

# A precise determination of the $B_c$ mass from dynamical lattice QCD

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We perform a precise calculation of the mass of the  $B_c$  meson using unquenched configurations from the MILC collaboration, including 2+1 flavours of improved staggered quarks. Lattice NRQCD and the Fermilab formalism are used to describe the  $b$  and  $c$  quarks respectively. We find the mass of the  $B_c$  meson to be 6.304(16)GeV.

## 1. Introduction

One of the results anticipated from run II [1] at the Tevatron is a measurement of the largely unexplored properties of the  $B_c(b\bar{c})$  meson. The current experimental determination comes from the *CDF* collaboration who quoted a value of 6.4(4) GeV [2] and from preliminary  $D\theta$  data, where 231 events have been identified, giving the  $B_c$  mass as  $M_{B_c} = 5.95(37)$  GeV [3]. A suitably precise lattice calculation puts us in a position to make a prediction of this mass, rather than confirm an experimental result.

In the past, the two main theoretical methods for finding  $M_{B_c}$  have been potential models [4,5] and lattice calculations in the quenched approximation [6,7]. Both of these techniques have found agreement with the experimental values, but both have drawbacks; potential models inevitably depend on the form of the potential used, while the effect of quenching in a lattice calculation is estimated to be about 100 MeV.

Now, however, it is possible to repeat the lattice calculation without the quenched approximation. The key is to use the unquenched ensembles for lattice gauge fields from the MILC collaboration [8]. These have 2+1 flavours of dynamical quarks. They use an improved gluon action,

and the Asqtad action for staggered light quarks, leaving discretisation errors of  $O(\alpha_s a^2)$ . With the MILC ensembles, lattice calculations agree with experimental measurements for a wide range of hadronic quantities [9].

## 2. Method

In bottomium ( $b\bar{b}$ ) systems the typical velocity of the quarks is  $v_b \approx 0.1$ , while in charmonium ( $c\bar{c}$ )  $v_c \approx 0.3$ . However, the unequal sharing of quark masses in the  $B_c$  changes these values, giving  $v_c \approx 0.5$  and  $v_b \approx 0.04$ . With this in mind, we choose to use different formalisms to describe the  $b$  and  $c$  quarks.

For the  $c$  quark we use the clover action, with the non-relativistic (Fermilab) interpretation [10]. The clover coupling is adjusted to its tadpole improved tree-level value. For the  $b$  quark we use  $O(v^4)$  lattice NRQCD [11] with coefficients fixed at tree level. Our reasoning for these choices comes from the charmonium spectrum, reproduced using Fermilab quarks [12], and from the  $\Upsilon$  spectrum, reproduced using the NRQCD action [13]. Both of these calculations were performed on the MILC 2+1 flavour ensembles which our calculations use.

In both the NRQCD and Fermilab formalisms, the hadron energy at zero momentum is offset from the mass by an energy shift. In appropriate

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mass differences, this shift cancels. Therefore,

$$M_{B_c} - \frac{1}{2}[M_\psi + M_\Upsilon] = E_{B_c} - \frac{1}{2}[E_\psi + E_\Upsilon], \quad (1)$$

and

$$M_{B_c} - [M_{B_s} + M_{D_s}] = E_{B_c} - [E_{B_s} + E_{D_s}], \quad (2)$$

yield the differences in binding energies. (Here  $M_X$  and  $E_X$  denote masses and energies calculated on the lattice respectively.) After computing these differences on the lattice, we obtain our result for  $M_{B_c}$  by adding back  $[M_\psi + M_\Upsilon]/2$  or  $M_{D_s} + M_{B_s}$ . These methods for extracting  $M_{B_c}$  will be referred to as the *quarkonium baseline* (1) and the *heavy-light baseline method* (2). When results are quoted for the quarkonium baseline method, the “charmonium particle” used is the spin average of the  $\eta_c$  and  $J/\psi$ . The absence of an experimental signal for  $\eta_b$  prevents us from doing this for the  $b$  quark as well.

### 3. Simulation Details

In our calculation, the configurations included a single flavour at around the strange quark mass ( $m_s$ ) and two degenerate light flavours over a range of masses down to  $m_s/5$ . The valence  $c$  and  $b$  quark masses were set using the kinetic masses of the  $D_s$  and  $\Upsilon$  states. The lattice spacing is set on each ensemble through the 1S–2S radial splitting of the  $\Upsilon$  system. But an important advantage of calculations on the MILC ensembles is that any of several quantities shown in [9] could have been used.

In addition to the majority of our calculations performed at  $a \approx 0.12$  fm we ran a single calculation on a finer ensemble, at  $a \approx 0.09$  fm.

### 4. Results

The result of calculating using different masses for the two (degenerate) light sea quarks is shown in Fig. 1. This plot also allows a comparison of the quarkonium and heavy-light baseline methods of Eqs. (1) and (2).

We plot our result from calculating at the “fine” lattice spacing ( $a \approx 0.09$  fm). We find on these configurations that  $M_{B_c} = 6.309(3)$  GeV, (quoting only the statistical error). We also plot

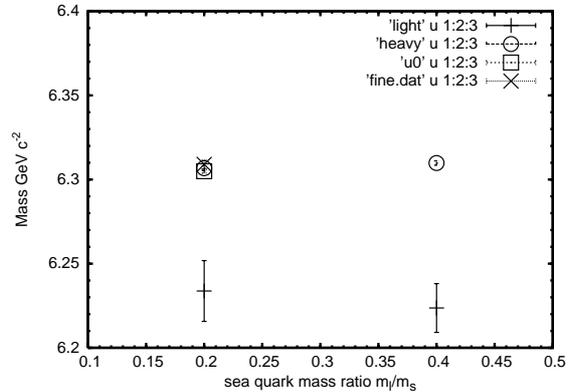


Figure 1. Chiral limit of quarkonium and heavy-light baseline methods. Only statistical errors are shown. The  $m_l/m_s$  ratio corresponds to the bare masses used for the dynamical quarks.

a single point where the tadpole improvement factor ( $u_0$ ) for the  $b$  quark has been calculated using the mean link in Landau gauge as opposed to the fourth root of the plaquette (used for all other points). Changing only this quantity allows us to assess the influence of the relativistic corrections in the NRQCD action. We see that the effect is negligible, as expected.

The effect of varying the valence quark masses on the kinetic masses of the  $J/\psi$  and  $\Upsilon$  was used to set the error on the  $B_c$  from these parameters. We find that this leads to a error of 10 MeV due to the uncertainty in the  $b$  quark mass and 5 MeV due to the uncertainty for the  $c$ .

We also estimate a systematic error arising from higher order terms in the Fermilab quark action. For the heavy quarks, the NRQCD action includes the  $O(v^4)$  terms at tree level, the Fermilab quark action also includes them but with some mismatch. Because of the form of Eqs. (1) and (2) we expect a more efficient cancellation of these differences using the quarkonium baseline method, where all quantities are in the heavy quark sector. Estimating the corrections, we get a  $\pm 10$  MeV uncertainty for both methods, and an additional downward shift of 30 – 50 MeV for the

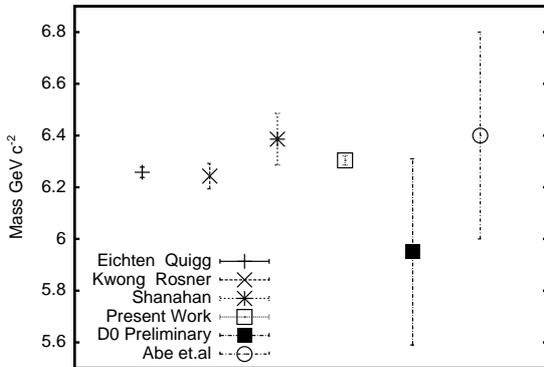


Figure 2. Mass of the  $B_c$  from potential models, Quenched lattice calculation and the present work (quarkonium baseline method).

heavy-light baseline method. Evidence for this shift can be seen in Fig. 1.

We give our final result using the quarkonium baseline method because of this shift. Our result is  $6.304(3)(10)(11)(5)(3)$  GeV, where the first error is statistical, the second is due to higher order corrections to the Fermilab quark action, the third is due to uncertainties in the valence quark masses, and the final number is an estimate of our chiral extrapolation error. Further error analysis is still underway, but combining the current errors in quadrature gives:

$$M_{B_c} = 6.304(16) \text{ GeV}. \quad (3)$$

This result is shown in context in Fig. 2. We also show points due to Eichten and Quigg [4] and Kwong and Rosner [5] who estimate results from a range of potentials.

The result due to Shanahan [6] et al. is the previous lattice determination. Their calculation used the same method as this work, but was carried out in the quenched approximation. We achieve a significant reduction in the size of the systematic error from the removal of the quenched approximation.

## 5. Conclusions

This calculation shows once again the importance of including dynamical quarks in lattice simulations. Having removed the ambiguity in setting the lattice spacing and fixing the quark masses, we are able to reach an unprecedented accuracy for a lattice calculation of the mass of the  $B_c$  particle. This sets a well defined target for the current experimental studies.

## 6. Acknowledgements

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