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CP asymmetry in flavour-specific B decays[a]

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I first discuss the phenomenology of a_{fs}^q ($q = d, s$), which is the CP asymmetry in flavour-specific B_q decays such as $B_d \to X\ell^+\overline{\nu}_\ell$ or $B_s \to D_s^-\pi^+$. a_{fs}^q can be obtained from the time evolution of *any* untagged B_q decay. Then I present recently calculated next-to-leading-order QCD corrections to a_{fs}^q, which reduce the renormalisation scheme uncertainties significantly. For the Standard Model we predict $a_{\mathrm{fs}}^d = -(5.0 \pm 1.1) \times 10^{-4}$ and $a_{\mathrm{fs}}^s = (2.1 \pm 0.4) \times 10^{-5}$. As a by-product we determine the ratio of the width difference in the B_d system and the average B_d width to $\Delta\Gamma_d/\Gamma_d = (3.0 \pm 1.2) \times 10^{-3}$ at next-to-leading order in QCD.

1 Preliminaries

The time evolution of the $B_d - \overline{B}_d$ system is determined by a Schrödinger equation:

$$i\frac{d}{dt}\begin{pmatrix} |B_d(t)\rangle \\ |\overline{B}_d(t)\rangle \end{pmatrix} = \left(M^d - i\,\frac{\Gamma^d}{2}\right)\begin{pmatrix} |B_d(t)\rangle \\ |\overline{B}_d(t)\rangle \end{pmatrix}, \tag{1}$$

which involves two Hermitian 2×2 matrices, the mass matrix M^d and the decay matrix Γ^d. Here $B_d(t)$ and $\overline{B}_d(t)$ denote mesons which are tagged as a B_d and $\overline{B}_d$ at time $t = 0$, respectively. By diagonalising $M^d - i\Gamma^d/2$ one obtains the mass eigenstates:

$$\begin{aligned} \text{Lighter eigenstate:} \quad |B_{d,L}\rangle &= p|B_d^0\rangle + q|\overline{B}{}_d^0\rangle. \\ \text{Heavier eigenstate:} \quad |B_{d,H}\rangle &= p|B_d^0\rangle - q|\overline{B}{}_d^0\rangle \qquad \text{with } |p|^2 + |q|^2 = 1. \end{aligned} \tag{2}$$

We discuss the mixing formalism for B_d mesons, the corresponding quantities for $B_s - \overline{B}_s$ mixing are obtained by the replacement $d \to s$. The coefficients q and p in Eq. (2) are also different for the B_d and B_s systems. The $B_d - \overline{B}_d$ oscillations in Eq. (1) involve the three physical quantities $|M_{12}^d|$, $|\Gamma_{12}^d|$ and $\phi_d = \arg(-M_{12}^d/\Gamma_{12}^d)$ (see e.g. [1]). The mass and width differences between $B_{d,L}$ and $B_{d,H}$ are related to them as

$$\Delta M_d = M_H^d - M_L^d = 2\,|M_{12}^d|, \qquad \Delta\Gamma_d = \Gamma_L^d - \Gamma_H^d = 2\,|\Gamma_{12}^d|\cos\phi_d, \tag{3}$$

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where M_L^d, Γ_L^d and M_H^d, Γ_H^d denote the masses and widths of $B_{d,L}$ and $B_{d,H}$, respectively.

The third quantity to determine the mixing problem in Eq. (1) is

$$a_{\rm fs}^d \;=\; {\rm Im}\, \frac{\Gamma_{12}^d}{M_{12}^d} \;=\; \frac{\Delta\Gamma_d}{\Delta M_d} \tan\phi_d. \tag{4}$$

$a_{\rm fs}^d$ is the CP asymmetry in *flavour-specific* $B_d \to f$ decays, which means that the decays $\overline{B}_d \to f$ and $B_d \to \overline{f}$ (with $\overline{f}$ denoting the CP-conjugate final state) are forbidden [2]. Next we consider flavour-specific decays in which the decay amplitudes $A_f = \langle f|B_d\rangle$ and $\overline{A}_{\overline{f}} = \langle \overline{f}|\overline{B}_d\rangle$ in addition satisfy

$$|A_f| \;=\; |\overline{A}_{\overline{f}}|. \tag{5}$$

Eq. (5) means that there is no direct CP violation in $B_d \to f$. Then $a_{\rm fs}^d$ is given by

$$a_{\rm fs}^d \;=\; \frac{\Gamma(\overline{B}_d(t)\to f) - \Gamma(B_d(t)\to \overline{f})}{\Gamma(\overline{B}_d(t)\to f) + \Gamma(B_d(t)\to \overline{f})}. \tag{6}$$

Note that the oscillatory terms cancel between numerator and denominator. The standard way to access $a_{\rm fs}^d$ uses $B_d \to X\ell^+\overline{\nu_\ell}$ decays, which justifies the name *semileptonic CP asymmetry* for $a_{\rm fs}^d$. In the B_s system one can also use $B_s \to D_s^-\pi^+$ to measure $a_{\rm fs}^s$. Yet, for example, Eq. (6) does not apply to the flavour-specific decays $B_d \to K^+\pi^-$ or $B_s \to K^-\pi^+$, which do not obey Eq. (5).

$a_{\rm fs}^d$ measures *CP violation in mixing*. Other commonly used notations involve the quantities $|q/p|$ or ϵ_B; they are related to $a_{\rm fs}^d$ as

$$1 - \left|\frac{q}{p}\right| \;=\; \frac{a_{\rm fs}^d}{2}, \qquad\qquad \frac{{\rm Re}\,\epsilon_B}{1+|\epsilon_B|^2} \;=\; \frac{a_{\rm fs}^d}{4}. \tag{7}$$

Here $\epsilon_B = (1+q/p)/(1-q/p)$ is the analogue of the quantity $\overline{\epsilon}_K$ in $K^0-\overline{K}^0$ mixing. Unlike $a_{\rm fs}^d$ it depends on phase conventions and should not be used. In Eq. (7) and future equations we neglect terms of order $(a_{\rm fs}^q)^2$.

$a_{\rm fs}^d$ is small for two reasons: First $|\Gamma_{12}^d/M_{12}^d| = O(m_b^2/M_W^2)$ suppresses $a_{\rm fs}^d$ to the percent level. Second there is a GIM suppression factor m_c^2/m_b^2 reducing $a_{\rm fs}^d$ by another order of magnitude. Generic new physics contributions to $\arg M_{12}^d$ (e.g. from squark-gluino loops in supersymmetric theories) will lift this GIM suppression. $a_{\rm fs}^s$ is further suppressed by two powers of the Wolfenstein parameter $\lambda \simeq 0.22$. Therefore $a_{\rm fs}^d$ and $a_{\rm fs}^s$ are very sensitive to new CP phases [1, 3], which can enhance $|a_{\rm fs}^d|$ and $|a_{\rm fs}^s|$ to 0.01. $|a_{\rm fs}^d|$ can be further enhanced by new contributions to Γ_{12}^d, which is doubly Cabibbo-suppressed in the Standard Model.

The experimental world average for $a_{\rm fs}^d$ is [4]

$$a_{\rm fs}^d \;=\; 0.002 \pm 0.013.$$

2 Measurement of $a_{\rm fs}^q$

2.1 Flavour-specific decays

We first discuss the flavour-specific decays without direct CP violation in the Standard Model. First note that the "right-sign" asymmetry vanishes:

$$\Gamma(B_q(t)\to f) - \Gamma(\overline{B}_q(t)\to \overline{f}) \;=\; 0. \tag{8}$$

Since we are hunting possible new physics in a tiny quantity, we should be concerned whether Eq. (5) still holds in the presence of new physics.[b] Further no experiment is exactly charge-symmetric, and the efficiencies for $\overline{B} \to \overline{f}$ and $B \to f$ may differ by a factor of $1+\delta_c$. One can use the "right-sign" asymmetry in Eq. (8) to calibrate for both effects: In the presence of a charge asymmetry δ_c one will measure

$$a_{\rm right}^{q,\delta_c} \equiv \frac{\Gamma(B_q(t)\to f)-(1+\delta_c)\Gamma(\overline{B}_q(t)\to \overline{f})}{\Gamma(B_q(t)\to f)+(1+\delta_c)\Gamma(\overline{B}_q(t)\to \overline{f})} = \frac{|A_f|^2-|\overline{A}_{\overline{f}}|^2}{|A_f|^2+|\overline{A}_{\overline{f}}|^2} - \frac{\delta_c}{2}. \tag{9}$$

Instead of the desired CP asymmetry in Eq. (6) one will find

$$a_{\rm fs}^{q,\delta_c} = \frac{\Gamma(\overline{B}_d(t)\to f)-(1+\delta_c)\Gamma(B_d(t)\to \overline{f})}{\Gamma(\overline{B}_d(t)\to f)+(1+\delta_c)\Gamma(B_d(t)\to \overline{f})} = a_{\rm fs}^q + a_{\rm right}^{q,\delta_c}. \tag{10}$$

Thus δ_c and the direct CP asymmetry $(|A_f|^2-|\overline{A}_{\overline{f}}|^2)/(|A_f|^2+|\overline{A}_{\overline{f}}|^2)$ enter Eq. (9) and Eq. (10) in the same combination and $a_{\rm fs}^q$ can be determined. Above we have kept only terms to first order in the small quantities $1-|\overline{A}_{\overline{f}}|^2/|A_f|^2$, δ_c and $a_{\rm fs}^q$.

It is well-known that the measurement of $a_{\rm fs}^q$ requires neither tagging nor the resolution of the B_q–$\overline{B}_q$ oscillations [2]. Since the right-sign asymmetry in Eq. (8) vanishes, the information on $a_{\rm fs}^q$ from Eq. (6) persists in the untagged decay rate

$$\Gamma[f,t] = \Gamma(B_q(t)\to f) + \Gamma(\overline{B}_q(t)\to f). \tag{11}$$

At a hadron collider one also cannot rule out a production asymmetry $\delta_p = N_{\overline{B}_q}/N_{B_q} - 1$ between the numbers $N_{\overline{B}_q}$ and N_{B_q} of $\overline{B}_q$'s and B_q's. An untagged measurement will give

$$a_{\rm fs,unt}^{q,\delta_c}(t) = \frac{\Gamma[f,t]-(1+\delta_c)\Gamma[\overline{f},t]}{\Gamma[f,t]+(1+\delta_c)\Gamma[\overline{f},t]} = a_{\rm right}^{q,\delta_c} + \frac{a_{\rm fs}^q}{2} - \frac{a_{\rm fs}^q+\delta_p}{2}\,\frac{\cos(\Delta M_q\, t)}{\cosh(\Delta\Gamma_q t/2)}. \tag{12}$$

The use of the larger untagged data sample to determine $a_{\rm fs}^d$ seems to be advantageous at the $\Upsilon(4S)$ B factories, where $\delta_p = 0$. Then the time evolution in Eq. (12) contains enough information to separate $a_{\rm fs}^d$ from $a_{\rm right}^{d,\delta_c} = a_{\rm fs,unt}^{d,\delta_c}(t=0)$.

Eqs. (6),(9) and (10) still hold, when the time-dependent rates are integrated over t. The time-integrated untagged CP asymmetry reads (for $|A_f| = |\overline{A}_{\overline{f}}|$, $\delta_c=\delta_p=0$):

$$A_{\rm fs,unt}^q \equiv \frac{\int_0^\infty dt[\Gamma[f,t]-\Gamma[\overline{f},t]]}{\int_0^\infty dt[\Gamma[f,t]+\Gamma[\overline{f},t]]} = \frac{a_{\rm fs}^q}{2}\,\frac{x_q^2-y_q^2}{x_q^2-1}, \tag{13}$$

where $x_q = \Delta M_q/\Gamma_q$, $y_q = \Delta\Gamma_q/(2\Gamma_q)$ and Γ_q is the average decay width in the B_q system. In particular a measurement of $a_{\rm fs}^s$ does not require to resolve the rapid $B_s-\overline{B}_s$ oscillations. In $\Upsilon(4S)$ B factories a common method to constrain $a_{\rm fs}^d$ is to compare the number N_{++} of decays $(B_d(t),\overline{B}_d(t)) \to (f,f)$ with the number N_{--} of decays to $(\overline{f},\overline{f})$, typically for $f = X\ell^+\nu_\ell$. Then one finds $a_{\rm fs}^d = (N_{++}-N_{--})/(N_{++}+N_{--})$.

We next exemplify the measurement of $a_{\rm fs}^s$ from time-integrated tagged $B_s \to f$ decays, having $f = X\ell^+\nu_\ell$ in mind. This approach should be feasible at the Fermilab Tevatron. We

[b] Direct CP violation requires the presence of a CP-conserving phase. In the case of $B_d \to D^-\ell^+\nu_\ell$ this phase comes from photon exchange and is small. Also somewhat contrived scenarios of new physics are needed to get a sizeable CP-violating phase in a semileptonic decay. Thus here one needs to worry about $|A_f| \neq |\overline{A}_{\overline{f}}|$ only, once $a_{\rm fs}^d$ is probed at the permille level.

allow the detector to be charge-asymmetric ($\delta_c \neq 0$) and also relax Eq. (5) to $|A_f| \approx |\overline{A}_{\overline{f}}|$. Let N_f denote the total number of observed decays of meson tagged as B_s at time $t = 0$ into the final state f. Further $\overline{N}_f$ denotes the analogous number for a meson initially tagged as a $\overline{B}_s$. The corresponding quantities for the decays $B_s(t) \to \overline{f}$ and $\overline{B}_s(t) \to \overline{f}$ are $N_{\overline{f}}$ and $\overline{N}_{\overline{f}}$. One has

$$\overset{(-)}{N}_f \;\propto\; \int_0^\infty dt\, \Gamma(\overset{(-)}{B}_s(t) \to f), \qquad\qquad \overset{(-)}{N}_{\overline{f}} \;\propto\; (1+\delta_c) \int_0^\infty dt\, \Gamma(\overset{(-)}{B}_s(t) \to \overline{f})$$

with the same constant of proportionality. The four asymmetries

$$\begin{aligned}
\frac{N_f - \overline{N}_{\overline{f}}}{N_f + \overline{N}_{\overline{f}}} &= a_{\rm right}^{s,\delta_c}\,, & \frac{\overline{N}_f - N_{\overline{f}}}{\overline{N}_f + N_{\overline{f}}} &= a_{\rm right}^{s,\delta_c} + a_{\rm fs}^{s}\,, \\
\frac{N_f - \overline{N}_f}{N_f + \overline{N}_f} &= \frac{1-y_s^2}{1+x_s^2} - \frac{a_{\rm fs}^s}{2}\,, & \frac{\overline{N}_{\overline{f}} - N_{\overline{f}}}{\overline{N}_{\overline{f}} + N_{\overline{f}}} &= \frac{1-y_s^2}{1+x_s^2} + \frac{a_{\rm fs}^s}{2}
\end{aligned} \tag{14}$$

then allow to determine $a_{\rm fs}^s$ and $(1-y_s^2)/(1+x_s^2)$. In the second line of Eq. (14) terms of order $a_{\rm fs}^s/x_s^2$ have been neglected. (Of course the last asymmetry in Eq. (14) is redundant.)

2.2 Any decay

Since q/p enters the time evolution of *any* neutral $B_q \to f$ decay, we can use any such decay to determine $a_{\rm fs}^q$. The time dependent decay rates involve

$$\lambda_f \;=\; \frac{q}{p}\frac{\langle f|\overline{B}_q\rangle}{\langle f|B_q\rangle}.$$

In Eq. (1.73)-(1.77) of [1] $\Gamma(B_q(t) \to f)$, $\Gamma(\overline{B}_q(t) \to f)$, $\Gamma(B_q(t) \to \overline{f})$and $\Gamma(\overline{B}_q(t) \to \overline{f})$ can be found for the most general case, including a non-zero $\Delta\Gamma_q$. For the untagged rate one easily finds

$$\begin{aligned}
\Gamma[f,t] \;\;\propto\;\; e^{-\Gamma_q t} \Bigg\{ & \left[1 + \frac{a_{\rm fs}^q}{2}\right] \left[\cosh \frac{\Delta\Gamma_q\, t}{2} + A^{\Delta\Gamma} \sinh \frac{\Delta\Gamma_q\, t}{2}\right] - \\
& \frac{a_{\rm fs}^q}{2} \left[A^{\rm dir} \cos(\Delta M_q\, t) + A^{\rm mix} \sin(\Delta M_q\, t)\right] \Bigg\}
\end{aligned} \tag{15}$$

with

$$A^{\rm dir} = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \qquad A^{\rm mix} = -\frac{2\,{\rm Im}\,\lambda_f}{1+|\lambda_f|^2} \qquad \text{and} \qquad A^{\Delta\Gamma} = -\frac{2\,{\rm Re}\,\lambda_f}{1+|\lambda_f|^2}\,. \tag{16}$$

Hence one can obtain $a_{\rm fs}^q$ from the amplitude of the tiny oscillations in Eq. (15), once $A^{\rm dir}$ and $A^{\rm mix}$ are determined from the $\cos\Delta M_q\, t$ and $\sin\Delta M_q\, t$ terms of the time evolution in the tagged $B_q(t) \to f$ decay. If f is a CP eigenstate, $A^{\rm dir}$ and $A^{\rm mix}$ are the direct and mixing-induced CP asymmetries. For example, in $B_d \to J/\psi K_S$ one has $\lambda_f = -\exp(-2i\beta) + \mathcal{O}(a_{\rm fs})$, so that one can set $A^{\rm dir} = 0$ and $A^{\rm mix} = -\sin(2\beta)$ in Eq. (15). The flavour-specific decays discussed in the previous section correspond to the special case $\lambda_f = 0$.

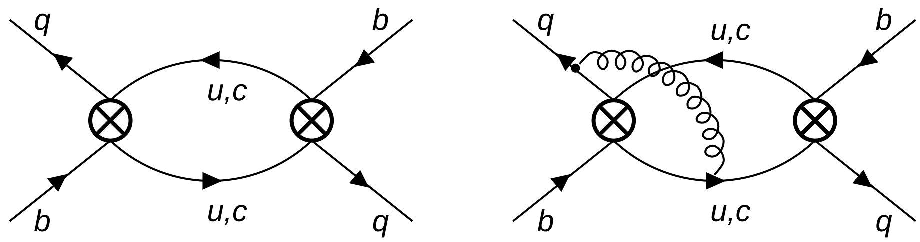

Figure 1: Leading order contribution to Γ_{12} (left) and a sample NLO diagram (right). The crosses denote effective $\Delta B = 1$ operators describing the W-mediated b decay. The full set of NLO diagrams can be found in [6].

3 QCD corrections to $a_{\rm fs}^q$

$a_{\rm fs}^q = \mathrm{Im}\,\Gamma_{12}^q/M_{12}^q$ is proportional to two powers of the charm mass m_c. A theoretical prediction in leading order (LO) of QCD cannot control the renormalisation scheme of m_c. Therefore the LO result $a_{\rm fs}^q$ suffers from a theoretical uncertainty which is not only huge but also hard to quantify. While next-to-leading order (NLO) QCD corrections to M_{12}^q are known for long [5], the computation of those to Γ_{12}^q has been completed only recently. The LO and a sample NLO diagram are shown in Fig. 1. The NLO result for the contribution with two identical up-type quark lines (sufficient for the prediction of $\Delta\Gamma_s$) has been calculated in [6] and was confirmed in [7]. The contribution with one up-quark and one charm-quark line was obtained recently in [7] and [8]. In order to compute Γ_{12}^q one exploits the fact that the mass m_b of the b-quark is much larger than the fundamental QCD scale $\Lambda_{\rm QCD}$. The theoretical tool used is the Heavy Quark Expansion (HQE), which yields a systematic expansion of Γ_{12}^q in the two parameters $\Lambda_{\rm QCD}/m_b$ and $\alpha_s(m_b)$ [9]. Γ_{12}^q and M_{12}^q involve hadronic "bag" parameters, which quantify the size of the non-perturbative QCD binding effects and are difficult to compute. The dependence on these hadronic parameters, however, largely cancels from $a_{\rm fs}^q$.

Including corrections of order α_s [6–8] and $\Lambda_{\rm QCD}/m_b$ [7, 8, 10] we predict [8]

$$a_{\rm fs}^d \;=\; 10^{-4}\left[-\frac{\sin\beta}{R_t}(12.0\pm 2.4)+\left(\frac{2\sin\beta}{R_t}-\frac{\sin 2\beta}{R_t^2}\right)(0.2\pm 0.1)\right].$$

Here β is the angle of the unitarity triangle measured in the CP asymmetry of $B_d \to J/\psi K_S$. If $(\overline{\rho},\overline{\eta})$ denotes the apex of the usual unitarity triangle, then $R_t \equiv \sqrt{(1-\bar\rho)^2+\bar\eta^2}$ is the length of one of its sides. For the Standard Model fit to the unitarity triangle with $\beta = 22.4^\circ \pm 1.4^\circ$ and $R_t = 0.91 \pm 0.05$ [11] one finds:

$$a_{\rm fs}^d \;=\; -(5.0\pm 1.1)\cdot 10^{-4}$$

The impact of a future measurement of $a_{\rm fs}^d$ on the unitarity triange is shown in Fig. 2. The result for the B_s system is

$$a_{\rm fs}^s \;=\; (12.0\pm 2.4)\cdot 10^{-4}\,|V_{us}|^2 R_t \sin\beta \;=\; (2.1\pm 0.4)\cdot 10^{-5}.$$

From Eq. (3) one finds that $\Delta\Gamma_q/\Delta M_q = -\mathrm{Re}(\Gamma_{12}^q/M_{12}^q)$. This ratio was predicted to NLO in [6] for the B_s system. With the new result of [7, 8] we can also predict $\Delta\Gamma_d/\Delta M_d$. Due to a numerical accident, the Standard Model prediction for the ratio $\Delta\Gamma_q/\Delta M_q$ is essentially the same for $q = d$ and $q = s$:

$$\frac{\Delta\Gamma_q}{\Delta M_q} = (4.0\pm 1.6)\times 10^{-3}\,, \qquad\qquad \frac{\Delta\Gamma_d}{\Gamma_d} = (3.0\pm 1.2)\times 10^{-3}. \tag{17}$$

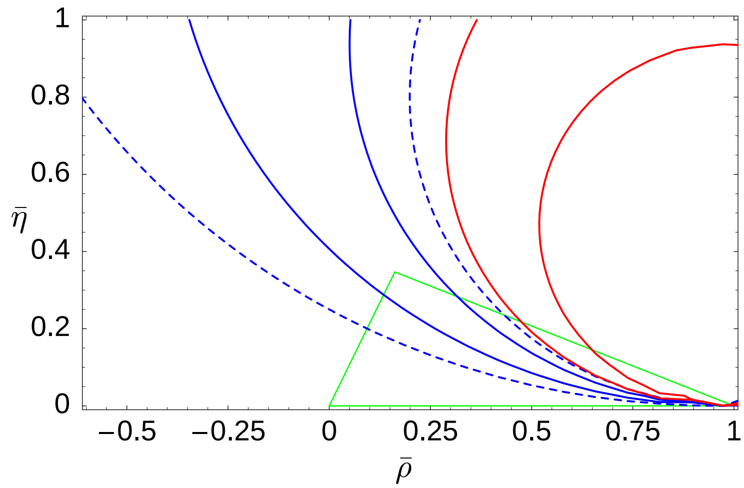

Figure 2: Constraint in the $(\bar{\rho}, \bar{\eta})$ plane from $a_{\rm fs}^d$. Area between solid pair of curves: NLO, for the cases $a_{\rm fs}^d = -5 \times 10^{-4}$ (left) and $a_{\rm fs}^d = -10^{-3}$ (right). Area between dashed curves: LO for $a_{\rm fs}^d = -5 \times 10^{-4}$. The current best fit to the unitarity triangle [11] is also shown.

The precise values for the quark masses, "bag" factors and α_s used for our numerical predictions can be found in Eq. (7) of [8].

We close our discussion with a remark about the B_s system. It is possible that new physics contributions render the $B_s - \overline{B}_s$ oscillations so large that a measurement of ΔM_s will be impossible. In general such new physics contribution will affect the CP phase ϕ_s and suppress $\Delta\Gamma_s$ in Eq. (3). Different measurements of $\Delta\Gamma_s$ can then determine $|\cos\phi_s|$ despite of the unobservably rapid B_s–$\overline{B}_s$ oscillations [12]. A measurement of the sign of $a_{\rm fs}^s \propto \sin\phi_s$ (which will then be enhanced, unless ΔM_s is extreme) through e.g. Eq. (13) will then reduce the four-fold ambiguity in ϕ_s from the measurement of $|\cos\phi_s|$ to a two-fold one.

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