\sigma &= 1.83 (1.78), & \epsilon &= 0.147 (0.145), \\
\delta &= 0.00946 (0.0086), & \delta' &= 0.00827 (0.0079), \\
\phi &= 119.4^\circ (126^\circ), & \eta &= 6 \times 10^{-6} (8 \times 10^{-6}),
\end{aligned} \tag{3}$$

defined at the GUT scale to fit the low scale observables after evolution downward from Λ_{GUT} :

$$\begin{aligned}
m_t(m_t) &= 165 \text{ GeV}, & m_\tau &= 1.777 \text{ GeV}, \\
m_c(m_c) &= 1.23 \text{ GeV}, & m_\mu &= 105.7 \text{ MeV}, \\
m_u(1 \text{ GeV}) &= 3.6 \text{ MeV}, & m_e &= 0.55 \text{ MeV}, \\
m_b(m_b) &= 4.25 \text{ GeV}, & V_{cb} &= 0.0410, \\
m_s(1 \text{ GeV}) &= 148 \text{ MeV}, & V_{us} &= 0.220, \\
m_d(1 \text{ MeV}) &= 7.9 \text{ MeV}, & |V_{ub}/V_{cb}| &= 0.090, \\
\delta_{CP} &= 64^\circ, & \sin 2\beta &= 0.72.
\end{aligned} \tag{4}$$

A better overall agreement with experiment [18] is obtained with the new parameter values aside from the electron mass which is most sensitive to small corrections. With no extra phases present other than the one appearing in the CKM mixing matrix, the vertex of the CKM unitary triangle occurs near the center of the presently allowed region with $\sin 2\beta \simeq 0.72$. The Hermitian matrices $M_U M_U^\dagger$, $M_D M_D^\dagger$, and $M_N M_N^\dagger$ are diagonalized with small left-handed rotations, while $M_L M_L^\dagger$ is diagonalized by a large left-handed rotation. This accounts for the small value of $V_{cb} = (U_{UL}^\dagger U_{DL})_{cb}$, while $|U_{\mu 3}| = |(U_{LL}^\dagger U_{\nu L})_{\mu 3}|$ will turn out to be large for any reasonable right-handed Majorana mass matrix, M_R [8].

The effective light neutrino mass matrix, M_ν , is obtained from the type I seesaw mechanism once the right-handed Majorana mass matrix, M_R , is specified. The large atmospheric neutrino mixing $\nu_\mu \leftrightarrow \nu_\tau$ arises primarily from the structure of the charged lepton mass matrix M_L , while the structure of the right-handed Majorana mass matrix M_R determines the type of $\nu_e \leftrightarrow \nu_\mu$, ν_τ solar neutrino mixing, so that the solar and atmospheric mixings are essentially decoupled in the model. The LMA solar neutrino solution is obtained with a special form of M_R , as will be seen in a moment. However, this special form can be explained by the structure of the Froggatt-Nielsen diagrams [16]. The most general form for the right-handed Majorana mass matrix considered in [16] which gives the large mixing angle (LMA) solar neutrino solution is

$$M_R = \begin{pmatrix} c^2 \eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_R, \tag{5}$$

where the parameters ϵ and η are those introduced in Eq. (2) for the Dirac sector. With $a \neq b = c$, for example, the structure of M_R arises in the following way. The VEV of one particular Higgs singlet that has $\Delta L = 2$ contributes to all nine matrix elements giving a factorized rank 1 form. The VEV of a second Higgs singlet also breaks lepton number, but contributes only to the 13 and 31 elements of M_R .

Given the right-handed Majorana mass matrix above, the seesaw formula results in

$$\begin{aligned}
M_\nu &= -M_N M_R^{-1} M_N^T \\
&= - \begin{pmatrix} 0 & \frac{1}{a-b}\epsilon & 0 \\ \frac{1}{a-b}\epsilon & \frac{b^2-c^2}{(a-b)^2}\epsilon^2 & \frac{b}{b-a}\epsilon \\ 0 & \frac{b}{b-a}\epsilon & 1 \end{pmatrix} m_U^2 / \Lambda_R.
\end{aligned} \tag{6}$$

As a numerical example, with just three additional input parameters: $a = 1$, $b = c = 2$ and $\Lambda_R = 2.65 \times 10^{14}$ GeV, one obtains

$$M_\nu = - \begin{pmatrix} 0 & -\epsilon & 0 \\ -\epsilon & 0 & 2\epsilon \\ 0 & 2\epsilon & 1 \end{pmatrix} m_U^2 / \Lambda_R, \quad (7)$$

leading to

$$\begin{aligned} m_1 &= 5.1 \times 10^{-3} \text{ eV}, & m_2 &= 9.1 \times 10^{-3} \text{ eV}, & m_3 &= 52 \times 10^{-3} \text{ eV}, \\ M_1 &\simeq M_2 \simeq 2.3 \times 10^8 \text{ GeV}, & M_3 &= 2.7 \times 10^{14} \text{ GeV}, \\ \Delta m_{32}^2 &= 2.6 \times 10^{-3} \text{ eV}^2, & \sin^2 2\theta_{\text{atm}} &= 0.991, \\ \Delta m_{21}^2 &= 5.6 \times 10^{-5} \text{ eV}^2, & \tan^2 \theta_{12} &= 0.48, \\ U_{e3} &= -0.0172 - 0.00094i, & \sin^2 2\theta_{13} &= 0.0012, \\ J &= 2.0 \times 10^{-4}, & \delta_{CP} &= 177^\circ, & \chi_1 &= -180^\circ, & \chi_2 &= 90^\circ, \end{aligned} \quad (8)$$

to be compared with the present atmospheric, solar and reactor data and best-fit point in the LMA region [19]

$$\begin{aligned} \Delta m_{32}^2 &\simeq 2.6 \times 10^{-3} \text{ eV}^2, & \sin^2 2\theta_{\text{atm}} &= 1.0 \quad (\geq 0.92 \text{ at } 90\% \text{ c.l.}), \\ \Delta m_{21}^2 &= 7.1 \times 10^{-5} \text{ eV}^2, & \tan^2 \theta_{12} &= 0.44. \end{aligned} \quad (9)$$

In fact, the whole presently-allowed LMA region can be covered with a thin strip in the $a-b$ plane given by $b = c \simeq 1.9 + 1.4a$. In this region, $\sin^2 2\theta_{13} = 0.0004 - 0.0012$. Although the prediction for U_{e3} is thus 3 - 5 times larger than the CKM mixing element V_{ub} , a Neutrino Factory would be required to reach that range of $\sin^2 2\theta_{13}$.

It is interesting to remark that the hierarchy exhibited by the light left-handed neutrinos is a weak and normal one as is typical in $SO(10)$ models. This is somewhat surprising, for both the Dirac and Majorana neutrino matrices, N and M_R , exhibit strong hierarchies. As is apparent these hierarchies nearly cancel each other in the type I seesaw mechanism.

It is a simple matter to compute the effective mass parameter for neutrino-less double beta decay. For this purpose, we first note that the MNS neutrino mixing matrix is given by the product of the two unitary matrices which diagonalize the charged lepton and light left-handed neutrino mass matrices, i.e., $V_{MNS} = U_{LL}^\dagger U_{\nu L}$. The diagonalization occurs as follows:

$$\begin{aligned} U_{LL}^T M_L M_L^\dagger U_{LL}^* &= \text{diag}(m_e^2, m_\mu^2, m_\tau^2), \\ U_{\nu L}^T M_{\nu L} U_{\nu L} &= \text{diag}(m_1, m_2, m_3), \end{aligned} \quad (10)$$

where the mass eigenvalues are taken to be positive. Clearly an arbitrary phase transformation can be made on U_{LL} but not on $U_{\nu L}$. Hence the mixing matrix V_{MNS} has the form

$$V_{MNS} = U_{MNS} \Phi, \quad \Phi = \text{diag}(\exp^{i\chi_1}, \exp^{i\chi_2}, 1), \quad (11)$$

where U_{MNS} has the standard form with the Dirac CP phase appearing in the 13 element, while χ_1, χ_2 are the two Majorana CP phases. In terms of the above, the effective mass in neutrino-less double beta decay can then be written as

$$\begin{aligned}\langle m_{ee} \rangle &= \left| \sum_j m_j (U_{MNS} \Phi)_{1j}^2 \right| \\ &= \left| \sum_j m_j \eta_j |U_{MNS,1j}|^2 \right|,\end{aligned}\tag{12}$$

where $\eta_j = \exp^{2i\chi_j}$ for $j = 1, 2$ is the CP-parity of the j th lepton. For the example illustrated, $\chi_1 = -180^\circ$, $\chi_2 = 90^\circ$ and $\langle m_{ee} \rangle = 0.57$ meV, well below the present or future limit of observability, typical for a normal hierarchy spectrum. The lightest two neutrino states have opposite CP-parity.

Note that the two lightest heavy right-handed neutrinos are nearly degenerate with

$$(M_2 - M_1)/M_2 = 1.21 \times 10^{-4}.\tag{13}$$

Actually they are nearly degenerate in magnitude only, their opposite relative signs that appear in the eigenvalues of M_R signify they have exactly opposite CP -parity, a feature which enables them to evolve downward from the GUT scale without their separation receiving large radiative corrections. The small relative separation of the quasi-degenerate states N_1 and N_2 suggests that some resonance enhancement of leptogenesis may result from the level crossing. As we shall see in the next Section, this separation is too large by more than an order of magnitude to produce enough leptogenesis. Moreover, at least one of the four parameters a, b, c and Λ_R in the right-handed Majorana matrix defined in (5) must be complex in order to generate a lepton asymmetry. We shall attempt to modify the parameter assignments in M_R to achieve satisfactory leptogenesis after discussing the required conditions in the next Section.

III. LEPTOGENESIS WITH TYPE I SEESAW

Here we present the basic formulas for calculation of the lepton asymmetry which results in baryogenesis with the type I seesaw mechanism. For this purpose, we can use as a guide the recent phenomenological studies carried out by the authors of ref. [6]. There is an important difference in our approaches, however. They chose to work in the charged lepton flavor basis with the right-handed Majorana mass matrix M_R diagonal, while our model was naturally developed in the $SO(10)$ flavor basis. Hence we must transform our neutrino matrices M_N and M_R to the basis in which M_R is diagonal in order to apply the conventional formulas in refs. [4–6].

The basic assumption is that a lepton asymmetry ϵ_i is generated by the CP -violating out-of-equilibrium decays of the right-handed neutrino N_i . Since the right-handed Majorana mass term violates lepton number by two units, N_i is identical to its charge conjugate state and can decay in two ways:

$$N_i \rightarrow \nu_j + \phi, \quad \bar{\nu}_j + \phi^\dagger; \quad i, j = 1, 2, 3\tag{14}$$

where ϕ is the Higgs field coupling the right-handed N_i to the light left-handed neutrino ν_j . The Yukawa coupling involved is given by the ij th element of the Dirac matrix M'_N in the

basis where the right-handed Majorana neutrino mass matrix M'_R is diagonal. Let us label that Yukawa coupling by h'_{ij} . The lepton asymmetry is then traced to the imaginary part of the interference arising from the direct decay diagram and the one-loop diagrams depicted in Fig. 1:

$$\begin{aligned}\epsilon_i &= \frac{\Gamma(N_i \rightarrow \nu_j \phi) - \Gamma(N_i \rightarrow \bar{\nu}_j \phi^\dagger)}{\Gamma(N_i \rightarrow \nu_j \phi) + \Gamma(N_i \rightarrow \bar{\nu}_j \phi^\dagger)} \\ &= \frac{1}{8\pi} \sum_{k \neq i} f(|M_k|^2/|M_i|^2) \mathcal{I}_{ki},\end{aligned}\tag{15}$$

where

$$\mathcal{I}_{ki} = \frac{\text{Im}[(h'^\dagger h')_{ik}^2]}{(h'^\dagger h')_{ii}},\tag{16}$$

$$f(x) = \sqrt{x} \left[\frac{1}{1-x} + 1 - (1+x) \log\left(\frac{1+x}{x}\right) \right].\tag{17}$$

When summed over both neutrino and charged lepton final states for all three families, the total lepton asymmetry is still given by the right-hand side of Eq. (15); hence we have omitted the subscript j on ϵ_i . The first term in $f(x)$ arises from the self-energy correction while the second and third terms arise from the one-loop vertex. In the limit of quasi-degenerate neutrinos, $f(x)$ for $x = |M_k|^2/|M_i|^2$ reduces to

$$\lim_{x \rightarrow 1} f(x) = -\frac{|M_i|}{2(|M_k| - |M_i|)}.\tag{18}$$

A resonance enhancement thus arises from the level crossing or near vanishing of the denominator of $f(x)$ in the case of nearly degenerate masses. The lepton asymmetry ϵ_1 associated with the decay of N_1 thus simplifies to

$$\epsilon_1 = -\frac{1}{16\pi} \frac{|M_1|}{|M_2| - |M_1|} \mathcal{I}_{21}.\tag{19}$$

The lepton asymmetry ϵ_2 associated with the decay of N_2 is found to be equal to that of ϵ_1 for nearly degenerate states. In either case, the enhancement is limited by the decay widths of the two states for $||M_2| - |M_1|| \sim \Gamma_1/2 \simeq \Gamma_2/2$, cf. [20], where

$$\Gamma_i = \frac{1}{8\pi} (h'^\dagger h')_{ii} |M_i|.\tag{20}$$

In Eq. (15) above, h' is the Yukawa coupling matrix for the Dirac neutrinos in the basis where the right-handed Majorana matrix M_R is diagonal, which we shall call the primed basis. Denoting mass matrices written in that basis with primes, one has then $h' = M'_N/(v \sin \beta)$ with $v = 174$ GeV. In Eq. (2) the mass matrices are given in the original $SO(10)$ flavor basis, which we call the unprimed basis.

If the various neutrino mass matrices are diagonalized as follows

$$\begin{aligned}
U_{N_L}^T M_N U_{N_R}^* &= \text{diag}(m_u, 9m_c, m_t), \\
U_{\nu_L}^T M_\nu U_{\nu_L} &= \text{diag}(m_1, m_2, m_3), \\
U_{M_R}^\dagger M_R U_{M_R}^* &= \text{diag}(M_1, M_2, M_3),
\end{aligned} \tag{21}$$

then one has that

$$M_N^\dagger M_N' = U_{M_R}^T M_N^\dagger M_N U_{M_R}^* \tag{22}$$

The transformation matrix U_{M_R} is uniquely determined provided we require the mass eigenvalues in Eq. (21) be real and positive. The factor of 9 in the diagonalized M_N can be understood by comparing M_N and M_U in Eq. (2).

The lepton asymmetry produced by the decay of N_i is partially diluted by the lepton number-violating processes themselves [21]. This washout is determined as a function of an effective mass given by

$$\tilde{m}_i = (M_N^\dagger M_N')_{ii} / |M_i|. \tag{23}$$

The washout factor κ_i for $i = 1, 2$ with \tilde{m}_i in the range $10^{-2} - 10^3$ eV is approximated by

$$\kappa_i(\tilde{m}_i) \simeq 0.3 \left(\frac{10^{-3} \text{eV}}{\tilde{m}_i} \right) \left(\log \frac{\tilde{m}_i}{10^{-3} \text{eV}} \right)^{-0.6}. \tag{24}$$

The lepton number asymmetry produced per unit entropy at temperature $T > M_{1,2}$, taking into account decays of both N_1 and N_2 and their nearly equal washout factors, is then given by [22]

$$\frac{n_L}{s} \simeq \frac{2\kappa_1 \epsilon_1 g_N T^3}{s \pi^2} \tag{25}$$

$$= \frac{90}{2\pi^4} \frac{g_N}{g_*} \kappa_1 \epsilon_1. \tag{26}$$

We have used the expression for the entropy of the co-moving volume, $s = (2/45)g_*\pi^2 T^3$. Here $g_N = 2$ refers to the two spin degrees of freedom of each decaying Majorana neutrino, while $g_* = 106.75$ refers to the effective number of relativistic degrees of freedom contributing to the entropy in the absence of supersymmetric particles. Hence we find

$$\left(\frac{n_L}{s} \right)^{\text{SM}} \simeq 8.66 \times 10^{-3} \kappa_1 \epsilon_1. \tag{27}$$

The corresponding $B - L$ asymmetry per unit entropy is just the negative of n_L/s , since baryon number is conserved in the right-handed Majorana neutrino decays. While $B - L$ is conserved by the electroweak interactions following those decays, the sphaleron processes violate $B + L$ conservation and convert the $B - L$ asymmetry into a baryon asymmetry. Following the work of Harvey and Turner [3] the connection is

$$\frac{n_B}{s} \simeq -\frac{24 + 4N_H}{66 + 13N_H} \frac{n_L}{s}, \tag{28}$$

where N_H is the number of Higgs doublets. Again in the absence of supersymmetric contributions, $N_H = 1$ and the proportionality factor is $-28/79$. With the entropy density $s = 7.04n_\gamma$ in terms of the photon density, the baryon asymmetry of the Universe, defined by the ratio η_B of the net baryon number to the photon number, is given in terms of the lepton asymmetry ϵ_i and the washout parameter κ_i by

$$\eta_B^{SM} \equiv \frac{n_B}{n_\gamma} \simeq -0.0216\kappa_1\epsilon_1. \quad (29)$$

Successful leptogenesis will require that the final result for η_B should lie in the observed range [23]

$$\eta_B = (6.15 \pm 0.25) \times 10^{-10}. \quad (30)$$

In the case where supersymmetric particles are considered to contribute to the decays, entropy and sphaleron interactions, the following modifications are in order: (a) The presence of sleptons and higgsinos in the self-energy and vertex loops will double the interference terms without affecting the tree-level decay rates, so the asymmetry is doubled. (b) The presence of sleptons and higgsinos in the decay products of the right-handed neutrinos will double the decay rates without affecting the asymmetry. (c) With the sneutrino counterparts of the decaying heavy neutrinos taken into account, the lepton asymmetry is further doubled. (d) The value of the effective number of relativistic degrees of freedom is now $g_* = 228.75$. (e) In Eq. (28) we must take $N_H = 2$ for the two Higgs doublets present in the supersymmetric case.

The result is that the lepton asymmetry is replaced by a factor of 4 times the value of ϵ_1 given in Eq. (26), but since the widths of the decaying states are doubled, the mass separation should be taken twice as large in the computation of the asymmetry. The washout factor as determined by the Boltzmann equations is also affected, but in a very complicated fashion with the supersymmetric particles present [24]. We shall assume that the expression in Eq. (24) is still a good approximation although this is open to question. The appropriate expressions now read

$$\left(\frac{n_L}{s}\right)^{SUSY} \simeq 1.62 \times 10^{-2} \kappa_1 \epsilon_1, \quad (31)$$

$$\left(\frac{n_B}{s}\right)^{SUSY} \simeq -5.62 \times 10^{-3} \kappa_1 \epsilon_1, \quad (32)$$

$$\eta_B^{SUSY} \simeq -0.0396 \kappa_1 \epsilon_1. \quad (33)$$

We now apply the above framework to the $SO(10)$ model of [9,16] to determine just how successful leptogenesis would be in inducing baryogenesis given the mass matrices and parameters that lead to the observed neutrino masses and mixings. In this model, most of the complex phases can be rotated away from the Dirac mass matrices M_U , M_D , M_L , and M_N ; in fact all but two can, which were called α and ϕ in [9]. The phase α gets set to zero by fitting the quark masses which is why it does not show up in Eq. (2). The phase ϕ is more important and is responsible for the CP violating phase δ_{CKM} in the CKM matrix. In order to get substantial leptogenesis in this model there must be a large phase in M_R .

Now, in [16] M_R was taken to be real, as it was in in Sect. II, simply for convenience of analysis, as the interest there was only atmospheric and solar neutrino oscillations rather than leptonic CP-violating effects. An example of the fit with real M_R is given in Table 1 as case (I-0). Note that with the type I seesaw mechanism only four parameters, a , b , c and Λ_R , are required in addition to the eight present in the four Dirac mass matrices to obtain excellent agreement with the neutrino mass and mixing data. However, there is no reason for M_R to be real, and so here we shall investigate complex values.

A simple choice of complex parameters for M_R is

$$\begin{aligned} a &= 1.2 - 0.45i, & b &= 2.0, \\ c &= 2.0, & \Lambda_R &= 2.65 \times 10^{14} \text{ GeV}. \end{aligned} \tag{34}$$

This we call case (I-1), and the resulting observables are shown also in Table 1. Note that while the neutrino mixing parameters are all within the present acceptable range, the baryon asymmetry η_B is much too small, as it is almost four orders of magnitude below the observed value. This can be traced to the fact that the relative separation of the two quasi-degenerate right-handed neutrino masses is too large compared with the widths of their levels, $(M_2 - M_1)/\Gamma_1 = 34.2$.

The best results we have been able to obtain in the model arise if we allow small non-zero values in the 12, 13, 21, and 31 entries of the Dirac neutrino matrix. Consider, for example, the choice

$$\begin{aligned} a &= 0.25 + 0.15i, & b &= 1.2 + 0.9i, \\ c &= 0.25 + 0.25i, & \Lambda_R &= 2.9 \times 10^{14} \text{ GeV}, \\ (M_N)_{12} = (M_N)_{21} &= -0.65 \times 10^{-5}, & (M_N)_{13} = (M_N)_{31} &= -1.0 \times 10^{-5}, \end{aligned} \tag{35}$$

which we call case (I-2). We have checked that the introduction of these non-zero values into the Dirac neutrino mass matrix, and likewise for the up quark mass matrix, does not destroy the good agreement for the quark masses and CKM mixings. We see from Table 1 that the neutrino mixing parameters for case (I-2) are also in very good agreement with the present known data. The splitting of two quasi-degenerate right-handed neutrino masses has now been reduced to half the width of either state, which maximizes the resonance enhancement. Note that the relative CP-parity of the two lightest neutrino states is no longer opposite but differs from that by about 4° , a necessary condition to get satisfactory leptogenesis. Moreover, for this type I seesaw model, the Dirac phase δ_{CP} is large and much closer to maximal than in case (I-1). But the baryon asymmetry is $\eta_B = 2.2 \times 10^{-10}$, which falls short of the observed value by a factor of three. The biggest improvement over case (I-1) occurs in the lepton asymmetry which has improved by three orders of magnitude, while the washout factor is only slightly larger.

The numbers given for the three cases discussed so far for cases (I-0), (I-1), and (I-2) all left out the contributions from the supersymmetric particles. In case (I-3) of Table 1, we have taken those contributions into account. For this case we take

$$\begin{aligned} a &= 0.2 + 0.1i, & b &= 1.15 + 0.9i, \\ c &= 0.25 + 0.3i, & \Lambda_R &= 2.84 \times 10^{14} \text{ GeV}, \\ (M_N)_{12} = (M_N)_{21} &= -0.65 \times 10^{-5}, & (M_N)_{13} = (M_N)_{31} &= -1.05 \times 10^{-5}, \end{aligned} \tag{36}$$

We see from Table 1 that although η_B given by Eq. (33) in the SUSY case appears to be twice as large as in the SM case, as calculated from Eq. (19) ϵ_1 is only half as large. The net baryon asymmetries are thus nearly equal for cases (I-2) and (I-3), provided one is justified in using the same washout formula in both cases. It thus appears that, in this realistic model of quark and lepton masses and mixings, obtaining sufficient baryon asymmetry through thermal leptogenesis is somewhat problematic.

IV. LEPTOGENESIS WITH TYPE III SEESAW

In a recent paper [12] a new type of seesaw mechanism was proposed for light neutrino masses that can be implemented in grand unified theories based on $SO(10)$ or larger groups. This was called the type III see-saw mechanism and can be implemented in the model of [9,16], which we have been considering. Indeed, in [13] it was argued that when implemented in this model, the type III see-saw can give both realistic neutrino masses and mixings and sufficiently large leptogenesis without fine-tuning of the forms of the matrices. Here we will look at type III leptogenesis in more detail, giving numerical examples.

The type III seesaw involves introducing, in addition to the left- and right-handed neutrinos (ν_i, N_i) contained in the $\mathbf{16}_i$, three $SO(10)$ -singlet neutrinos $\mathbf{1}_i$. Thus the neutrino mass matrix is not 6×6 but 9×9 , and is given by

$$W_{neut} = (\nu_i, N_i^c, S_i) \begin{pmatrix} 0 & (M_N)_{ij} & F'_{ij}u \\ (M_N^T)_{ij} & 0 & F_{ij}\Omega \\ (F'^T)_{ij}u & (F^T)_{ij}\Omega & \mathcal{M}_{ij} \end{pmatrix} \begin{pmatrix} \nu_j \\ N_j^c \\ S_j \end{pmatrix}, \quad (37)$$

where the $\nu_i \in \bar{\mathbf{5}}(\mathbf{16}_i)$ are the usual left-handed neutrinos, the $N_i^c = \mathbf{1}(\mathbf{16}_i)$ are conjugates of the usual right-handed neutrinos, and the $S_i = \mathbf{1}_i$ are the $SO(10)$ -singlet neutrinos. The index i runs over families. The M_N submatrix represents the usual Dirac mass matrix contribution involving the doublet and singlet neutrinos in the $\mathbf{16}_i$'s as in the previous Sect. The new terms in the third row and third column arise from couplings involving the three singlet neutrinos S_i as given by the additional contributions to the Yukawa superpotential:

$$W_{RH\nu} = F_{ij}^a(\mathbf{16}_i\mathbf{1}_j)\overline{\mathbf{16}}_H^a + \mathcal{M}_{ij}\mathbf{1}_i\mathbf{1}_j. \quad (38)$$

The superscript a distinguishes the different Higgs $\overline{\mathbf{16}}_H$ representations, if there are more than one. The F' matrix in Eq. (37) arises when at least one of the Higgs fields, $\mathbf{5}(\overline{\mathbf{16}}_H^a)$, gets an electroweak VEV, u_a , in the $\mathbf{5}$ $SU(5)$ subgroup direction. Likewise, the F matrix appears when at least one of the $\mathbf{1}(\overline{\mathbf{16}}_H^a)$ fields get a superheavy VEV, Ω_a , in the $SU(5)$ singlet direction. More explicitly, we mean

$$\begin{aligned} F'_{ij}u &= \sum_a F_{ij}^a u_a, \\ F_{ij}\Omega &= \sum_a F_{ij}^a \Omega_a. \end{aligned} \quad (39)$$

With the submatrix \mathcal{M} also superheavy, W_{neut} can be diagonalized to yield the 3×3 matrix in the light left-handed neutrino $\nu\nu$ block

$$M_\nu = -M_N M_R^{-1} M_N^T - (M_N H + H^T M_N^T) \frac{u}{\Omega}, \quad (40)$$

where

$$\begin{aligned} M_R &= (F\Omega)\mathcal{M}^{-1}(F^T\Omega), \\ H &\equiv (F'F^{-1})^T. \end{aligned} \quad (41)$$

The first term in Eq. (40) is the usual type I seesaw contribution. The second term is the new type III seesaw contribution and arises when the $\overline{\mathbf{16}}_H^a$ fields develop an electroweak VEV, so $F' \neq 0$. If the elements of the matrix \mathcal{M} are small compared to those of $F\Omega$, then it is easy to see from Eqs. (40) and (41) that the type I contribution becomes negligible compared to the type III contribution. In the limit that $\mathcal{M} = 0$ one sees from Eq. (37) that the superheavy neutrinos have simply the mass term $F_{ij}\Omega(N_i^c S_j)$. That is, the N_i^c and S_i pair up to form three Dirac neutrinos. On the other hand, if \mathcal{M} is small (compared to $F\Omega$) but not zero, then these three Dirac neutrinos get slightly split into six eigenstates forming three nearly degenerate pseudo-Dirac neutrinos. It is this fact that can be exploited to enhance leptogenesis.

In Ref. [13] it was assumed that the matrices F , F' and \mathcal{M} in the original flavor basis all have elements of the order

$$\begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}, \quad (42)$$

where $\lambda \equiv \eta/\epsilon = 4.1 \times 10^{-5}$. This form is suggested by that of M_N in Eq. (2), where the 11 element is much smaller than the other non-zero elements, possibly due to an Abelian flavor symmetry. It is convenient to go to a basis where F is diagonalized. This is done by a biunitary transformation. We indicate quantities in this basis by the tilde symbol. These are related to those in the original flavor basis by

$$\tilde{F} = U^T F V, \quad N^c = U \tilde{N}^c, \quad S = V \tilde{S}. \quad (43)$$

Then

$$\tilde{F}\Omega = \begin{pmatrix} \lambda^2 F_1 & 0 & 0 \\ 0 & F_2 & 0 \\ 0 & 0 & F_3 \end{pmatrix} \Omega = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}, \quad (44)$$

where the F_i are of order unity. And

$$\tilde{\mathcal{M}} = \begin{pmatrix} \lambda^2 g_{11} & \lambda g_{12} & \lambda g_{13} \\ \lambda g_{12} & g_{22} & g_{23} \\ \lambda g_{13} & g_{23} & g_{33} \end{pmatrix} M_S, \quad \tilde{F}'u = \begin{pmatrix} \lambda^2 f'_{11} & \lambda f'_{12} & \lambda f'_{13} \\ \lambda f'_{21} & f'_{22} & f'_{23} \\ \lambda f'_{31} & f'_{32} & f'_{33} \end{pmatrix} m_U. \quad (45)$$

Again we assume f'_{ij} , $g_{ij} \sim 1$. Here $M_S \ll \Omega$ in order to obtain three superheavy quasi-Dirac neutrino pairs.

Finally we transform the Dirac neutrino mass matrix according to $\tilde{M}_N = M_N U$. Because of the assumed form of F , the matrix U has the form

$$U = \begin{pmatrix} u_{11} & \lambda u_{12} & \lambda u_{13} \\ \lambda u_{21} & u_{22} & u_{23} \\ \lambda u_{31} & u_{32} & u_{33} \end{pmatrix}, \quad (46)$$

where $u_{ij} \sim 1$ and by unitarity $u_{11} \simeq 1$. The transformed Dirac neutrino matrix then becomes

$$\tilde{M}_N \cong \begin{pmatrix} \eta u_{11} & \eta \lambda u_{12} & \eta \lambda u_{13} \\ \epsilon \lambda u_{31} & \epsilon u_{32} & \epsilon u_{33} \\ \lambda u_{31} & u_{32} & u_{33} \end{pmatrix} m_U \equiv \tilde{Y} m_U. \quad (47)$$

The type III seesaw mechanism given in Eq. (40) with dominance of the second term then yields for the light left-handed neutrino mass matrix (recall that $\eta/\lambda = \epsilon$)

$$M_\nu \cong - \begin{bmatrix} 2\eta \left(\frac{u_{11} f'_{11}}{F_1} \right) & \epsilon \left(\frac{u_{11} f'_{21}}{F_1} \right) & \epsilon \left(\frac{u_{11} f'_{31}}{F_1} \right) \\ \epsilon \left(\frac{u_{11} f'_{21}}{F_1} \right) & 2\epsilon \sum_j \left(\frac{u_{3j} f'_{2j}}{F_j} \right) & \sum_j \left[\left(\frac{u_{3j} f'_{2j}}{F_j} \right) + \epsilon \left(\frac{u_{3j} f'_{3j} - u_{2j} f'_{2j}}{F_j} \right) \right] \\ \epsilon \left(\frac{u_{11} f'_{31}}{F_1} \right) & \sum_j \left[\left(\frac{u_{3j} f'_{2j}}{F_j} \right) + \epsilon \left(\frac{u_{3j} f'_{3j} - u_{2j} f'_{2j}}{F_j} \right) \right] & 2 \sum_j \left(\frac{u_{3j} f'_{3j}}{F_j} - \epsilon \frac{u_{2j} f'_{3j}}{F_j} \right) \end{bmatrix} \left(\frac{m_U^2}{\Omega} \right). \quad (48)$$

This clearly has a well-defined hierarchical form which is similar to the corresponding M_ν determined in Sect. III. for the type I seesaw mechanism.

Leptogenesis is almost exclusively produced by the decays of the lightest pair of the six superheavy neutrinos. Neglecting, as is justified, the mixing of the lightest pair of superheavy neutrinos with the two heavier pairs of superheavy neutrinos, we find the effective two-by-two mass matrix for the lightest pair to be

$$(\tilde{N}_1^c, \tilde{S}_1) \lambda^2 \begin{pmatrix} 0 & F_1 \Omega \\ F_1 \Omega & g_{11} M_S \end{pmatrix} \begin{pmatrix} \tilde{N}_1^c \\ \tilde{S}_1 \end{pmatrix}. \quad (49)$$

If, as we assume, $M_S \ll \Omega$, these form an almost degenerate pseudo-Dirac pair, or equivalently two Majorana neutrinos with nearly equal and opposite masses. These Majorana neutrinos are $N_{1\pm} \cong (\tilde{N}_1^c \pm \tilde{S}_1)/\sqrt{2}$, with masses $M_{1\pm} \cong \pm M_1 + \frac{1}{2} \tilde{\mathcal{M}}_{11} = \lambda^2 (\pm F_1 \Omega + \frac{1}{2} g_{11} M_S)$. These can decay into light neutrino plus Higgs boson via the term $Y_{i\pm} (N_{1\pm} \nu_i) H$, where

$$Y_{i\pm} \cong (\tilde{Y}_{i1} \pm \tilde{F}'_{i1})/\sqrt{2} \mp \frac{\tilde{\mathcal{M}}_{11}}{4M_1} (\tilde{Y}_{i1} \mp \tilde{F}'_{i1})/\sqrt{2}. \quad (50)$$

Here \tilde{Y} is the Dirac Yukawa coupling matrix given in Eq. (47).

It is straightforward to show that the lepton asymmetry per decay produced by the decays of $N_{1\pm}$ is given by [25,5]

$$\epsilon_1 = \frac{1}{4\pi} \frac{\text{Im}[\sum_j (Y_{j+} Y_{j-}^*)]^2}{\sum_j [|Y_{j+}|^2 + |Y_{j-}|^2]} f(M_{1+}^2/M_{1-}^2), \quad (51)$$

where $f(M_{1+}^2/M_{1-}^2)$ comes from the absorptive part of the decay amplitude of N_{\pm} and was given earlier in Eq. (17). For the application here $f(M_{1+}^2/M_{1-}^2) \cong -M_1/2\tilde{\mathcal{M}}_{11} =$

$-(F_1/2g_1)(\Omega/M_S)$ which can be large if $M_S \ll \Omega$. The expression for $f(M_{1+}^2/M_{1-}^2)$ given above is only valid when the mass splitting $|M_+| - |M_-| = \tilde{\mathcal{M}}_{11}$ is larger than half the widths of the $N_{1\pm}$, which from Eq. (20) are given by $\Gamma_{\pm} \cong \frac{1}{8\pi} M_1 \sum_k |Y_{k\pm}|^2$. From Eqs. (45), (47), and (50), one sees that $\Gamma_{1\pm} \sim \lambda^2 M_1/8\pi$. As we shall see with our numerical examples, this condition that the splitting of $N_{1\pm}$ be comparable to or greater than their widths, is easily satisfied.

Making use of Eqs. (50) and (51) one obtains

$$\epsilon_1 = \frac{1}{4\pi} \frac{\sum_j (|\tilde{Y}_{j1}|^2 - |\tilde{F}'_{j1}|^2) \text{Im}(\sum_k \tilde{Y}_{k1}^* \tilde{F}'_{k1})}{\sum_j (|\tilde{Y}_{j1}|^2 + |\tilde{F}'_{j1}|^2)} f(M_{1+}^2/M_{1-}^2), \quad (52)$$

This can be evaluated in terms of the parameters of the model using Eqs. (45) and (47), giving

$$\epsilon_1 \cong \frac{\lambda^2}{4\pi} \left[\frac{(|u_{31}|^2 - |f'_{31}|^2) \text{Im}(u_{31}^* f'_{31})}{|u_{31}|^2 + |f'_{31}|^2 + |f'_{21}|^2} \right] f(M_{1+}^2/M_{1-}^2). \quad (53)$$

The washout parameter is given approximately as before by Eq. (24) where now

$$\tilde{m}_1 \equiv \frac{8\pi v_u^2 \Gamma_{N_{1\pm}}}{M_{N_{1\pm}}^2} \cong \lambda^2 \frac{v_u^2}{M_1} (|u_{31}|^2 + |f'_{31}|^2 + |f'_{21}|^2). \quad (54)$$

The lepton asymmetry is then translated into the baryon asymmetry for the Universe by the same formulas which appeared earlier in Sect. III.

We now wish to give some numerical examples for this type III seesaw mechanism. The first requirement, of course, is that we choose values of the parameters that reproduce the neutrino mass and mixing data. One could search over the whole space of parameters, but this is a cumbersome task, as in the type III mechanism there are many parameters involved in the sector of superheavy singlet neutrinos, including the parameters u_{ij} and f'_{ij} . A more convenient approach is to find values of u_{ij} and f'_{ij} that make the matrix M_ν (given approximately in Eq. (48)) have a form close to that shown in Eq. (6), since we already know that that form can reproduce the neutrino mass and mixing data for suitable a , b , and c . First let us choose values of u_{ij} that are simple and such that U in Eq. (46) is unitary. They can not be too simple, i.e., have too many zeros, or else there will not be sufficient leptogenesis. We choose the following form for simplicity:

$$U = \begin{pmatrix} 1 & -\lambda(1 + \sqrt{2})i & \lambda \\ -\lambda(1 + \sqrt{2})i & 1/\sqrt{2} & i/\sqrt{2} \\ \lambda & i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \quad (55)$$

where $\lambda = \eta/\epsilon = 4.1 \times 10^{-5}$ as before. We can make M_ν very close numerically to the form in Eq. (6) by taking

$$\begin{aligned}
f'_{11} &\simeq 0, \text{ with } f'_{12} \text{ and } f'_{13} \text{ arbitrary,} \\
f'_{21} &= -F_1/(b-a), \\
f'_{22} &= -\left[(1+\sqrt{2})i + \frac{1}{2\sqrt{2}}\left(\frac{c^2-a^2}{b-a}\right) - i\frac{\epsilon}{2\sqrt{2}}\left(\frac{c^2-b^2}{b-a}\right) - (b-a)\frac{1-\sqrt{2}}{\sqrt{2}}\frac{f'_{31}}{F_1}\right]F_2/(b-a), \\
f'_{23} &= -\left[1 - \frac{1}{2\sqrt{2}}i\left(\frac{c^2-a^2}{b-a}\right) + \frac{\epsilon}{2\sqrt{2}}\left(\frac{c^2-b^2}{b-a}\right) + (b-a)\frac{1-\sqrt{2}}{\sqrt{2}}i\frac{f'_{31}}{F_1}\right]F_3/(b-a), \\
f'_{31} &\simeq 0, \\
f'_{32} &= \frac{1}{2\sqrt{2}}\left[-i + 2(2+\sqrt{2})i\frac{f'_{31}}{F_1}\right]F_2, \\
f'_{33} &= \frac{1}{2\sqrt{2}}\left[1 + 2\sqrt{2}\frac{f'_{31}}{F_1}\right]F_3,
\end{aligned} \tag{56}$$

For simplicity we take the $\tilde{\mathcal{M}}$ matrix of Eq. (45) to be diagonal and set $g_{11} = g_{22} = g_{33} = 1$. Three examples are illustrated in Table 2, with the values of f'_{21} , f'_{22} , f'_{23} determined from Eq. (56) by setting a , b , c set equal to $1.2 - 0.45i$, 2.0 , 2.0 , respectively, as in case (I-1) of Table 1; in addition Ω appearing in the type III seesaw is set equal to $\Lambda_R = 2.7 \times 10^{14}$ appearing in the case (I-1). In these examples, F_1 , F_2 , F_3 and M_S are allowed to vary. We have considered only the contributions to leptogenesis from the SM particles, since the previous results indicated little difference with the inclusion of the supersymmetric contributions, given the uncertainty in the corresponding washout factor.

By using the type III formulas beginning with the transformed version of the light neutrino mass matrix given in Eq. (40), we clearly reproduce the neutrino mass and mixing results obtained with the type I seesaw mechanism in Table 1 for Case (I-1), where no modification of the original Dirac neutrino mass matrix was introduced. In fact, these results are independent of the actual values taken for F_1 , F_2 , F_3 and M_S , and their values are not repeated in Table 2. In particular, the Dirac CP-violating phase remains far from maximal at 172° . On the other hand, one sees that successful leptogenesis and observed baryogenesis are easily obtained in all three examples for those four input parameters chosen. The masses of the lightest pseudo-Dirac pair of heavy neutrinos range from 4.5×10^4 GeV up to 4.5×10^6 GeV. For this mass range, overproduction of gravitinos is not an issue [26]. The mass separation of the two Majorana neutrinos in the lightest pair is well above their decay widths. Hence we see that the type III seesaw mechanism has a much easier time simultaneously giving realistic light neutrino masses and mixings and successful leptogenesis. In the realistic model we have examined, we see that it fails by at least a factor of three to give enough leptogenesis with the conventional seesaw mechanism. By contrast, with the type III seesaw mechanism as we have seen, the light neutrino mixing issue is decoupled from the issue of leptogenesis; moreover, there are ready-made pairs of nearly degenerate heavy neutrinos. Consequently, there is no difficulty in the same realistic model getting successful leptogenesis with the type III seesaw mechanism.

V. CONCLUSIONS

In this paper we have explored the issue of resonant leptogenesis in a very predictive $SO(10)$ grand unified model which leads to reliable numerical results. Phenomenological studies [6] by Ellis, Raidal, and Yanagida, as well as by Akhmedov, Frigerio, and Smirnov

have previously suggested that successful leptogenesis can be easily obtained with resonant enhancement, though they had not looked at any specific realistic models in detail to be sure that both the leptogenesis and neutrino mass and mixing results can be simultaneously satisfied. In order to obtain the desired LMA solar neutrino solution in the model considered here, it was found that near degeneracy of the lightest two right-handed neutrinos was required. This feature neatly favors the resonant enhancement scenario for leptogenesis which can survive washout and be converted into the observed baryon excess by sphaleron interactions in thermal equilibrium above the critical electroweak symmetry-breaking temperature.

We have first studied this problem in the conventional type I seesaw framework. The seven model parameters, including one complex one, in the Dirac matrices are fixed by the charged lepton and quark mass and mixing data. The lopsided nature of the down quark and charged lepton mass matrices neatly accounts for both the small value of the V_{cb} quark mixing parameter and the near maximal mixing of the mu- and tau-neutrinos. As originally proposed, the right-handed Majorana neutrino mass matrix depended on just four real parameters which, in conjunction with the Dirac neutrino and charged lepton mass matrices, then leads to a sub-maximal mixing of the solar neutrinos but no leptogenesis.

By allowing three of the four M_R parameters to be complex and introducing two additional very small parameters into the Dirac neutrino mass matrix, we were able to achieve a sizeable amount of leptogenesis; however, it falls short of the observed value by a factor of three. The limiting factor is the requirement that the heavy neutrino $M_1 - M_2$ mass separation must be comparable to or larger than half the decay width of either neutrino. For this application the two masses are found to be of the order of 3×10^8 GeV and separated by 600 GeV. In the supergravity scenario of SUSY breaking, however, the gravitino problem which requires an upper bound on the reheating temperature of $T_R \lesssim 10^7$ GeV at best makes this solution somewhat problematic. This can be alleviated, however, if SUSY breaking occurs via the gauge-mediated scenario.

We then considered a type III seesaw mechanism by which three $SO(10)$ singlet neutrinos are added to the spectrum, so the neutrino matrix is 9×9 . With the singlet neutrinos at some large intermediate scale, the effective “double seesaw” results in three pairs of quasi-Dirac neutrinos in place of three right-handed Majorana neutrinos. In this scenario resonant-enhanced leptogenesis is achieved by the lightest pair of quasi-Dirac neutrinos, for which their mass separation is typically one hundred times the widths of the two states. Hence successful leptogenesis is easily obtained. With this type III seesaw the leptogenesis is completely decoupled from the neutrino mass and mixing issues, so the good agreement of the latter with present-day observations is preserved as observed earlier by us in [13]. Here we have seen that with one choice of parameters the masses of the lightest neutrino pair can be of the order of 5×10^4 GeV, so the gravitino problem is no longer an issue. The only drawback for this type III seesaw mechanism is the appearance of a large number of new parameters needed to specify the expanded 9×9 neutrino matrix.

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TABLES

TABLE I. Type I seesaw results for the four cases considered in the text.

	Case (I-0): (SM)	Case (I-1): (SM)	Case (I-2): (SM)	Case (I-3): (SUSY)
Input: a	1.0	1.20 - 0.45i	0.25 + 0.15i	0.20 + 0.10i
b	2.0	2.0	1.20 + 0.90i	1.15 + 0.90i
c	2.0	2.0	0.25 + 0.25i	0.25 + 0.3i
Λ_R (GeV)	2.65×10^{14}	2.70×10^{14}	2.90×10^{14}	2.84×10^{14}
$(M_N)_{12}, (M_N)_{21}$	0.0	0.0	-0.65×10^{-5}	-0.65×10^{-5}
$(M_N)_{13}, (M_N)_{31}$	0.0	0.0	-1.0×10^{-5}	-1.05×10^{-5}
Output: M_1 (GeV)	2.31×10^8	2.16×10^8	3.06×10^8	3.08×10^8
M_2 (GeV)	2.31×10^8	2.16×10^8	3.06×10^8	3.08×10^8
M_3 (GeV)	2.71×10^{14}	2.76×10^{14}	2.90×10^{14}	2.86×10^{14}
$\Delta M_{21}/M_2$	1.21×10^{-4}	1.31×10^{-4}	1.96×10^{-6}	3.87×10^{-6}
Γ_1/M_1	3.83×10^{-6}	3.83×10^{-6}	3.84×10^{-6}	3.83×10^{-6}
m_1 (meV)	5.1	5.3	2.8	2.8
m_2 (meV)	9.1	9.8	8.8	8.8
m_3 (meV)	52.0	52.0	51.0	52.0
Δm_{32}^2 (eV ²)	2.6×10^{-3}	2.6×10^{-3}	2.5×10^{-3}	2.6×10^{-3}
Δm_{21}^2 (eV ²)	5.6×10^{-5}	6.9×10^{-5}	6.9×10^{-5}	6.9×10^{-5}
$\sin^2 2\theta_{atm}$	0.991	0.988	0.979	0.981
$\sin^2 2\theta_{sol}$	0.87	0.86	0.80	0.80
$\tan^2 \theta_{12}$	0.48	0.46	0.39	0.39
$\sin^2 2\theta_{13}$	0.0012	0.0006	0.0012	0.0014
$ U_{e3} $	0.017	0.012	0.018	0.019
δ_{CP}	177°	172°	76°	71°
χ_1, χ_2	-180°, 90°	24°, 117°	102°, -170°	102°, -170°
$\langle m_{ee} \rangle$ (meV)	0.57	0.58	0.44	0.44
ϵ_1	0.0	-6.4×10^{-7}	-1.2×10^{-3}	-6.1×10^{-4}
\tilde{m}_1 (eV)	12.6	13.5	9.5	9.5
κ_1	6.2×10^{-6}	5.8×10^{-6}	8.3×10^{-6}	8.4×10^{-6}
η_B	0.0	0.80×10^{-13}	2.2×10^{-10}	2.0×10^{-10}

TABLE II. Type III seesaw results for three examples patterned after case (I-1) in Table 1. The light neutrino mass and mixings results obtained are identical to those in case (I-1) of Table 1 and are not repeated here.

		Case (III-1): (SM)	Case (III-2): (SM)	Case (III-3): (SM)
Input:	Ω (GeV)	2.7×10^{14}	2.7×10^{14}	2.7×10^{14}
	F_1	1.0	10.	0.1
	F_2	1.0	0.1	0.1
	F_3	1.0	1.0	1.0
	M_S	4.3×10^5	8.5×10^8	1.0×10^5
	f_{11}/F_1	0.0	0.0	0.0
	f_{12}/F_2	1.0	1.0	1.0
	f_{13}/F_3	1.0	1.0	1.0
	f_{21}/F_1	$-0.950 + 0.534i$	$-9.496 + 5.341i$	$-0.095 + 0.053i$
	f_{22}/F_2	$-2.279 - 1.537i$	$-0.227 - 0.154i$	$-0.228 - 0.154i$
	f_{23}/F_3	$-0.194 + 1.523i$	$-0.194 + 1.523i$	$-0.194 + 1.523i$
	f_{31}/F_1	0.0	0.0	0.0
	f_{32}/F_2	$-0.354i$	$-0.035i$	$-0.035i$
	f_{33}/F_3	0.354	0.354	0.354
	g_{11}	1.0	1.0	1.0
	g_{22}	1.0	1.0	1.0
	g_{33}	1.0	1.0	1.0
Output:	M_1 (GeV)	$\pm 4.50 \times 10^5$	$\pm 4.50 \times 10^6$	$\pm 4.50 \times 10^4$
	M_2 (GeV)	$\pm 2.70 \times 10^{14}$	$\pm 2.70 \times 10^{13}$	$\pm 2.70 \times 10^{13}$
	M_3 (GeV)	$\pm 2.70 \times 10^{14}$	$\pm 2.70 \times 10^{14}$	$\pm 2.70 \times 10^{14}$
	$(M_{1+} + M_{1-})/M_{1+}$	1.6×10^{-9}	3.15×10^{-7}	3.7×10^{-9}
	Γ_1/M_1	6.9×10^{-11}	3.88×10^{-9}	3.82×10^{-11}
	χ_1, χ_2	$-156^\circ, 113^\circ$	$24^\circ, -67^\circ$	$-156^\circ, 113^\circ$
	ϵ_1	2.5×10^{-5}	1.6×10^{-4}	-1.4×10^{-4}
	\tilde{m}_1 (eV)	0.10	0.57	0.48
	κ_1	1.2×10^{-3}	1.8×10^{-4}	2.1×10^{-4}
	η_B	-6.2×10^{-10}	-6.1×10^{-10}	6.3×10^{-10}

FIGURES

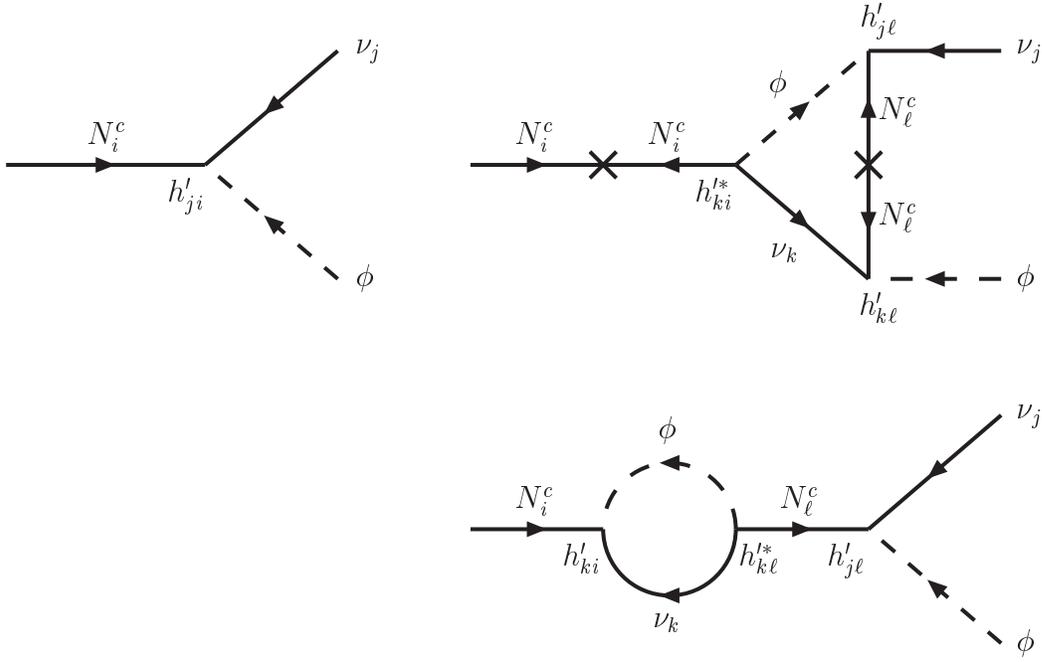


Figure 1: Tree-level and one-loop Feynman diagrams in the computation of the lepton asymmetry. A similar set of diagrams exists for the decays into the antineutrino channels.