

# Phase Motion in the Scalar Low-Mass $\pi^- \pi^+$ Amplitude in $D^+ \rightarrow \pi^- \pi^+ \pi^+$

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## Abstract.

This work is a direct and model-independent measurement of the low mass  $\pi^+ \pi^-$  phase motion in the  $D^+ \rightarrow \pi^- \pi^+ \pi^+$  decay. The results show a strong phase variation, compatible with an isoscalar  $\sigma(500)$  meson. This result confirms the previous Fermilab E791 result which found evidence for the existence of this scalar particle using a full Dalitz-plot analysis.

## INTRODUCTION

Charm meson decays are a natural place for studying light scalar mesons. These decays are a clean environment: well defined initial state, very small non-resonant component and large coupling to scalars. In addition, D decays can provide insight into the quark content of these controversial particles, since the bulk of the hadronic width comes from modes for which there is a W-radiation amplitude.

Studies of light scalars using charm decays started a few years ago, when E791[1] presented results of Dalitz-plot analyses of the decays  $D_s^+, D^+ \rightarrow \pi^- \pi^+ \pi^+$  [2, 3] and  $D^+ \rightarrow K^- \pi^+ \pi^+$  [4]. In particular, the analysis of the decays  $D^+ \rightarrow \pi^- \pi^+ \pi^+$  and  $D^+ \rightarrow K^- \pi^+ \pi^+$  showed evidence for the  $\sigma$  and  $\kappa$  mesons. In the case of  $D^+ \rightarrow \pi^- \pi^+ \pi^+$ , the  $\sigma$  appears as an accumulation of signal events in low  $\pi^+ \pi^-$  mass. The E791 data could only be described by an amplitude having an  $s$  dependent phase ( $s$  being the  $\pi^- \pi^+$  mass squared).

Several years later, the same effect - an accumulation of signal events in low  $\pi^+ \pi^-$  mass - was also observed in other decays from different experiments, for, instance,  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$  from CLEO[5] and Belle[6], and  $J/\psi \rightarrow \omega \pi^+ \pi^-$  from BES[7]. Again, the description of these data requires amplitudes with  $s$ -dependent phase.

Since E791 publication of the  $D_s^+, D^+ \rightarrow \pi^- \pi^+ \pi^+$  results, two kinds of criticisms have been made: the quoted values of both  $\sigma$  and  $\kappa$  parameters are not correct because the simple Breit-Wigner formula is inadequate to describe broad scalars, especially when near the threshold; the other criticism is that one should not claim the existence of a resonance without showing the phase motion of the corresponding amplitude.

The Breit-Wigner formula, although being in this case only a naive approximation, has the key ingredient: an  $s$ -dependent phase. On the other hand, there is no single agreed way to treat broad scalars near threshold. The  $\sigma$  parameters depend strongly on the assumed functional form of its line shape.

In this work[8] the second criticism is addressed. The phase variation of the low-mass  $\pi^+ \pi^-$  amplitude is extracted in a model independent way.

# Phase Motion in the Scalar Low-Mass $\pi^-\pi^+$ Amplitude in $D^+ \rightarrow \pi^-\pi^+\pi^+$

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## Abstract.

This work is a direct and model-independent measurement of the low mass  $\pi^+\pi^-$  phase motion in the  $D^+ \rightarrow \pi^-\pi^+\pi^+$  decay. The results show a strong phase variation, compatible with an isoscalar  $\sigma(500)$  meson. This result confirms the previous Fermilab E791 result which found evidence for the existence of this scalar particle using a full Dalitz-plot analysis.

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## THE AMPLITUDE DIFFERENCE METHOD

In Dalitz-plot analysis, resonant amplitudes are complex functions written in the general form  $\mathcal{A} = f(s)e^{i\delta(s)}$ . Non-resonant amplitudes, in contrast, have  $\delta(s) = \text{constant}$ . If two resonant amplitudes cross in some region of the Dalitz-plot, they will interfere. The interference pattern in this crossing region depends on the  $s$ -dependent phases of both resonances. One can extract the phase motion of an unknown amplitude that crosses a well known resonance provided that:

- the contribution of other amplitudes is negligible in the crossing region between the amplitude under study and the known resonance;
- the integrated amplitude of the known resonance is symmetric with respect to an effective mass squared ( $m_{eff}^2$ ).

The second condition ensures that, by comparing the amplitude below and above  $m_{eff}^2$ , we end up with an expression that involves only the desired phase  $\delta(s_{13})$ . Then, we can write the approximate amplitude of this phase space region in a simple way,

$$\mathcal{A}(s_{12}, s_{13}) \simeq a_R \mathcal{B}\mathcal{W}(s_{12}) \mathcal{M}(s_{12}, s_{13}) + a_s / (p^* / \sqrt{s_{13}}) \sin\delta(s_{13}) e^{i(\delta(s_{13}) + \gamma)} \quad (1)$$

where  $\gamma$  is the overall relative final state interaction (FSI) phase difference between the two amplitudes,  $a_R$  and  $a_s$  are respectively the real magnitudes of the known resonance and the under-study complex amplitude,  $\sin\delta(s_{13})e^{i\delta(s_{13})}$  represents the most general amplitude for a two-body elastic scattering;  $p^* / \sqrt{s_{13}}$  is a phase space factor to make this description compatible with  $\pi\pi$  scattering and  $\mathcal{M}(s_{12}, s_{13})$ ,  $\mathcal{B}\mathcal{W}(s_{12})$  are the angular function and Breit-Wigner for the known resonance, respectively.

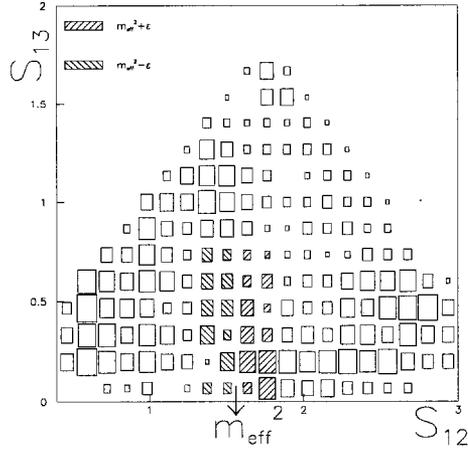
The quantity  $\Delta |\mathcal{A}(s_{13})|^2 \equiv |\mathcal{A}(m_{eff}^2 + \varepsilon, s_{13})|^2 - |\mathcal{A}(m_{eff}^2 - \varepsilon, s_{13})|^2$ , which is the difference of the amplitudes squared after integration over  $s_{12}$ , is computed in bins of  $s_{13}$ . It takes the form

$$\Delta |\mathcal{A}(s_{13})|^2 = \frac{-4a_s a_R / (p^* / \sqrt{s_{13}}) \varepsilon m_0 \Gamma_0}{\varepsilon^2 + m_0^2 \Gamma_0^2} (\sin(2\delta(s_{13}) + \gamma) - \sin\gamma) \mathcal{M}(s_{13}) / (p^* / \sqrt{s_{13}}) \quad (2)$$

If  $\delta(s_{13})$  is an analytical function of  $s_{13}$ , then there will be maximum and minimum values of  $\Delta |\mathcal{A}(s_{13})|^2$ , which we can use to determine both the constant term in the above equation and the phase  $\gamma$ ,

$$\frac{-4a_s a_R / (p^* / \sqrt{s_{13}}) \varepsilon m_0 \Gamma_0}{\varepsilon^2 + m_0^2 \Gamma_0^2} \equiv \mathcal{C} = (\Delta |\mathcal{A}'|^2_{max} - \Delta |\mathcal{A}'|^2_{min}) / 2 \quad (3)$$

$$\gamma = \sin^{-1} \left( \frac{\Delta |\mathcal{A}'|^2_{max} + \Delta |\mathcal{A}'|^2_{min}}{\Delta |\mathcal{A}'|^2_{min} - \Delta |\mathcal{A}'|^2_{max}} \right) \quad (4)$$



**FIGURE 1.**  $D^+ \rightarrow \pi^- \pi^+ \pi^+$  Dalitz-plot folded distribution. The events used by the AD analysis are in the hatched region. The size of the area of each bin in the plot corresponds to the number of events in that bin.

where  $\mathcal{A}' \equiv \mathcal{A} / (\mathcal{M} \sqrt{s_{13}} / p^*)$ .

Finally, considering  $\delta(s_{13})$  an increasing function of  $s_{13}$ , we have

$$\delta(s_{13}) = \frac{1}{2} (\sin^{-1}(\frac{1}{\mathcal{E}} \Delta | \mathcal{A}'(s_{13})|^2 + \sin(\gamma)) - \gamma) \quad (5)$$

This is, in essence, the idea of the amplitude difference method [9]. The method was shown to work in a "calibration" exercise using the  $f_0(980)$  resonance in the  $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$  decay [10]. In this case we have the  $f_0(980)$  contribution in both  $s_{12}$  and  $s_{13}$  axes. We were able to get a phase motion  $\delta(s_{13})$  compatible with the  $f_0(980)$  using the  $f_0(980)$  in  $s_{12}$ .

## PHASE MOTION OF SCALAR LOW-MASS $\pi^- \pi^+$ AMPLITUDE

The folded Dalitz-plot distribution of the  $D^+ \rightarrow \pi^- \pi^+ \pi^+$  decay is shown Fig. 1. The horizontal and vertical axes are the squares of the  $\pi^+ \pi^-$  invariant mass high ( $s_{12}$ ) and low ( $s_{13}$ ) combinations.

To study the low mass region in  $s_{13}$ , there are three possible well known resonances in  $s_{12}$  to act as a probe in this decay:  $\rho(770)$ ,  $f_0(980)$  and  $f_2(1270)$ . Figure 1 shows that the  $\rho(770)$  and  $f_0(980)$  are located in regions where other amplitudes can not be considered negligible. On the other hand, the tensor  $f_2(1270)$ ,  $m_0^2 = 1.61 \text{ GeV}^2/c^4$ , is placed where the  $\rho(770)$ , in the crossed channel reaches a minimum due to its decay angular distribution.

With the proper choice of  $m_{eff}^2 = 1.535 \text{ GeV}^2/c^4$ , the integral over  $s_{12}$  of the  $f_2(1270)$  amplitude squared is symmetrical: the number of events between  $m_{eff}^2$  and  $m_{eff}^2 + \varepsilon$  ( $\varepsilon = 0.26 \text{ GeV}^2/c^4$ ) is equal to the number of events between  $m_{eff}^2$  and  $m_{eff}^2 - \varepsilon$ . Moreover, in this region there is no significant contribution other than the  $\pi\pi$  complex amplitude under study in  $s_{13}$  (the amount of  $\rho(770)$  within this mass region was estimated to be  $\sim 5\%$ ). The choice of the  $f_2(1270)$  as the analyser amplitude satisfies the necessary conditions for the amplitude difference method.

The acceptance and the background must be similar between  $m_{eff}^2$  and  $m_{eff}^2 + \varepsilon$  and  $m_{eff}^2$  and  $m_{eff}^2 - \varepsilon$ , otherwise there would be biases in  $\delta(s_{13})$ . Monte Carlo simulations show that the acceptance is nearly uniform in this region. The background in this region comes mostly from random combinations of three pions, and it is also uniformly distributed. Since we are subtracting two similar distributions, we considered the background only in the size of the statistical error.

The  $f_2(1270)$  angular function has a zero at about  $s_{13} \simeq 0.48 \text{ GeV}^2/c^4$ . This means a singularity in  $\mathcal{A}'$ . This singularity is handled in the following way. The data is divided into ten  $s_{13}$  bins. The binning is such that the singularity is placed in the middle of one bin. Doing this, we isolate the singularity in a single bin (bin 6) and discard its further use in the analysis.

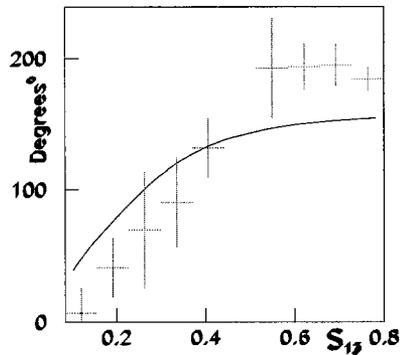
The values of  $\gamma$  and  $\mathcal{C}$  were obtained solving Equations 3 and 4. The value of the phase  $\gamma$  from the amplitude difference method is in agreement with that of the full Dalitz-plot analysis,  $\gamma_{AD} = 2.78 \pm 0.38 \pm 0.40$  and  $\gamma_{Dalitz} = 2.59 \pm 0.19$ [3].

Having  $\gamma$  and  $\mathcal{C}$ , the phase  $\delta(s_{13})$  is obtained for each  $s_{13}$  bin using Equation 5. There are ambiguities arising from the  $\sin^{-1}$  operation, so  $\delta(s_{13})$  was determined with the assumption that the phase difference starts at zero at threshold and is an increasing, monotonic, smooth function of  $s_{13}$ .

The phase motion of the low  $\pi^+\pi^-$  mass amplitude, including systematic and statistical errors, is shown in Fig. 2. In spite of the limited statistics, a strong phase variation is clearly observed. Starting from zero at the threshold the phase varies by about  $180^\circ$  and saturates at around  $s_{13} = 0.6 \text{ GeV}^2/c^4$ . This is the expected behavior of resonance. The observed phase motion supports the interpretation of the  $\sigma(500)$  as a true resonance. A constant or slowly varying phase would disfavor this interpretation. With more statistics we could have more bins and the pole position could be inferred. Fig. 2 shows also the phase motion of the simple Breit-Wigner (solid line) used in [3]. Even considering the Breit-Wigner as a naive approximation, there is a qualitative agreement between its phase motion and the directly extracted  $\delta(s_{13})$ : the description of the data requires an amplitude with a strong phase variation.

## CONCLUSIONS

A direct, model-independent measurement of the phase motion of the low mass  $\pi^+\pi^-$  scalar amplitude was discussed. Using the well known  $f_2(1270)$  tensor meson in the crossing channel as the base resonance, from the  $D^+ \rightarrow \pi^-\pi^+\pi^+$  decay, the  $\delta(s_{13})$  phase motion was extracted. We obtain a  $\delta(s_{13})$  variation of about  $180^\circ$ , which is the



**FIGURE 2.** Phase motion vs  $s_{13}$ , with errors shown (systematic and statistical in quadrature). The continuous line is the Breit-Wigner phase motion, with the E791 parameters for the  $\sigma(500)$ .

expected behavior of a resonant amplitude. This result supports the interpretation of the  $\sigma(500)$  as a true resonance, in agreement with the conclusions from our previous analysis of full  $D^+ \rightarrow \pi^- \pi^+ \pi^+$  Dalitz-plot. We could not extract the  $\sigma$  pole position with this method due to the limited statistics. The measurement of the correct  $\sigma$  pole position from a full Dalitz-plot analysis needs the correct functional form (theorists should agree on what it is). Whether the  $\sigma$  pole is the same in charm decay and scattering remains an open question.

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