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Towards Better Constrains on the Higgs Boson Mass: Two-Loop Fermionic Corrections to $\sin^2\theta_{\rm eff}^{\rm lept}$ [a]

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The complete two-loop electroweak fermionic corrections to the effective leptonic weak mixing angle, $\sin^2\theta_{\rm eff}^{\rm lept}$, are now available. Here we shortly present the methods applied and illustrate the implications on indirect prediction for the Higgs boson mass within the standard model.

1 Introduction

Present efforts on both the experimental and theoretical sides are focused on the Higgs boson. As LEPII did not give a significant signal of its existence, either a positive announcement at the Tevatron or the running of LHC are awaited. In the meantime the standard model is being scrutinized in order to indirectly predict the mass of the Higgs boson. The most valuable information comes from the W boson mass, $M_{\rm W}$, and $\sin^2\theta_{\rm eff}^{\rm lept}$but also to some smaller extent from other observables, *e.g.* the width of the Z boson. As far as $M_{\rm W}$ is concerned it is now known with one of the best precisions, both from experiment and from theory [1]. On the other hand, the weak mixing angle is measured with a relative precision that is almost a factor two worse and the two most accurate measurements differ from each other by 2.9σ. Still, its dependence on the Higgs boson mass, $M_{\rm H}$, is three times more pronounced than in the $M_{\rm W}$ case, demanding better theoretical precision for the goal of explicit tests of the model and for the prediction of $M_{\rm H}$.

Recently we performed a calculation of the two-loop fermionic corrections to $\sin^2\theta_{\rm eff}^{\rm lept}$ [2]. In this contribution we clarify some of the aspects which were not discussed previously. In order to avoid errors which cannot be identified with the help of general properties of the theory (gauge invariance and UV/IR finiteness), two independent calculations were performed for most parts. Be-

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low we sketch several details of both methods. Finally we demonstrate the consequences of the new result on the Higgs boson mass prediction.

2 Calculation

The effective weak mixing angle, $\sin^2\theta_{\rm eff}^{\rm lept}$, can be defined through the vector and axial vector couplings (g_V and g_A respectively) of an on-mass shell Z boson to a pair of fermions, such that

$$\sin^2\theta_{\rm eff}^{\rm lept} = \frac{1}{4}\left(1 - {\rm Re}\left(\frac{g_V}{g_A}\right)\right). \tag{1}$$

The proper vector and axial-vector structure of the Feynman amplitude can be extracted by a projector. Namely, using the Dirac equation we find that the needed quantity is determined if we apply the following operator:

$$\mathcal{N}_\mu = \hat{p}_2(g_1\gamma_\mu + g_2\gamma_\mu\gamma_5)\hat{p}_1 \tag{2}$$

and then take a trace over the amplitude. Here $\hat{p}_1$ and $\hat{p}_2$ denote momenta of the external fermions multiplied by Dirac matrices,

$$g_1 = -\frac{1}{g_A^{(0)}\,2\,(d-2)\,p^2}, \qquad g_2 = \frac{g_V^{(0)}}{(g_A^{(0)})^2\,2\,(d-2)\,p^2}, \tag{3}$$

where $p^2 = (p_1+p_2)^2$, $g_V^{(0)}$ and $g_A^{(0)}$ being the tree level values of the couplings and d is the space-time dimension.

This calculation was performed in the on-shell scheme, for which the two-loop counter-terms were already known and thoroughly tested in the past [3]. The only complications come from the two-loop one-particle irreducible vertex diagrams. The evaluation of them was performed not only by two independent calculations but whenever possible also by different methods.

2.1 Method I

The needed two-loop vertex diagrams may depend on two dimensionless variables: $M_{\rm W}/M_{\rm Z}$ and/or $m_{\rm t}/M_{\rm Z}$ (the Higgs boson mass does not appear due to CP conservation). We considered two cases: diagrams with light fermions only, which depend on one variable; and diagrams with top quark loops, which depend on both variables.

The light fermion contributions can be reduced to a set of master integrals with the help of standard methods of Integration By Parts and Lorenz Invariance Identities. Still, at the two-loop level this is a nontrivial task, therefore

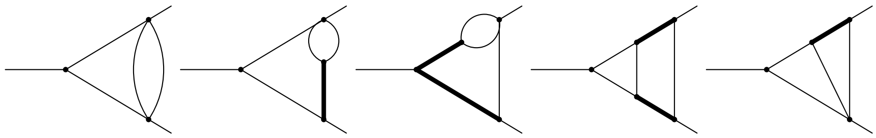

Figure 1: Set of master integrals required for the calculation of the light fermion contributions. The solid lines denote massive particles, W or Z bosons, the thin lines inside a diagram denote massless particles.

it has been assigned to a newly written C++ library, IdSolver[4]. The set of the identified master integrals, pictured in Fig. 1, was evaluated using differential equations in the external momentum. This allowed to obtain analytical results represented by polylogarithms of weight four at worst.

Obtaining an exact result for the heavy fermion contributions would be more problematic. However, due to large scale differences we can safely apply the heavy top mass expansion, which reduces the diagrams to two-loop tadpoles at most. The series shows good convergence and in practice we used an expansion up to m_t^{-10}, reaching sufficient precision.

For completeness we should also mention that the diagram generation was performed by DiaGen [4] and most of the algebraic manipulations were done with FORM [5]. More details can be found in [2].

2.2 Method II

We have also performed an independent calculation based on numerical integrations for the master integrals. This method is based on a dispersion representation of the one-loop self-energy function B_0,

$$B_0(p^2, m_1^2, m_2^2) \;=\; -\int_{(m_1+m_2)^2}^{\infty} \mathrm{d}s\, \frac{\Delta B_0(s, m_1^2, m_2^2)}{s-p^2}, \tag{4}$$

$$\Delta B_0(s, m_1^2, m_2^2) \;=\; (4\pi\mu^2)^{4-d}\, \frac{\Gamma(d/2-1)}{\Gamma(d-2)}\, \frac{\lambda^{(d-3)/2}(s, m_1^2, m_2^2)}{s^{d/2-1}}, \tag{5}$$

where $\lambda(a,b,c) = (a-b-c)^2 - 4bc$. Using this relation, any scalar two-loop integral T with a self-energy sub-loop as in Fig. 2 (a) can be expressed as [6]

$$\begin{aligned} T_{N+1}(p_i; m_i^2) = -\int_{s_0}^{\infty} \mathrm{d}s\, \Delta B_0(s, m_N^2, m_{N+1}^2) \\ \times \int \mathrm{d}^4 q\, \frac{1}{q^2-s}\, \frac{1}{(q+p_1)^2-m_1^2} \cdots \frac{1}{(q+p_1+\ldots+p_{N-1})^2-m_{N-1}^2}. \end{aligned} \tag{6}$$

Here the integral in the second line is an N-point one-loop function, and the integration over s is performed numerically. While in principle it is also possible

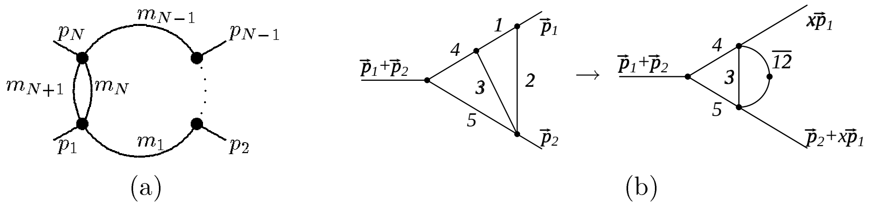

Figure 2: (a) General representation of a two-loop scalar diagram with self-energy sub-loop. (b) Reduction of triangle sub-loop to self-energy sub-loop by means of Feynman parameters.

to introduce dispersion relations for triangle sub-loops, it is technically easier to reduce them to self-energy sub-loops by introducing Feynman parameters [7],

$$\begin{aligned}&[(q+p_1)^2-m_1^2]^{-1}\,[(q+p_2)^2-m_2^2]^{-1}=\int_0^1 \mathrm{d}x\;[(q+\bar{p})^2-\overline{m}^2]^{-2}\\ &\bar{p}=x\,p_1+(1-x)p_2, \qquad \overline{m}=x\,m_1+(1-x)m_2-x(1-x)(p_1-p_2)^2.\end{aligned} \tag{7}$$

This is indicated diagrammatically in Fig. 2 (b). The integration over the Feynman parameters is also performed numerically. As a result, all master integrals for the vertex topologies can be evaluated by at most 3-dim. numerical integrations. Similar to before, the reduction of integrals with irreducible numerators to a small set of master integrals in the case of propagator subloops was accomplished by using Integration By Parts and Lorentz Invariance identities, which were implemented in an independent realization of the Laporta algorithm [8] within Mathematica.

3 Results and Conclusion

Our new fitting formula, presented in [2], contains all the recent results on two- and three- loop corrections to $\sin^2\theta_{\rm eff}^{\rm lept}$ (for references see [2]). Using this formula, instead of the old one [9], the central value of the Higgs boson mass is shifted by about +18.6 GeV if $M_{\rm H}$ is determined from $\sin^2\theta_{\rm eff}^{\rm lept}$ alone. [b]. This should be compared with an almost as large shift of +20 GeV given by the previous formula only, which was generated by the recent change in the measured top quark mass. These effects are shown in Fig. 3. We do not plot the uncertainty on the theory curves; the theoretical error on $\sin^2\theta_{\rm eff}^{\rm lept}$ due to the neglect of higher order contributions is estimated to be 4.9×10^{-5} whereas the error on the standard model parameters, mainly $m_{\rm t}$ and $\Delta\alpha_{had}$, gives around 2.6×10^{-4} for $M_{\rm H}$ in the range from 100 to 600 GeV. In the global standard model analysis

[b]We use the same parameters as in [2], except for the new experimental value on $\sin^2\theta_{\rm eff}^{\rm lept}$.

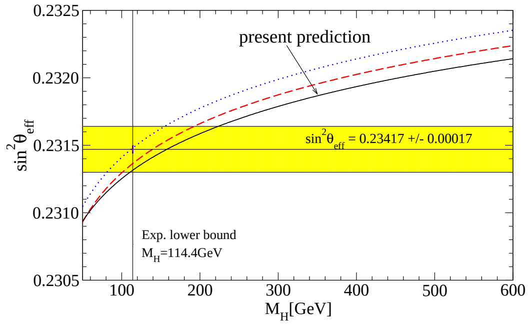

Figure 3: $\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}$ obtained with the previous and most recent fitting formulas. The blue dotted and red dashed curves denote the results of [8] with $m_t = 174.4$ GeV and $m_t = 178$ GeV respectively. The black solid curve represents the newest prediction of [2] for $m_t = 178$ GeV.

of EWWG (see "blue-band plot" in[10]) the net effect of these results is not so strong setting the central value of the M_H approximately at 117 GeV with upper limit around 260 GeV (95%CL).

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