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The effect of cluster magnetic field on the Sunyaev Zeldovich power spectrum

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ABSTRACT
Precision measurements of the Sunyaev Zeldovich (SZ) effect in upcoming blank sky surveys require theoretical understanding of all physical processes with $\gtrsim 10\%$ effects on the SZ power spectrum. We show that, observed cluster magnetic field could reduce the SZ power spectrum by $\sim 20\%$ at $l \sim 4000$, where the SZ power spectrum will be precisely measured by the Sunyaev Zeldovich array (SZA) and the Atacama cosmology telescope (ACT). At smaller scale, this effect is larger and could reach a factor of several. Such effect must be considered for an unbiased interpretation of the SZ data. Though the magnetic effect on the SZ power spectrum is very similar to that of radiative cooling, it is measurable by multi-band CMB polarization measurement.

Key words: cosmology-large scale structure-theory:magnetic field-clusters:cosmic microwave background

1 INTRODUCTION

Our universe is almost completely ionized at $z \lesssim 6$. Ionized electron scatters off CMB photons by its thermal motion and so generates secondary CMB temperature fluctuations. This effect is known as the thermal Sunyaev Zeldovich (SZ) effect. Since all free electrons participate in the inverse Compton scattering and contribute to the SZ effect, the SZ effect is an unbiased probe of the thermal energy of the universe. Its precision measurement and interpretation are of great importance to understand the thermal history of the universe.

Several detections of the CMB excess power over the primary CMB at $\sim 10'$ scale (CBI at $l \lesssim 3500$ (Bond et al. 2002; Mason et al. 2003), BIMA at $l \sim 6800$ (Dawson et al. 2002) and a marginal detection by ACBAR at $l \sim 3000$ (Kuo et al. 2002)) may signal the first detections of the SZ effect in a blank sky survey. Several upcoming CMB experiments such as ACT, Planck and SZA are likely able to measure the SZ power spectrum with $\sim 1\%$ accuracy[1].

Such precision measurement of the SZ effect requires an accurate understanding of it. Since upcoming SZ experiments are mainly interferometers, most works have focused on the prediction of the SZ power spectrum. Lots effort has been made to predict the SZ effect in an adiabatically evolving universe, both analytically (Cole & Kaiser 1988; Makino & Suto 1993; Atrio-Barandela & Mucket 1999; Komatsu & Kitayama 1999; Cooray, Hu & Tegmark 2000; Molnar & Birkinshaw 2000; Majumdar 2001; Zhang & Pen 2001; Komatsu & Seljak 2002) and simulationally (da Silva et al. 2000; Refregier et al. 2000; Seljak, Burwell & Pen 2001; Springel, White & Hernquist 2001; Zhang, Pen & Wang 2002). But various processes could introduce $\gtrsim 10\%$ uncertainties to the predicted power spectrum. The most significant ones may be feedback, preheating and radiative cooling, as favored by observations of clusters (Xue & Wu (2003) and reference therein) and the soft X-ray background (Pen 1999). The injection of non-gravitational energy heats up gas, makes it less clumpy and decreases the SZ power spectrum (da Silva et al. 2001; Lin et al. 2002; White, Hernquist & Spingel 2002). An energy injection of $\sim$ 1KeV per nucleon could decrease the SZ power spectrum by a factor of 2. Radiative cooling efficiently removes hot gas in the core of clusters and groups and reduces the SZ power significantly, especially at small scales (da Silva et al. 2001; Zhang & Wu 2003). Supernova remnants generated by first stars cool mainly through Compton scattering over CMB photons. The high efficiency of such energy injection into CMB could introduce a SZ effect comparable to that of low redshift gas (Oh, Cooray & Kamionkowski 2003).

In this paper, we discuss the influence of cluster magnetic field on the SZ effect. As we will find, cluster magnetic field can suppress the SZ power spectrum by $\sim 20\%$ at $l \sim 4000$ and a factor of 2 at $1'$.

Micro-gauss magnetic field universally exists in intr-

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[1] Zhang, Pen & Trac (2003) estimate the accuracy of the kinetic SZ effect measurement by ACT, which could reach 1%. Since the thermal SZ effect is about 30 times stronger than the kinetic SZ effect, a 1% accuracy measurement of the thermal SZ effect by ACT is highly likely.

acluster medium (ICM) (refer to Carilli & Taylor (2002) for a recent review). The strength of magnetic field in the core of non-cooling flow clusters is generally several μG, as inferred from Faraday rotation measure (Carilli & Taylor (2002); Eilek & Owen (2002); Taylor, Fabian & Allen (2002), but see Newman, Newman & Rephaeli (2002); Rudnick & Blundell (2003) for the discussion of smaller values). The magnetic pressure in the center of cluster could reach $\sim$ 1-10% of the gas thermal pressure. This extra pressure offsets part of the gravity and prohibits ICM to further fall in and so results in a less clumpy gas core. The cluster SZ temperature decrement could be then reduced by a factor 10% (Dolag & Schindler 2000; Koch, Jetzer & Puy 2003). However, this effect is only non-trivial for low mass clusters and groups in which gravity is weaker. Since such clusters are difficult to detect, the detection of the magnetic field effect on individual clusters is highly challenging.

The SZ power spectrum, on the other hand, avoids this problem. Three characteristics of the SZ power spectrum amplify the effect of magnetic field comparing to that of individual clusters. (1) Less massive clusters and groups are more populous, which amplifies their contribution to the SZ effect. (2) The contribution of less massive clusters and groups to the SZ power spectrum concentrates on smaller angular scales, comparing to more massive clusters. (3) At $l \gtrsim 4000$, the contribution to the SZ effect is mainly from $z \gtrsim 0.5$ (Zhang & Pen 2001), where less massive clusters are more dominant comparing to nearby universe. Combining these three points, the small scale SZ power is dominated by low mass clusters and groups. Since the influence of magnetic field on those clusters and groups is larger, one expects, if there is no strong decrease in the strength of magnetic field at $z \sim 1$, as suggested by high redshift source rotation measures (Carilli & Taylor (2002) and reference therein), magnetic field could change the SZ power spectrum at small scale by a significant fraction. Such effect is likely observable in future SZ surveys, thus the study of the effect of magnetic field on the SZ power spectrum serves for both the precision modeling of the SZ effect and a better understanding of cluster magnetic field and so deserves a detailed analysis.

Koch, Jetzer & Puy (2003) build an analytical model to estimate the effect of magnetic field on individual clusters adopting an isothermal β model for the ICM state and solve the magneto-hydrostatic equilibrium equation perturbatively. Our goal is to investigate its collective effect on the SZ power spectrum. For this purpose, we build a more detailed and more consistent model, based on the model of *universal gas density profile* (Komatsu & Seljak 2001). Since for low mass clusters and groups, magnetic pressure is comparable to gas thermal pressure, the perturbative method breaks down. So we solve the magneto-hydrostatic equilibrium equation non-perturbatively. We develop our model in §2 and apply it to the SZ effect in §3. We discuss and conclude in §4. Throughout this paper, we adopt a WMAP-alone cosmology: $\Omega_m = 0.268$, $\Omega_\Lambda = 0.732$, $\Omega_b = 0.044$, $\sigma_8 = 0.84$ and $h = 0.71$ (Spergel et al. 2003).

2 THE EFFECT OF MAGNETIC FIELD ON CLUSTERS

In this section, we will solve the hydrostatic equilibrium equation of individual clusters for the gas density and temperature profile, when magnetic field is present or not. First we need to know the gravitational potential well, which is mainly determined by dominant dark matter. The dark matter density profile can be well approximated by the NFW profile (Navarro, Frenk & White 1996, 1997)

$$\rho_{\rm dm} = \rho_s y_{\rm dm}(x) = \rho_s \frac{1}{x(1+x)^2}. \tag{1}$$

Here, $x \equiv r/r_s$ is the radius in unit of core radius r_s. The core radius is related to the virial radius by the compact factor $c \equiv r_{\rm vir}/r_s$. According to the definition of virial radius, $r_{\rm vir} \equiv [M/(4\pi\Delta_c(z)\rho_c(z)/3)]^{1/3}$ where $\Delta_c \sim 100$ is the mean density of a halo with mass M inside of its virial radius in unit of the critical density $\rho_c(z)$ at redshift z. We adopt the predicted Δ_c from Eke, Cole & Frenk (1996). The compact factor we adopt is (Seljak 2000):

$$c = 6\left(\frac{M}{10^{14}M_\odot/h}\right)^{-0.2}. \tag{2}$$

The gas density ρ_g and temperature T_g can then be solved by the hydrostatic equilibrium condition and one extra assumption on the gas state. In the literature, one either assumes (a) that gas follows dark matter ($\rho_g \propto \rho_{\rm dm}$. e.g. Wu & Xue (2002); Xue & Wu (2003)), or (b) isothermality (T_g = cons.), or (c) a polytropic gas ($T_g \propto \rho_g^{\gamma-1}$. e.g. Komatsu & Seljak (2001)). We will follow the procedure of Komatsu & Seljak (2001) and adopt the assumption (c). We will outline its basic idea in §2.1 and extend it to the case when magnetic field exists (§2.2).

2.1 $B = 0$

The three key ingredients of Komatsu & Seljak (2001) are

- The hydrostatic equilibrium condition:

$$\frac{dp_g}{dr} = -\frac{GM(\leqslant r)}{r^2}\rho_g. \tag{3}$$

Here, $M(\leqslant r)$ is the total mass contained in the sphere with radius r. It can be approximated as

$$M(\leqslant r) \equiv 4\pi\rho_s(1+\frac{\Omega_b}{\Omega_{\rm dm}})r_s^3 m(x). \tag{4}$$

Here, $m(x) \equiv \int_0^x u^2 y_{\rm dm}(u)du$.

- A polytropic form of the gas equation of state:

$$p_g \propto \rho_g^\gamma. \tag{5}$$

Then Eq. (3) becomes

$$y_g^{\gamma-1} = 1 - 3\frac{T_{\rm vir}}{T_g(r=0)}\frac{\gamma-1}{\gamma}\frac{c}{m(c)}\int_0^x du\frac{m(u)}{u^2}. \tag{6}$$

Here, $y_g(x) \equiv \rho_g(r)/\rho_g(r=0)$ is the relative gas density profile. $T_{\rm vir}$ is the virial temperature defined as

$$T_{\rm vir} \equiv \frac{GM\mu m_H}{3k_B r_{\rm vir}} = 5.23\Omega_m\frac{M/M_8}{r_{\rm vir}/({\rm Mpc/h})}{\rm KeV}. \tag{7}$$

Here $M_8 = 5.96\Omega_m M_\odot/h$ is the average mass contained

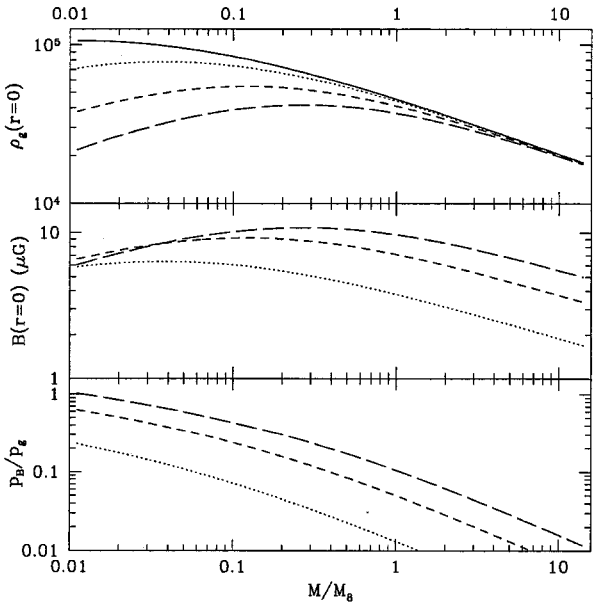

Figure 1. The effect of magnetic field on intracluster gas state. We assume $B(r) = B_*[\rho_g/10^4\bar{\rho}_g]^{0.9}$. The central gas density (in unit of mean gas density), strength of central magnetic field and the ratio between the magnetic pressure and the thermal pressure are shown in top, middle and bottom panel, respectively. For solid, dot, short dash and long dash lines, $B_* = 0, 1, 2, 3\ \mu G$, respectively. More massive clusters have stronger gravity and larger gas pressure, so the effect of magnetic field is weaker. For less massive clusters, magnetic gas pressure is comparable with the thermal pressure and $\rho_g(r=0)$ can be greatly suppressed by a factor of unity. This density suppression in turn suppresses the strength of the magnetic field by the B-ρ_g correlation. So the resulting strength of central magnetic field has only a weak dependence on halo mass and falls in the observed range of 1-10μG over a broad range of halo mass, from galaxy-size halos to most massive clusters. On the other hand, magnetic field only weakly changes the gas temperature.

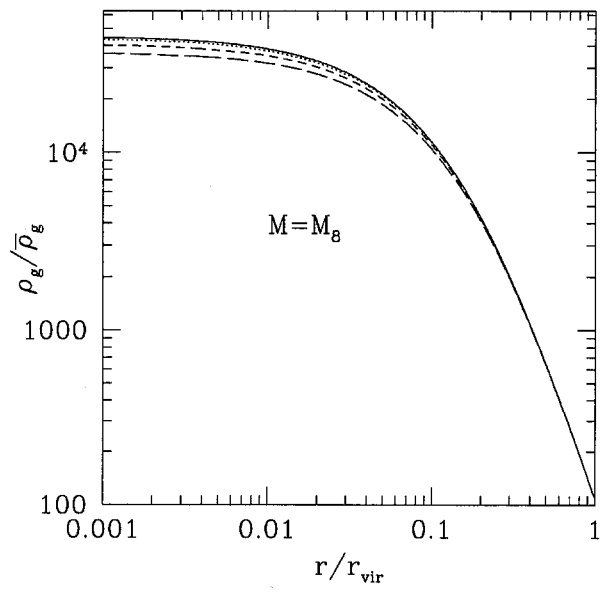

Figure 2. The effect of magnetic file on the gas density profile. We assume $B(r) = B_*[\rho_g)/10^4\bar{\rho}_g]^{0.9}$. For solid, dot, short dash and long dash lines, $B_* = 0, 1, 2, 3\ \mu G$ respectively. Since the magnetic field is strongest in the core, its effect is most significant in the core and decreases dramatically outward.

within a 8 Mpc/h comoving radius. Thus the gas state is solved up to three constants $\rho_g(r=0)$, $T_g(r=0)$ and γ.

- Gas density distribution follows dark matter density distribution in the outer region of each cluster. So, $y_g(x)$ and $y_{\rm dm}(x)$ must have the same slope at $x \gtrsim c/2$. This requirement simultaneously fixes $T_g(r=0)$ and γ:

$$\frac{T_g(r=0)}{T_{\rm vir}} = \frac{3}{\gamma}\frac{c}{m(c)}|S_*|^{-1} \times \left(\frac{m(c)}{c} + (\gamma-1)|S_*|\int_0^c du\frac{m(u)}{u^2}\right), \quad (8)$$

$$\gamma = 1.15 + 0.01(c - 6.5). \quad (9)$$

Here, $S_* \equiv d\ln y_{\rm dm}(x)/\ln(x)|_c$ is the slope of the dark matter density profile at $x = c$. $\rho_g(r=0)$ is fixed by requiring that the baryon-dark matter density ratio in the outer region follows its universal ratio:

$$\rho_g(r=0) = \rho_s\frac{\Omega_b}{\Omega_{\rm dm}}\frac{y_g(c)}{y_{\rm dm}(c)}. \quad (10)$$

2.2 $B \neq 0$

Magnetic pressure ($p_B = B^2/8\pi$) provides extra force to offset gravity and suppresses the falling of gas into the gravitational potential well. One then expects a less clumpy gas core. Clusters generally have a magnetic field of the order μG, whose pressure is $\sim 10\%$ of the thermal pressure:

$$\frac{p_B}{p_g} = 0.0252\left(\frac{1000}{1+\delta_g}\right)\left(\frac{5\text{KeV}}{k_BT_g}\right)\left(\frac{B}{1\mu G}\right)^2\left(\frac{0.02}{\Omega_b h^2}\right). \quad (11)$$

With the existence of magnetic field, Eq. 3 changes to

$$\frac{dp_g}{dr} + \frac{dp_B}{dr} = -\frac{GM(\leqslant r)}{r^2}\rho_g. \quad (12)$$

If we assume that $B \propto \rho^\alpha$, we obtain

$$y_g^{\gamma-1} + \eta y_g^{2\alpha-1} = (1+\eta)y_g^{\gamma-1}(B=0), \quad (13)$$

where

$$\eta = \frac{p_B(B, r=0)}{p_g(B, r=0)}\frac{2\alpha}{2\alpha-1}\frac{\gamma-1}{\gamma} \quad (14)$$

is determined by the central density and temperature.

There is no definite prediction of α, but one can infer its lower limit. Since most, if not all, magnetic field generation mechanisms, such as galactic winds and hierarchical mergers of cluster formation, produce magnetic field positively correlated with gas density (Carilli & Taylor (2002) and reference therein), the further amplification of adiabatic compression will produce $\alpha \geqslant 2/3$. For example, the hierarchical merger produces $\alpha \simeq 0.9$ (Dolag et al. 2001). In this paper, we consider two cases of α, $\alpha = 0.9$ and $\alpha = 2/3$.

Given $\alpha \geqslant 2/3$, the magnetic pressure drops faster than the thermal pressure, so in the outer regions near virial ra-

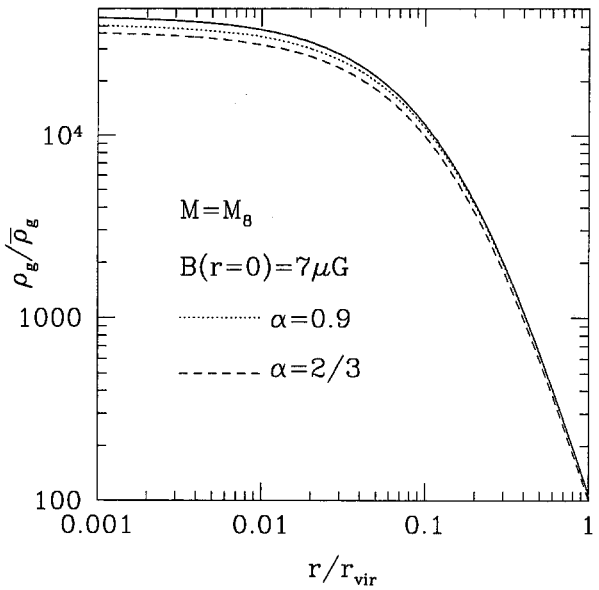

Figure 3. The dependence of magnetic effect on individual clusters on the B-ρ_g correlation ($B \propto \rho_g^\alpha$). We choose suitable coefficient in this scaling relation such that the strength of central magnetic field ($7\mu G$ in this figure) is identical for various α. Since the magnetic pressure of α drops more slowly than that of $\alpha = 0.9$, its effect is observable from the core to cluster outer region. So the overall effect of magnetic field of $\alpha = 2/3$ is larger than that of $\alpha = 0.9$.

dius, magnetic field can be neglected. One can then easily show that γ does not change due to the presence of magnetic field. The two constant $T_g(r=0)$ and $\rho_g(r=0)$ are related to their corresponding values when $B=0$ by:

$$\frac{T_g(r=0,B)}{T_g(r=0,B=0)} = (1+\eta)^{-1}, \tag{15}$$

and

$$\frac{\rho_g(r=0,B)}{\rho_g(r=0,B=0)} = (1+\eta)^{\frac{-1}{\gamma-1}}. \tag{16}$$

η is fixed by

$$\eta(1+\eta)^{(2\alpha-\gamma)/(\gamma-1)} = \eta_0, \tag{17}$$

where η_0 is the value of η by substituting $\rho_g(r=0,B)$ and $T_g(r=0,B)$ in Eq. 14 with $\rho_g(r=0,B=0)$ and $T_g(r=0,B=0)$.

We adopt a parametric form of $B(r)$

$$B(r) = B_* \left(\frac{\rho_g(r)}{10^4\bar{\rho}_g(z=0)}\right)^\alpha. \tag{18}$$

The results for various B_* and α are shown in Fig. 1, 2 and 3. As expected, magnetic field has weaker effect on more massive clusters since their gravity is stronger. For M_8 clusters, the magnetic pressure can reach 10% of the thermal pressure and can suppress the central gas density by $\sim 10\%$.

Strong magnetic field suppresses the gas infall (top panel, Fig. 1) and therefore the strength of magnetic field in turn by the B-ρ_g correlation. Such back-reaction is dominant in low mass clusters and groups and so causes the strength of magnetic field to cease to increase toward low mass end (middle panel, Fig. 1). Our parametric model predicts a several μG magnetic field over a wide range of halo mass, from galaxy-size halo mass ($\lesssim 0.01M_8$) to massive cluster mass. We do not find any strong dependence of magnetic field on halo mass, which is consistent with observations of galaxy (Beck et al. (1996) and references therein) and cluster magnetic field. The agreement between our predicted galaxy magnetic field strength with observations suggests that our parametric treatment of magnetic field may extend to galaxy scale. If so, magnetic field may suppress gas infall to low mass halos by a factor of several. Such suppression may have a significant effect on star formation in such halos and may be partly responsible for the formation of dark satellite halos due to inefficient gas accretion. This issue may deserve further investigation.

We focus on a M_8 cluster to investigate the dependence of magnetic effect on B-ρ_g correlation. To single out its effect, we choose corresponding B_* for $\alpha = 0.9$ and $\alpha = 2/3$ such that the magnetic field in the core of a M_8 cluster is the same for two cases. In order to satisfy this requirement, B_* for $\alpha = 2/3$ must be larger than that of $\alpha = 0.9$. So magnetic field of $\alpha = 2/3$ has a larger pressure gradient and thus a larger effect on clusters (Fig. 3). We have adopted $B_*(\alpha = 2/3) = 2.94\ \mu G$ while $B_*(\alpha = 0.9) = 2.0\ \mu G$ in Fig. 3. Since for $\alpha = 2/3$, magnetic pressure drops more slowly toward outer region than $\alpha = 0.9$ case, its effect extends to a larger radius (Fig. 3). So, its overall effect is significantly larger than $\beta = 0.9$ case. Since α is likely bigger than 2/3, this case should be treated as the upper limit of the magnetic effect.

3 THE SUNYAEV ZELDOVICH EFFECT

In the Rayleigh-Jeans regime, the SZ temperature decrement is given by (Zeldovich & Sunyaev 1969)

$$\begin{aligned}\frac{\Delta T}{T_{\rm CMB}} &= -2\int \sigma_T \frac{n_e k_B T_g}{m_e c^2} a d\chi \\ &= -2.37\times 10^{-4}\Omega_b h \int \frac{(1+\delta_g)k_B T}{\rm keV} a^{-2} d\tilde{\chi}.\end{aligned} \tag{19}$$

Here, χ and a are the comoving distance and scale factor, respectively. $\tilde{\chi} \equiv \chi/(c/H_0)$ is the dimensionless comoving distance while H_0 is the present Hubble constant. We define a dimensionless gas pressure $p \equiv (1+\delta_g)k_B T_g/\mathrm{KeV}$. Then the mean SZ decrement is determined by the gas density weighted temperature $\bar{T}_g \equiv \langle p \rangle$. In the halo model, each halo has a pressure distribution $p(r,M)$, whose integral over the halo volume gives the contribution of each halo to the mean SZ temperature decrement. Once one knows the halo mass function $n(M,z)$, one can calculate $\bar{T}_g$ by

$$\bar{T}_g = \int \frac{dn}{dM} dM \left[\int_0^{r_{\rm vir}} p(r,M) a^{-3} 4\pi r^2 dr\right]. \tag{20}$$

The halo mass function n is well described by the Press-Schechter formalism (Press & Schechter 1974):

$$\frac{dn}{dM} = \sqrt{\frac{2}{\pi}} \frac{\rho_0}{M^2} \frac{\delta_c}{\sigma(M)} \left|\frac{d\ln\sigma(M)}{d\ln M}\right| \exp\left[-\frac{\delta_c^2}{2\sigma^2(M)}\right]. \tag{21}$$

Here, ρ_0 is the present mean matter density of the universe. $\sigma(M,z)$ is the linear theory rms density fluctuation in a

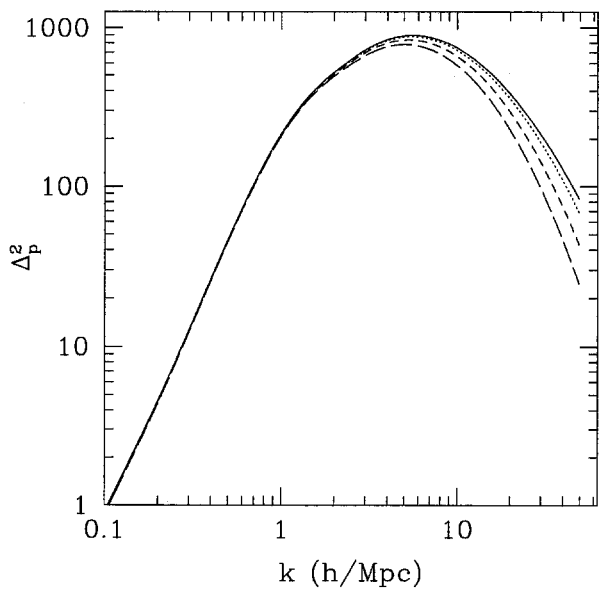

Figure 4. The effect of magnetic field on the gas thermal pressure power spectrum Δ_p^2. $p \equiv (1+\delta_g)T_g/\mathrm{KeV}$ is the dimensionless thermal pressure. This definition differs from the usual definition of the pressure power spectrum by a factor $\bar{T}_g^2$. We assume $B(r) = B_*[(\rho_g/10^4\bar{\rho}_g(z=0)]^{0.9}$. For solid, dot, short dash and long dash lines, $B_* = 0, 1, 2, 3\ \mu G$, respectively. The smaller scale power is mainly contributed by less massive clusters since they have smaller core radius. So, the effect of magnetic field increases toward smaller scale.

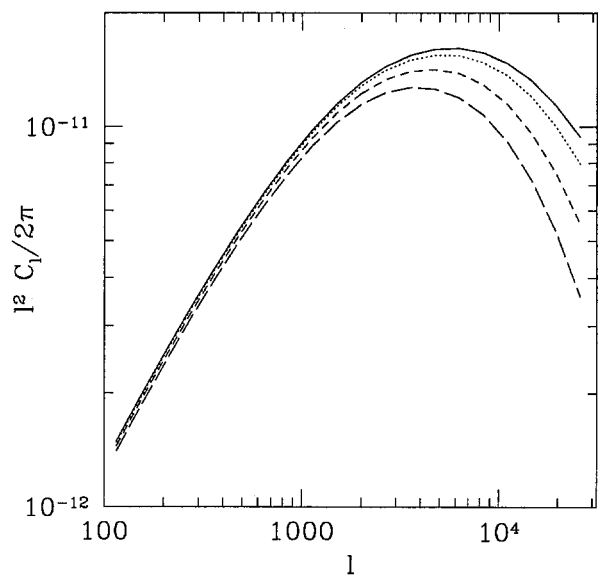

Figure 5. The effect of magnetic field on the SZ effect power spectrum C_l. We assume $B(r) = B_*[(\rho_g/10^4\bar{\rho}_g(z=0)]^{0.9}$. For solid, dot, short dash and long dash lines, $B_* = 0, 1, 2, 3\ \mu G$, respectively. Similar to the case of gas pressure power spectrum, the effect of B increases toward small scales where most contribution comes from less massive clusters.

sphere containing mass M at redshift z. δ_c is the linearly extrapolated over-density at which an object virializes. For a $\Omega_m = 1$ universe, $\delta_c = 1.686$. Since its dependence on cosmology is quite weak (Eke, Cole & Frenk 1996), we fix $\delta_c = 1.686$. This simplification introduces at most 1% error.

The contribution of each halo to the SZ power spectrum is determined by the Fourier component of its pressure profile:

$$p(k, M) = \int_0^{r_{\rm vir}} p(r, M) \frac{\sin(kr/a)}{kr/a} a^{-3} 4\pi r^2 dr. \tag{22}$$

The collective contribution of all halos at a certain redshift is the pressure power spectrum

$$\begin{aligned} p^2(k) &= \int p^2(k, M) \frac{dn}{dM} dM \\ &+ P_{\rm dm}(k) \left(\int p(k, M) b(M) \frac{dn}{dM} dM \right)^2 . \end{aligned} \tag{23}$$

Here, $b(M)$ is the linear bias of the halo number overdensity with respect to dark matter overdensity such that the halo-halo power spectrum $P(k, M_1, M_2) = P_{\rm dm}(k) b(M_1) b(M_2)$. The linear dark matter power spectrum $P_{\rm dm}(k)$ is calculated by the BBKS transfer function fitting formula (Bardeen et al. 1986). We adopt the Mo & White (1996) formula to calculate $b(M)$.

Throughout this paper, we alternatively refer the pressure variance $\Delta_p^2 \equiv p^2(k) k^3/2\pi$ as the pressure power spectrum. $\Delta_p^2(k, z=0)$ for $\alpha = 0.9$ is shown in Fig. 4. As expected, the effect of magnetic field concentrates on small scales. At $k \sim 6h/\mathrm{Mpc}$, the peak of $\Delta_p^2(k)$, it reduces Δ_p^2 by $\sim 10\%$. At smaller scales dominated by less massive halos, magnetic field can suppress Δ_p^2 by as large as a factor of 2.

The Limber's integral of Δ_p^2 over comoving distance is the 2D SZ power spectrum:

$$\frac{l^2 C_l}{2\pi} = \pi \left(2.37 \times 10^{-4} \Omega_b h\right)^2 \int \Delta_p^2(\frac{l}{\chi}, z) a^{-4} \frac{\tilde{\chi}}{l} d\tilde{\chi}. \tag{24}$$

To do this integral, one needs to know the evolution of cluster magnetic field. The origin of cluster magnetic field (Carilli & Taylor (2002) and reference therein) is quite unclear, so in the literature, there is no consensus reached on the evolution of magnetic field. For example, the scenario of hierarchical merger of cluster formation predicts a very strong evolution $B(z) \propto 10^{-2.5z}$ (Dolag, Bartelmann & Lesch 2002), while magnetization mechanism associated with magnetized galactic winds generated by starbursts predicts a much slower evolution (Volk & Atoyan 2000). Given such divergent predictions, the most reliable way to infer cluster magnetic field evolution may be observations. The measured large Faraday rotation measures of high z sources suggest that μG magnetic field may have existed in (proto-)cluster atmospheres (Carilli & Taylor (2002) and reference therein) at $z \gtrsim 2$. Such magnetic field strength is comparable to that of the present clusters. So, we assume no evolution in the magnetic field and apply Eq.(18) to all z.

We show the SZ power spectrum of $\alpha = 0.9$ under such assumption in Fig. 5. Since at higher redshifts, there are fewer massive clusters, the effect of magnetic field is stronger

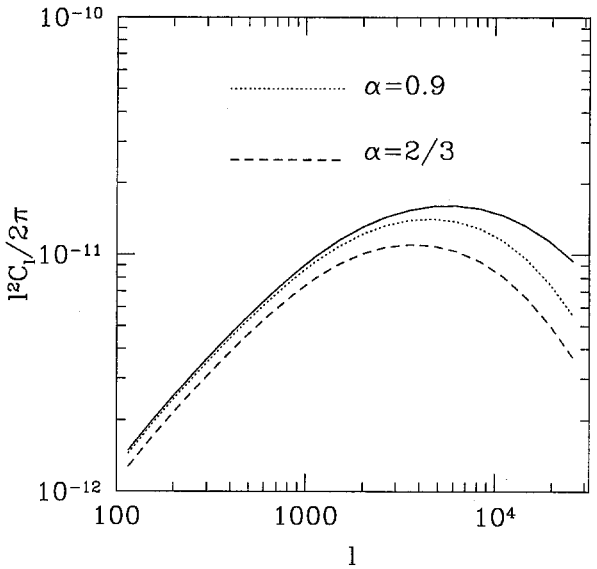

Figure 6. The effect of B-ρ_g correlation on the SZ power spectrum. We have chosen $B_* = 2.94\ \mu G$ for $\alpha = 2/3$ such that the central magnetic field of a M_8 cluster at $z = 0$ is the same as that of the case with $\alpha = 0.9$ and $B_* = 2.0\ \mu G$. Since magnetic pressure drops more slowly toward cluster outer region in the $\alpha = 2/3$ case, its effect extends in a wider spacial range and so becomes observable even at angular scales as large as $l \sim 100$. Since α is likely bigger than 2/3, $\alpha = 2/3$ case may show the upper limit of the magnetic effect.

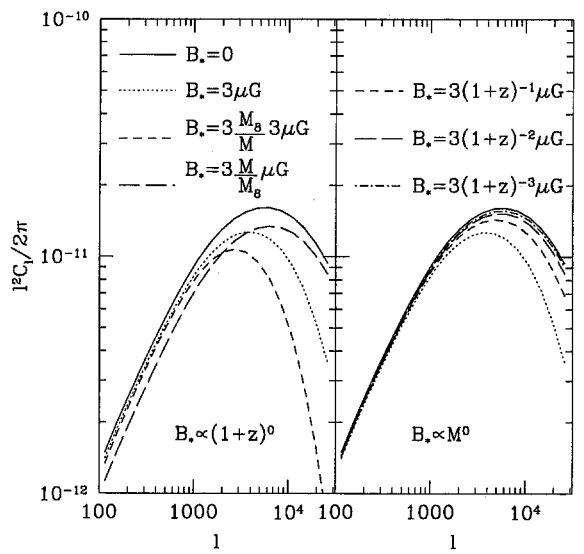

Figure 7. The effect of the B_* dependence on cluster mass and redshift. We parameterize this dependence as a power law $B_* \propto M^a(1+z)^b$. C_l at larger scales is determined by more massive clusters. For $a > 0$, more massive clusters have stronger magnetic field, which suppresses C_l at larger scales. At smaller scales C_l is determined by less massive clusters and groups. For $a > 0$, less massive clusters have weaker magnetic field and the magnetic field effect on C_l at small scales is suppressed. For $a < 0$, the suppression on C_l at small scales is much more significant. If B_* evolves faster than $(1+z)^{\sim -2}$, the magnetic effect on C_l is effectively negligible.

on C_l than on $\Delta_p^2(z=0)$. At $l \sim 4000$, the peak of the SZ power spectrum, magnetic field can suppress the SZ power spectrum by $\sim 20\%$. At large angular scale $l \lesssim 1000$, where nearby massive clusters dominate, the effect of magnetic field is negligible. At $l \gtrsim 4000$, the magnetic suppression is significant and can reach a factor of several. Such effect must be taken into account for a precision understanding of the SZ effect.

For the choice[2] of $(\alpha, B_*) = (0.9, 2.0\mu\mathrm{G})$ and $(2/3, 2.94\mu\mathrm{G})$, we compare the resulted SZ power spectra. Since $\alpha = 2/3$ suppresses more on cluster gas density in a wider range (Fig. 3), its effect on the SZ power spectrum is larger and extends to larger angular scales (Fig. 6). For the same reason, we find a 30% decrease in the mean temperature decrement comparing to $B = 0$ case and a 20% decrease comparing to $\alpha = 0.9$ case. Since α is likely bigger than 2/3, $\alpha = 2/3$ case should be treated as an upper limit of the magnetic effect.

The above discussion has omitted any dependence of B_* on cluster mass and redshift. Though this special choice is consistent with observations, since there is no solid predictions or measurements on B_*, one has to be aware of other possibilities. In order to discuss the effect of possible dependence of B_* on M and z, we parameterize this dependence as $B_* \propto M^a(1+z)^b$ and try several choices of a and b. $a \gtrsim 1$ produces too weak B for small groups while $a \lesssim -1$ produces too weak B for massive clusters. So, we only discuss the cases of $a = -1$ and 1. For $b \lesssim -3$, cluster magnetic field at $z > 1$ is too weak and contradicts with observations, so, we only discuss the cases of $b = -3, -2, -1$. The result is shown in fig. 7. If the main growth mechanism of cluster magnetic field is hierarchical merger, one then expects $a > 0$ since more massive clusters emerge from more mergers. Such correlation increases the magnetic field of more massive clusters and thus decreases C_l at larger scales comparing to the $a = 0$ case. Cluster B generation mechanism associated with star formation tends to have $a < 0$ since star forming galaxies mainly reside in field (or equivalently, small halos) instead of massive clusters. For this case, we expect a larger suppression at small scales due to stronger magnetic field in less massive clusters and groups. For the redshift dependence, we find that if B_* evolves faster than $(1+z)^{\sim -2}$, the effect of magnetic field on C_l is negligible. Due to large uncertainties in both a and b, we postpone further discussion in this paper.

[2] As a reminder, such choice of B_* produces the same central magnetic field for M_8 clusters at $z = 0$

4 DISCUSSION

Cluster magnetic field affects not only the SZ effect. Here we address its influence on cluster entropy, X-ray luminosity and the soft X-ray background (XRB).

Magnetic field may be partly responsible for entropy floors observed in clusters (e.g. Xue & Wu (2003) and reference therein). A conventional definition of cluster entropy is $K = T_g \rho_g^{-2/3}$. Magnetic field does not change entropies of very massive clusters, but it increases entropies of less massive clusters by decreasing their gas densities. But for a reasonable magnetic field, such effect is not sufficient to explain the entire entropy floors observed and could only be a minor cause comparing to preheating, feedback and radiative cooling (Xue & Wu 2003).

The X-ray luminosity of clusters have a strong dependence on gas density ($\propto \rho_g^2$) and mainly comes from central regions of clusters. So one expects a larger suppression to cluster X-ray luminosity than to cluster SZ effect. Since less massive halos are more clumpy, their contribution to the soft XRB is larger. So the suppression of magnetic field to individual cluster X-ray luminosity is further amplified in the soft XRB. One then expects a order of unity suppression to the mean flux and power spectrum of the soft XRB contributed by clusters and groups. This effect can be quantitatively investigated following a similar procedure, as applied to the XRB in the literature (Wu & Xue 2003; Zhang & Pen 2003).

Besides its dynamic effect, magnetic field can further change the ICM state by the suppression of the thermal conduction (Malyshkin (2001) and reference therein). This could produce non-negligible effect on the Sunyaev Zeldovich effect, especially when preheating, feedback or radiative cooling are present. This effect does not change the mean SZ temperature decrement, but would likely increase the small scale SZ power by hot and cool patches caused by inefficient heat conduction. Since this issue requires a detailed understanding of the distribution of entangled magnetic field in a cluster, which is not available at present, we postpone such estimation.

Here we address several subtleties in our SZ calculation. One is how to deal with the gas loss caused by magnetic field. Since magnetic field suppresses the gas infall and we only integrate over the virialized volume (see Eq. 20 and 22), the gas mass in such integration of each cluster is less than that of $B = 0$ case. One can always increase the integral upper limit in Eq. 20 and 22 to compensate the gas mass loss. Unfortunately, the fate of such gas is unclear. But because gas outside of virial radius is several times cooler than gas in the core, such effect is minor, so we neglect such calculation in this paper. Another issue is γ. We have assumed a constant γ across each clusters, but the effect of magnetic field on the gas polytropic state is unclear. Though these issues have to be scrutinized in magneto-hydrodynamic (MHD) simulations, the consistency between our analytical predictions and MHD simulations (Dolag & Schindler 2000) suggests that our model should be appropriate at the first order approximation.

We then conclude that, cluster magnetic field suppresses the SZ power spectrum around its peak by $\sim 20\%$ and by a factor of ~ 2 at $\sim 1'$. Such effect must be considered for an accurate theoretical understanding of the SZ effect and an unbiased interpretation of the SZ measurement in future blank sky surveys. The effect of magnetic field to the SZ power spectrum is similar to that of radiatively cooling (Zhang & Wu 2003), which also decreases the SZ power spectrum at small scales significantly while leaves the large scale power spectrum barely touched. Such similarity brings extra difficulty to extract the magnetic effect from the SZ observation and therefore the interpretation of the SZ data. The CMB polarization measurement can be applied to recover cluster magnetic fields (Ohno et al. 2003) and helps for a robust prediction of the SZ effect under the presence of magnetic field.

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