Fermilab FERMILAB-Pub-02/406 January 2004

CLNS 02/1790
SLAC-PUB-9230
hep-ph/0205274

May 24, 2002

Constraints on Light Bottom Squarks from Radiative B-Meson Decays

Thomas Becher
Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309, USA

Stephan Braig, Matthias Neubert
Newman Laboratory of Elementary-Particle Physics, Cornell University
Ithaca, NY 14853, USA

Alexander L. Kagan[1]
Fermi National Accelerator Laboratory
Batavia, IL 60510, USA

Abstract

The presence of a light $\tilde{b}$ squark ($m_{\tilde{b}} \sim 4\,\mathrm{GeV}$) and gluino ($m_{\tilde{g}} \sim 15\,\mathrm{GeV}$) might explain the observed excess in b-quark production at the Tevatron. Though provocative, this model is not excluded by present data. The light supersymmetric particles can induce large flavor-changing effects in radiative decays of B mesons. We analyse the decays $B \to X_s\gamma$ and $B \to X_{sg}$ in this scenario and derive restrictive bounds on the flavor-changing quark-squark-gluino couplings.

[1]On leave from: Department of Physics, University of Cincinnati, Cincinnati, Ohio 45221, USA

1 Motivation

The measured b-quark production cross section at hadron colliders exceeds next-to-leading order (NLO) QCD predictions by more than a factor of two. While it is conceivable that this discrepancy is due to higher-order corrections, the disagreement is surprising since NLO calculations have been reliable for other processes in this energy range. Berger et al. have analysed b-quark production in the context of the Minimal Supersymmetric Standard Model (MSSM) and find that the excess in the cross section could be attributed to gluino pair-production followed by gluino decay into pairs of b quarks and $\tilde{b}$ squarks, if both the gluino and the $\tilde{b}$ squark are sufficiently light [1]. In order to reproduce the transverse-momentum distribution of the b quarks, the masses of the gluino and light $\tilde{b}$-squark mass eigenstate should be in the range $m_{\tilde{g}} = 12$–$16\,\text{GeV}$ and $m_{\tilde{b}} = 2$–$5.5\,\text{GeV}$. The masses of all other supersymmetric (SUSY) particles are assumed to be large, of order several hundred GeV, so as to have evaded detection at LEP2. Interestingly, a renormalization-group analysis in the framework of the unconstrained MSSM shows that a light $\tilde{b}$ squark is most natural if it is accompanied by a light gluino with mass of order $10\,\text{GeV}$ [2].

Berger et al. have further observed that a light $\tilde{b}$ squark could have escaped direct detection. For example, the additional contribution to the $e^+e^- \to$ hadrons cross section at large energy would only be about 2% and hence difficult to disentangle. The pair-production of light scalars would alter the angular distribution of hadronic jets in e^+e^- collisions, but the present data are not sufficiently precise to rule out the existence of this effect [1]. On the other hand, there are important Z-pole constraints on the parameters of this model. Most importantly, production of the light $\tilde{b}$ squark at the Z pole has to be suppressed, which implies a stringent constraint on the mixing angle θ relating the sbottom mass and weak eigenstates [3]. More recently, several authors have studied loop effects of the light SUSY particles on electroweak precision measurements [4, 5, 6], finding potentially large contributions to the quantity R_b. However, a conflict with existing data can be avoided by having some of the superpartner masses near current experimental bounds, or by allowing for a new CP-violating phase in the SUSY sector [6].

The null result of a CLEO search for the semileptonic decays $\tilde{B} \to D^{(*)} l \pi$ and $\tilde{B} \to D^{(*)} l \tilde{\chi}^0$ of sbottom hadrons implies that the branching ratios for the decays $\tilde{b} \to c\, l$ induced by R-parity violating couplings, or $\tilde{b} \to c\, l \chi^0$ with an ultra-light neutralino χ^0, must be highly suppressed [7]. However, a light $\tilde{b}$ squark would be allowed to decay promptly via hadronic R-parity violating couplings in the modes $\tilde{b} \to \bar{c}\bar{q}$ or $\tilde{b} \to \bar{u}\bar{q}$ (with $q = u, s$). Alternatively, it could be long-lived, forming $\tilde{b}$-hadrons. An interesting consequence of hadronic R-parity violating decays would be the abundant production of light baryons. This could significantly alter the thrust-axis angular distribution for continuum events containing baryons at the B factories.

A striking manifestation of the light $\tilde{b}$-squark scenario would be the production of like-sign charged B mesons at hadron colliders, because the Majorana nature of the gluino allows for the production of $bb\tilde{b}^*\tilde{b}^*$ and $\bar{b}\bar{b}\tilde{b}\tilde{b}$ final states [1]. Another potential signature at hadron colliders is an enhanced yield of $t\bar{t}b\bar{b}$ events [8]. It has also been pointed out that sbottom pairs would be copiously produced in $\Upsilon(nS) \to \tilde{b}\tilde{b}^*$ and $\chi_{bJ} \to \tilde{b}\tilde{b}^*$ decays [9, 10]. Precise measurements of bottomonium decays could lead to new constraints on the squark and gluino masses.

The presence of light SUSY particles alters the running of α_s, and it is often argued that this would exclude the existence of light gluinos. This argument is incorrect. First, a gluino with mass $m_{\tilde{g}} \sim 15\,\text{GeV}$ would have a relatively small effect on the evolution of α_s. Taking, for instance, $\alpha_s(m_b) = 0.21$ (a value in agreement with all low-energy determinations of the QCD coupling) and including the contribution of the gluino octet to the β function above the scale $\mu_{\tilde{g}} = m_{\tilde{g}}$ yields $\alpha_s(m_Z) = 0.126$, which is about three standard deviations higher than the canonical value $\alpha_s(m_Z) = 0.118 \pm 0.003$. However, considering that at leading order only virtual gluino pairs contribute to the β function, a more realistic treatment would include the gluino contribution above a scale $\mu_{\tilde{g}} = 2m_{\tilde{g}} \sim 30\,\text{GeV}$, in which case $\alpha_s(m_Z) = 0.121$, in good agreement with the standard value. Secondly, it is important to realize that even a value of $\alpha_s(m_Z)$ significantly above 0.118 would not rule out the model, the reason being that the characteristic scale μ inherent in all determinations of $\alpha_s(\mu)$ is typically much smaller than the total energy. This is true, in particular, for the determinations based on event-shape variables. In practice, the measurements fix $\alpha_s(\mu)$ somewhere between a fraction of the Z mass down to several GeV, where the gluino contribution to the β function is negligible. Using these determinations to quote values of $\alpha_s(m_Z)$ (as is routinely done) assumes implicitly that the coupling runs as predicted in the SM. Finally, a careful analysis of the running of α_s in the presence of light SUSY particles would have to include, for each observable, the modifications in the theoretical formulae due to virtual and real emissions of the new particles. These corrections could be significant, and could partially compensate effects arising from the modification of the β function.

If we are to take the possibility of a light $\tilde{b}$ squark and light gluinos seriously, then the theoretical study of their impact must be extended to the phenomenology of weak decays of the b quark. New sources of flavor violation arise from s-$\tilde{b}$-$\tilde{g}$ and d-$\tilde{b}$-$\tilde{g}$ couplings. The overall scale of SUSY flavor-changing interactions originating from gluino exchange is set by the factor $g_s^2/m_{\tilde{g}}^2$, which is much larger than the corresponding factor $G_F \sim g_W^2/m_W^2$ for weak decays in the Standard Model (SM). Consequently, the new flavor-changing couplings must be much smaller than the CKM mixing angles in order for this model to be phenomenologically viable. The most stringent bounds arise from the radiative decay $B \to X_s\gamma$, which we discuss in the present work. (Contributions of light $\tilde{b}$ squarks to kaon decays, K-$\bar{K}$ mixing, and D-$\bar{D}$ mixing are strongly suppressed.) The presence of such tight bounds implies stringent constraints on model building.

If the light $\tilde{b}$ squark is sufficiently light to be pair-produced in b decays, new unconventional decay channels would be opened up, which could affect the phenomenology of B mesons and beauty baryons. Examples of potentially interesting consequences include modifications of beauty lifetime ratios, an enhancement of the semileptonic branching ratio of B mesons via production of charmless final states containing $\tilde{b}$ squarks, an enhancement of $\Delta\Gamma(B)$ and of the semileptonic CP asymmetry A_{SL}, and wrong-sign kaon production via $b \to \bar{s}\tilde{b}\tilde{b}$ transitions allowed by the Majorana nature of the gluino. The phenomenology of such effects will be discussed elsewhere. If the light $\tilde{b}$ squark is too heavy to be pair-produced, it would still give rise to potentially large virtual effects in B decays. Their study is the main purpose of this Letter.

2 The low-energy effective $\Delta B = 1$ Hamiltonian

We denote by $\tilde{d}_i$ with $i = 1,..,6$ the down-squark mass eigenstates, and by $\tilde{q}_L$ and $\tilde{q}_R$ with $q = d, s, b$ the interaction eigenstates (the superpartners of the left-handed and right-handed down quarks). They are related by a unitary transformation $\tilde{q}_L = \Gamma_{qi}^{L\dagger}\,\tilde{d}_i$ and $\tilde{q}_R = \Gamma_{qi}^{R\dagger}\,\tilde{d}_i$. We identify $\tilde{d}_3$ with the light sbottom mass eigenstate and define a sbottom-sector mixing angle θ through $\Gamma_{b3}^R = \cos\theta$ and $\Gamma_{b3}^L = \sin\theta$. The fact that the light $\tilde{b}$ squarks are not produced in Z decays implies

$$\sin\theta \approx \pm\sqrt{\frac{2}{3}}\,\sin\theta_W \approx \pm\sqrt{\frac{2}{3}\left(1 - \frac{m_W^2}{m_Z^2}\right)}\,. \tag{1}$$

The phenomenologically favored range for the $Z\tilde{d}_3\tilde{d}_3$ coupling is $|\sin\theta| = 0.3$–0.45 [3], meaning that the light sbottom is predominantly the superpartner of the right-handed bottom quark. In our numerical analysis we will assume a vanishing tree-level coupling to the Z and thus use $\sin\theta = \pm 0.395$.

The flavor-changing couplings involving the light $\tilde{b}$ and $\tilde{g}$ fields can be parameterized by dimensionless quantities

$$\epsilon_{qb}^{AB} = \Gamma_{q3}^{A\dagger}\,\Gamma_{b3}^{B}\,, \qquad (\text{with } \epsilon_{qb}^{AL} = \epsilon_{qb}^{AR}\,\tan\theta) \tag{2}$$

where $A, B = L, R$, and $q = s$ or d for $b \to s$ or $b \to d$ transitions, respectively. In general the parameters ϵ_{qb}^{AB} are complex, which can lead to new CP-violating effects. These parameters are invariant under a phase redefinition of the light $\tilde{b}$-squark state, and they transform in the same way as the products $V_{iq}^* V_{ib}$ (with $i = u, c, t$) of CKM matrix elements under a phase redefinition of the down-type quark fields. It follows that ratios of the type $\epsilon_{qb}^{AB}/\epsilon_{qb}^{CD}$ and $\epsilon_{qb}^{AB}/(V_{iq}^* V_{ib})$ are invariant under phase redefinitions, and thus can carry an observable, CP-violating phase.

Flavor-changing hadronic processes in the model with a light gluino and a very light $\tilde{b}$ squark are most transparently described by means of an effective "weak" Hamiltonian. If we neglect effects that are suppressed by inverse powers of the heavy SUSY scale, the relevant energy scales are the electroweak scale, at which the usual SM flavor-changing operators are generated by integrating out the top quark and the W and Z bosons, and the scale $m_{\tilde{g}}$, at which new flavor-changing operators are generated by integrating out the gluinos. We start by discussing the construction of the effective theory below the gluino scale, focusing on the new interactions proportional to ϵ_{qb}^{AB} induced by gluino exchange, as illustrated in Figure 1. SUSY modifications of the renormalization-group (RG) evolution of the standard weak-interaction operators will be discussed later. The remaining light degrees of freedom in the low-energy theory are the quarks u, d, s, c, b, the photon and

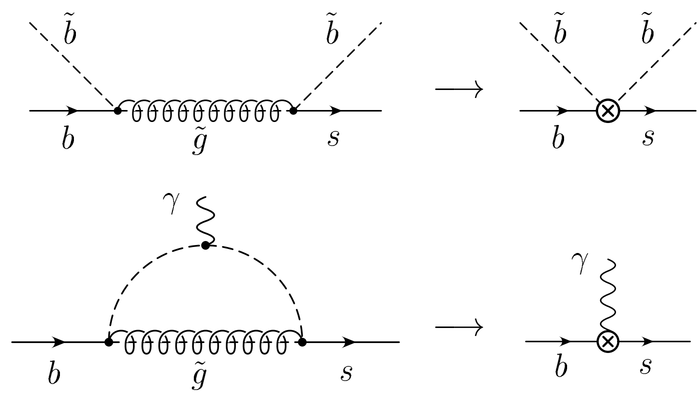


Figure 1: Examples of $b \to s$ transitions induced by gluino exchange. The diagrams on the right show the corresponding contributions in the effective theory where the gluinos are integrated out.

gluons, and the light $\tilde{b}$ squark. Operators in the effective Hamiltonian can be organized in an expansion in inverse powers of the gluino mass. For $m_{\tilde{g}} \approx 15\,\text{GeV} \gg m_b$, it is a good approximation to keep the leading terms in this expansion, which have mass dimension five. These operators comprise the usual electromagnetic and chromomagnetic dipole operators, and new operators containing two scalar $\tilde{b}$ fields. The effective Hamiltonian for $b \to s$ transitions is (here and below, $m_{\tilde{g}} \equiv m_{\tilde{g}}(m_{\tilde{g}})$ denotes the running gluino mass at the gluino matching scale)

$$H_{\text{eff}}^{\text{SUSY}} = \frac{4\pi\alpha_s(m_{\tilde{g}})}{m_{\tilde{g}}} \sum_i \mathcal{C}_i(\mu) \left[\epsilon_{sb}^{LR}\, O_i^{LR}(\mu) + (L \leftrightarrow R) \right] + O(1/m_{\tilde{g}}^2)\,, \tag{3}$$

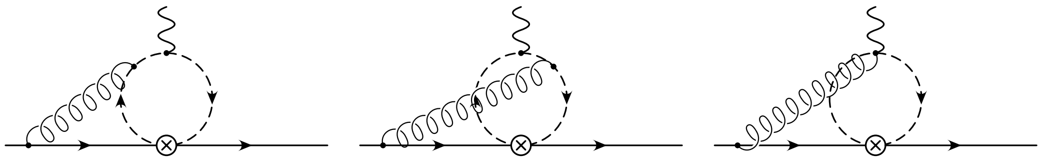
where

$$\begin{aligned} O_1^{LR} &= \bar{s}_L t_a \tilde{b}\, \tilde{b}^* t_a b_R\,, & O_2^{LR} &= \bar{s}_L \tilde{b}\, \tilde{b}^* b_R\,, \\ O_7^{LR} &= -\frac{e}{16\pi^2}\, \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R\,, & O_8^{LR} &= -\frac{g_s}{16\pi^2}\, \bar{s}_L \sigma_{\mu\nu} G^{\mu\nu} b_R\,. \end{aligned} \tag{4}$$

We work with the covariant derivative $iD^\mu = i\partial^\mu + eQ_d A^\mu + g_s A_a^\mu t_a$, where $Q_d = -\frac{1}{3}$ is the electric charge of a down-type (s)quark. (To facilitate comparison with the literature, which usually adopts the opposite sign convention for the couplings, we have included a factor of -1 in the definition of the dipole operators O_7 and O_8.) In addition, there are dimension-five fermion-number violating interactions of the form $\bar{s}^c(1 \pm \gamma_5) b\, \tilde{b}^* \tilde{b}^*$, which mediate $b \to \bar{s}\tilde{b}\tilde{b}$ transitions. They are irrelevant to our discussion here.

The Wilson coefficients at a scale $\mu \sim m_{\tilde{g}}$ are obtained by matching the effective theory to the full theory. At leading order we find

$$\mathcal{C}_1(m_{\tilde{g}}) = 2\,, \quad \mathcal{C}_2(m_{\tilde{g}}) = 0\,, \quad \mathcal{C}_7(m_{\tilde{g}}) = Q_d\, \frac{N^2-1}{4N}\,, \quad \mathcal{C}_8(m_{\tilde{g}}) = -\frac{N^2+1}{4N}\,, \tag{5}$$



		$O_1 \to O_7$	$O_2 \to O_7$	$O_1 \to O_8$	$O_2 \to O_8$
D_1	$-\frac{1}{8\epsilon^2}+\frac{3}{16\epsilon}$	$Q_d\,(-\frac{1}{4}+\frac{1}{4N^2})$	$Q_d\,(\frac{N}{2}-\frac{1}{2N})$	$\frac{1}{4N^2}$	$\frac{N}{2}-\frac{1}{2N}$
D_2	$-\frac{1}{8\epsilon^2}+\frac{3}{16\epsilon}$	$Q_d\,(-\frac{1}{4}+\frac{1}{4N^2})$	$Q_d\,(\frac{N}{2}-\frac{1}{2N})$	$\frac{1}{4}+\frac{1}{4N^2}$	$-\frac{1}{2N}$
D_3	$\frac{1}{4\epsilon^2}-\frac{1}{8\epsilon}$	$Q_d\,(-\frac{1}{4}+\frac{1}{4N^2})$	$Q_d\,(\frac{N}{2}-\frac{1}{2N})$	$\frac{1}{8}+\frac{1}{4N^2}$	$\frac{N}{4}-\frac{1}{2N}$

Figure 2: Two-loop diagrams relevant to the mixing of $O_{1,2}$ into $O_{7,8}$, and corresponding results, in units of $\alpha_s/4\pi$, after the subtraction of subdivergences. Mirror-symmetric graphs with the gluon attached to the s-quark line give identical contributions. Results in the first column of the table have to be multiplied by the color and charge factors in the remaining columns.

where $N = 3$ is the number of colors. In order to use the effective Hamiltonian for calculating B-decay amplitudes, we compute the values of the Wilson coefficients at a low scale $\mu \sim m_b$ by solving the RG equation $(d/d\ln\mu - \boldsymbol{\gamma}^T)\,\vec{C}(\mu) = 0$. At leading order, the anomalous dimension matrix $\boldsymbol{\gamma}$ receives contributions from the one-loop mixing of the operators (O_1, O_2) and (O_7, O_8) among themselves, and from the two-loop mixing of $O_{1,2}$ into $O_{7,8}$. This is analogous to the case of the SM, in which one needs to consider the two-loop mixing of the current-current operators into the dipole operators at leading order [11]. In our case, only the three two-loop diagrams shown in Figure 2 give a nonvanishing contribution. All other graphs vanish after their subdivergences are removed. The calculation of the UV divergences of these diagrams can be reduced to the evaluation of massive tadpole integrals [12]. The resulting anomalous dimension matrix in the operator basis (O_1, O_2, O_7, O_8) reads

$$\boldsymbol{\gamma} = \frac{\alpha_s}{4\pi}\begin{pmatrix} -6N+\frac{9}{N} & -\frac{3}{2}+\frac{3}{2N^2} & Q_d\,(\frac{1}{4}-\frac{1}{4N^2}) & -\frac{1}{8}-\frac{1}{4N^2} \\ -6 & -3N+\frac{3}{N} & Q_d\,(-\frac{N}{2}+\frac{1}{2N}) & -\frac{N}{4}+\frac{1}{2N} \\ 0 & 0 & N-\frac{1}{N} & 0 \\ 0 & 0 & Q_d\,(4N-\frac{4}{N}) & N-\frac{5}{N} \end{pmatrix} + O(\alpha_s^2)\,. \qquad (6)$$

The scale dependence of the Wilson coefficients is now readily obtained by solving the RG equation. Setting $N=3$, we find

$$\begin{aligned}
\mathcal{C}_1(\mu) &= \frac{16}{9}\,\eta^{-8} + \frac{2}{9}\,\eta^{-7/2}\,, \qquad \mathcal{C}_2(\mu) = \frac{8}{27}\left(\eta^{-8} - \eta^{-7/2}\right),\\
\mathcal{C}_7(\mu) &= Q_d\left(-\frac{4}{273}\,\eta^{-8} - \frac{4}{145}\,\eta^{-7/2} + \frac{438}{65}\,\eta^{2/3} - \frac{1224}{203}\,\eta^{4/3}\right),\\
\mathcal{C}_8(\mu) &= \left(\frac{1}{39}\,\eta^{-8} - \frac{1}{60}\,\eta^{-7/2} - \frac{219}{260}\,\eta^{2/3}\right). \qquad (7)
\end{aligned}$$

Here $\eta = [\alpha_s(m_{\tilde g})/\alpha_s(\mu)]^{1/\beta_0(5,0,1)}$, and

$$\beta_0(n_f, n_g, n_s) = \frac{11N}{3} - \frac{2}{3}\,n_f - \frac{2N}{3}\,n_g - \frac{1}{6}\,n_s \qquad (8)$$

is the first coefficient of the generalized QCD β function in the presence of n_f light Dirac fermions, n_g light gluino octets, and n_s light complex scalars. Numerical results for the coefficients $\mathcal{C}_i(\mu)$ will be given in Table 1 below. The scale dependence of the Wilson coefficients below the scale $m_{\tilde g}$ arises mainly from the mixing of O_1 with O_2 and O_7 with O_8. The mixing of O_1 and O_2 into the dipole operators turns out to be small numerically.

The presence of light SUSY particles also affects the RG evolution of the SM contributions to the effective weak Hamiltonian below the electroweak scale. We will now discuss these effects for the operators of relevance to radiative B decays.

3 The radiative decay $B \to X_s\gamma$

The inclusive radiative decay $B \to X_s\gamma$ is one of the most sensitive probes of physics beyond the SM. Indeed, we will see that this decay provides very stringent bounds on the flavor-changing couplings ϵ_{sb}^{LR} and ϵ_{sb}^{RL}. The SM prediction for the $B \to X_s\gamma$ decay rate is known at NLO [13, 14, 15, 16] and, within errors, agrees with the data. The change of this prediction due to the light SUSY particles present in our model is fourfold:

1. The main effects are the genuine SUSY flavor-changing interactions due to quark-squark-gluino couplings. For $\mu < m_{\tilde g}$, these interactions are described by the effective Hamiltonian constructed in the previous section.

2. Even in the absence of flavor-changing couplings in the SUSY sector (i.e., for vanishing ϵ_{qb}^{AB}), the SM operator basis gets enlarged by dimension-six penguin operators with field content $\bar s b\,\tilde b^* i\overleftrightarrow{D}^\mu \tilde b$ and $\bar s b\,\bar{\tilde g}\tilde g$. These new operators have small Wilson coefficients and yield a negligible contribution to the $B \to X_s\gamma$ decay rate. In our analysis, we will neglect these as well as all four-quark penguin operators.

3. The presence of a light gluino octet (and, to lesser extent, of a light $\tilde b$ scalar) modifies the running of the strong coupling constant. We use the two-loop expression for $\alpha_s(\mu)$, modified to account for the effects of the light SUSY particles.

4. There is a SUSY contribution to the b-quark wave-function renormalization, which adds to the anomalous dimensions of the SM operators. Also, the masses of the b quark and the gluino mix under renormalization, thus altering the scale dependence of $m_b(\mu)$. This is a novel effect due to the decoupling of the heavy $\tilde{b}_H$ squark at the SUSY scale.

The last two effects change the evolution of the Wilson coefficients of the SM operators between the electroweak scale and the scale $\mu \sim m_{\tilde{g}}$, where the gluino degrees of freedom are integrated out. (Beyond leading order, the anomalous dimensions of the SM operators are also changed due to internal loops involving SUSY particles.)

The effective weak Hamiltonian governing $B \to X_s\gamma$ decays in the SM is

$$H_{\rm eff}^{\rm weak} = -\frac{4G_F}{\sqrt{2}}\, V_{ts}^* V_{tb} \sum_i C_i(\mu_b)\, Q_i(\mu_b)\,. \tag{9}$$

The operators relevant to our calculation are

$$Q_1 = \bar{s}_L^i \gamma_\mu c_L^j\, \bar{c}_L^j \gamma^\mu b_L^i\,, \quad Q_2 = \bar{s}_L \gamma_\mu c_L\, \bar{c}_L \gamma^\mu b_L\,, \quad Q_{7\gamma} = m_b O_7^{LR}\,, \quad Q_{8g} = m_b O_8^{LR}\,. \tag{10}$$

To an excellent approximation, the contributions of other operators can be neglected. To obtain the values of the corresponding Wilson coefficients $C_i(\mu_b)$ in our model, we first evolve them from the electroweak scale $\mu = m_W$ down to a scale $\mu_{\tilde{g}} \sim m_{\tilde{g}}$, and in a second step from $\mu_{\tilde{g}}$ to a scale $\mu_b \sim m_b$.

Above the gluino scale, there are SUSY contributions to the wave-function renormalization constants of left- and right-handed b-quark fields from gluino-squark loops. At one-loop order, we obtain the gauge-independent results

$$\delta Z_2(b_L) = -\frac{C_F \alpha_s}{4\pi\epsilon}\, \sin^2\theta\,, \qquad \delta Z_2(b_R) = -\frac{C_F \alpha_s}{4\pi\epsilon}\, \cos^2\theta\,. \tag{11}$$

Next, by calculating the self-energies of b quarks and gluinos we find that their masses mix under renormalization. The corresponding anomalous dimension matrix in the basis $(m_b, m_{\tilde{g}})$, defined such that $dm_i/d\ln\mu = -(\boldsymbol{\gamma}_m)_{ij}\, m_j$, reads

$$\boldsymbol{\gamma}_m = \frac{\alpha_s}{4\pi} \begin{pmatrix} \frac{5N}{2} - \frac{5}{2N} & \left(N - \frac{1}{N}\right)\sin 2\theta \\ \sin 2\theta & 6N - \frac{1}{2} \end{pmatrix} + O(\alpha_s^2)\,. \tag{12}$$

Note that the off-diagonal entries are sensitive to the sign of the mixing angle θ. The RG evolution of the operators Q_i in (9) is complicated by the effects of mass mixing. To compute the resulting modifications of the Wilson coefficients we work in the extended operator basis $(Q_1, Q_2, m_b O_7^{LR}, m_b O_8^{LR}, m_{\tilde{g}} O_7^{LR}, m_{\tilde{g}} O_8^{LR})$. Using (11), (12), and the well-known anomalous dimensions of the SM operators [11], we obtain for the anomalous dimension matrix $\boldsymbol{\gamma}_Q = \frac{\alpha_s}{4\pi}\,\boldsymbol{\gamma}_Q^{(0)} + O(\alpha_s^2)$, where (setting $N=3$ and $Q_d = -\frac{1}{3}$)

$$\boldsymbol{\gamma}_Q^{(0)} = \begin{pmatrix} \frac{4}{3}\sin^2\theta - 2 & 6 & 0 & 3 & 0 & 0 \\ 6 & \frac{4}{3}\sin^2\theta - 2 & \frac{416}{81} & \frac{70}{27} & 0 & 0 \\ 0 & 0 & \frac{32}{3} - \frac{4}{3}\sin^2\theta & 0 & \frac{8}{3}\sin 2\theta & 0 \\ 0 & 0 & -\frac{32}{9} & \frac{28}{3} - \frac{4}{3}\sin^2\theta & 0 & \frac{8}{3}\sin 2\theta \\ 0 & 0 & \sin 2\theta & 0 & \frac{43}{2} - \frac{4}{3}\sin^2\theta & 0 \\ 0 & 0 & 0 & \sin 2\theta & -\frac{32}{9} & \frac{121}{6} - \frac{4}{3}\sin^2\theta \end{pmatrix}. \tag{13}$$

The solution of the RG equation in this basis yields coefficients $(c_1, c_2, c_3, c_4, c_5, c_6)$ at a scale between m_W and $m_{\tilde{g}}$. Their initial values at the electroweak scale are given by $(0, 1, C_{7\gamma}(m_W), C_{8g}(m_W), 0, 0)$. The relevant β-function coefficient in this range is $\beta_0(5,1,1)$. From these solutions, we obtain the SM Wilson coefficients at the scale $\mu_{\tilde{g}} \sim m_{\tilde{g}}$ by means of the relations $C_{1,2}(\mu_{\tilde{g}}) = c_{1,2}(\mu_{\tilde{g}})$ and

$$C_{7\gamma}(\mu_{\tilde{g}}) = c_3(\mu_{\tilde{g}}) + \frac{m_{\tilde{g}}(\mu_{\tilde{g}})}{m_b(\mu_{\tilde{g}})}\,c_5(\mu_{\tilde{g}})\,, \qquad C_{8g}(\mu_{\tilde{g}}) = c_4(\mu_{\tilde{g}}) + \frac{m_{\tilde{g}}(\mu_{\tilde{g}})}{m_b(\mu_{\tilde{g}})}\,c_6(\mu_{\tilde{g}})\,. \tag{14}$$

The sign of the coefficients $c_{5,6}$ depends on the sign of the mixing angle θ. At leading order, the running b-quark mass at the gluino scale is obtained from $m_b(\mu_{\tilde{g}}) = m_b(m_b)\,[\alpha_s(\mu_{\tilde{g}})/\alpha_s(m_b)]^{4/\beta_0(5,0,1)}$.

Once we have determined the SM contributions to the Wilson coefficients at the scale $\mu_{\tilde{g}}$, their evolution down to lower scales is governed by the well-known evolution equations of the SM. The corresponding 4×4 anomalous dimension matrix coincides with the upper left 4×4 corner of the extended matrix in (13) evaluated at $\theta = 0$. The resulting formulae are more complicated than in the SM, because in our case the coefficient $C_1(\mu_{\tilde{g}})$ does not vanish at the matching scale (whereas $C_1(m_W) = 0$ for the standard evolution).

Table 1 shows the results for the Wilson coefficients at different values of the renormalization scale. (The coefficient C_1 does not enter the $B \to X_s\gamma$ branching ratio and is omitted here.) The values of $C_{7\gamma}$ and C_{8g} depend on the sign of the mixing angle θ, although this effect is numerically small. For comparison, the values obtained at $\mu = m_b$ in the SM (using $\alpha_s(m_Z) = 0.118$) are $C_2 \simeq 1.12$, $C_{7\gamma} \simeq -0.32$ and $C_{8g} \simeq -0.15$. They are very close to the values found in the presence of the light SUSY particles. In addition, there are the extra contributions proportional to the new coefficients $\mathcal{C}_i$. The second column in the table shows the running b-quark mass at the various scales. Note that the value of m_b above the gluino scale is very sensitive to the sign of θ. The result $m_b(m_W) = 2.75\,\text{GeV}$ corresponding to positive θ appears to be favored by the DELPHI measurement $m_b(m_W) = (2.67 \pm 0.50)\,\text{GeV}$ obtained from three-jet production of heavy quarks at LEP [17]. However, here a similar comment as in our discussion of the running of α_s applies, namely that the DELPHI analysis implicitly assumes that m_b runs as predicted in the SM.

Table 1: Results for the Wilson coefficients and the running b-quark mass for different values of μ. Input parameters are $m_b(m_b) = 4.2\,\text{GeV}$, $m_t(m_W) = 174\,\text{GeV}$, $m_{\tilde{g}}(m_{\tilde{g}}) = 15\,\text{GeV}$, and $\alpha_s(m_b) = 0.21$. In the upper portion of the table the gluino is integrated out at $\mu = m_{\tilde{g}}$, in the lower portion at $\mu = 2m_{\tilde{g}}$. If two signs are shown, the upper (lower) one refers to positive (negative) mixing angle θ.

Scale	$m_b(\mu)$ [GeV]	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_7$	$\mathcal{C}_8$	C_2	$C_{7\gamma}$	C_{8g}
m_W	3.17 ∓ 0.42	—	—	—	—	1	-0.195	-0.097
$m_{\tilde{g}}$	3.59	2	0	-0.222	-0.833	1.040	-0.255 ± 0.028	-0.124 ± 0.014
m_b	4.20	2.691	0.066	-0.264	-0.804	1.104	-0.313 ± 0.023	-0.143 ± 0.011
m_W	3.13 ∓ 0.25	—	—	—	—	1	-0.195	-0.097
$m_{\tilde{g}}$	3.59	2.274	0.025	-0.241	-0.821	1.042	-0.255 ± 0.015	-0.124 ± 0.008
m_b	4.20	3.064	0.104	-0.280	-0.791	1.106	-0.313 ± 0.013	-0.143 ± 0.006

We are now ready to present our results for the $B \to X_s\gamma$ decay rate, including both the SM and the new SUSY flavor-changing contributions. It is convenient to define new coefficients

$$
\begin{aligned}
C_7^{LR}(\mu) &= C_{7\gamma}(\mu) - \frac{\sqrt{2}\pi\alpha_s(m_{\tilde{g}})}{G_F m_{\tilde{g}}}\,\frac{\epsilon_{sb}^{LR}}{V_{ts}^* V_{tb}}\,\frac{\mathcal{C}_7(\mu)}{m_b(\mu)}\,, \\
C_7^{RL}(\mu) &= -\frac{\sqrt{2}\pi\alpha_s(m_{\tilde{g}})}{G_F m_{\tilde{g}}}\,\frac{\epsilon_{sb}^{RL}}{V_{ts}^* V_{tb}}\,\frac{\mathcal{C}_7(\mu)}{m_b(\mu)}\,,
\end{aligned} \tag{15}
$$

and analogous coefficients C_8^{LR} and C_8^{RL}. These expressions exhibit the general features of our model as described earlier. The SUSY contributions are enhanced relative to the SM contributions by a large factor $\frac{\sqrt{2}\pi\alpha_s(m_{\tilde{g}})}{G_F m_{\tilde{g}} m_b} \approx 10^3$, meaning that the ratio of flavor-changing couplings, $\epsilon_{sb}^{AB}/(V_{ts}^* V_{tb})$, must be highly suppressed so as not to spoil the successful SM prediction for the branching ratio. The resulting leading-order expression for the $B \to X_s\gamma$ decay rate is

$$
\Gamma(B \to X_s\gamma) = \frac{G_F^2\alpha M_b^3 m_b^2(m_b)}{32\pi^4}\,|V_{ts}^* V_{tb}|^2 \left[|C_7^{LR}(\mu_b)|^2 + |C_7^{RL}(\mu_b)|^2\right], \tag{16}
$$

where $\mu_b \sim m_b$ is the renormalization scale. M_b is a low-scale subtracted quark mass, which naturally enters the theoretical description of inclusive B decays once the pole mass is eliminated so as to avoid bad higher-order perturbative behavior.

It is well known that NLO corrections have a significant impact on the $B \to X_s\gamma$ decay rate in the SM, which is largely due to NLO corrections to the matrix elements of the operators Q_i in the effective weak Hamiltonian. (NLO corrections to the Wilson coefficient $C_{7\gamma}$ have a much smaller effect.) In order to capture the bulk of these corrections, we include the $O(\alpha_s)$ contributions to the matrix elements but neglect SUSY NLO corrections to the coefficients C_7^{LR} and C_7^{RL}. We also neglect two-loop contributions to

Table 2: Results for the coefficients B_0 and $A_{1,2}$ for the SM (first row), and for the SUSY scenarios with positive (middle portion) and negative (lower portion) mixing angle θ, for $E_\gamma > 2\,\mathrm{GeV}$. The quoted errors refer to the variations of the theoretical parameters within the ranges specified in the text. The renormalization scale is varied between 2.5 and 7.5 GeV. Other input parameters are $V_{ts}^* V_{tb} = -0.04$, $\alpha_s(m_b) = 0.21$ for the SUSY scenario, and $\alpha_s(m_b) = 0.225$ for the SM.

	Default	$\Delta((m_b^{1S})^3\, m_b^2)$	$\Delta(m_c/m_b)$	$\Delta\mu_b$
$B_0^{\rm SM}$	3.44	± 0.21	∓ 0.11	${}^{+0.09}_{-0.16}$
B_0	2.93	± 0.18	∓ 0.09	${}^{+0.06}_{-0.13}$
$10^{-4}\, A_1$	3.60	0	± 0.05	${}^{+0.12}_{-0.02}$
$10^{-8}\, A_2$	3.12	0	± 0.10	${}^{+0.28}_{-0.12}$
B_0	3.68	± 0.23	∓ 0.11	${}^{+0.04}_{-0.11}$
$10^{-4}\, A_1$	3.19	0	± 0.04	${}^{+0.06}_{-0.01}$
$10^{-8}\, A_2$	2.48	0	± 0.07	${}^{+0.18}_{-0.08}$

the matrix elements involving $\tilde{b}$-squark loops. This is justified because of the relatively large mass of the $\tilde{b}$ squark, and because our two-loop anomalous dimension calculation has shown that there is very little mixing of the squark operators into the dipole operators. At NLO our results become sensitive to the precise definition of the mass parameter M_b, which we identify with the so-called Upsilon mass [18], for which we take the value $m_b^{1S} = 4.72 \pm 0.06\,\mathrm{GeV}$ [19]. (Up to a small nonperturbative contribution, m_b^{1S} is one half of the mass of the $\Upsilon(1S)$ resonance.) We also introduce a cutoff $E_\gamma^{\rm min} = \frac{1}{2}(1-\delta)\, m_b^{1S}$ on the photon energy in the B-meson rest frame, which is required in the experimental analysis of radiative B decays. We then obtain

$$\mathrm{Br}(B \to X_s\gamma)\big|_{E_\gamma > E_\gamma^{\rm min}} = \tau_B\, \frac{G_F^2 \alpha (m_b^{1S})^3 m_b^2(m_b)}{32\pi^4}\, |V_{ts}^* V_{tb}|^2\, K_{\rm NLO}(\delta)\,, \tag{17}$$

where $K_{\rm NLO}(\delta)$ is obtained from the formulae in [15] by obvious modifications to include the effects of the new SUSY contributions, and by a change in some of the NLO terms due to the introduction of the Upsilon mass in (16) [20]. The dependence of the branching ratio on the SUSY flavor-changing couplings can be made explicit by writing

$$\mathrm{Br}(B \to X_s\gamma)\big|_{E_\gamma > E_\gamma^{\rm min}} = 10^{-4}\, B_0(\delta) \Big[1 + A_1(\delta)\, \mathrm{Re}(\epsilon_{sb}^{LR}) + A_2(\delta)\, \big(|\epsilon_{sb}^{LR}|^2 + |\epsilon_{sb}^{RL}|^2 \big) \Big]\,. \tag{18}$$

In Table 2, we give results for the coefficients B_0 and $A_{1,2}$ including the dominant theoretical uncertainties. Following [16], we use a running charm-quark mass in the penguin-loop diagrams rather than the pole mass. This is justified, because the photon-energy cut imposed in the experimental analysis prevents the intermediate charm-quark propagators from being near their mass shell. Specifically, we work with the mass ratio

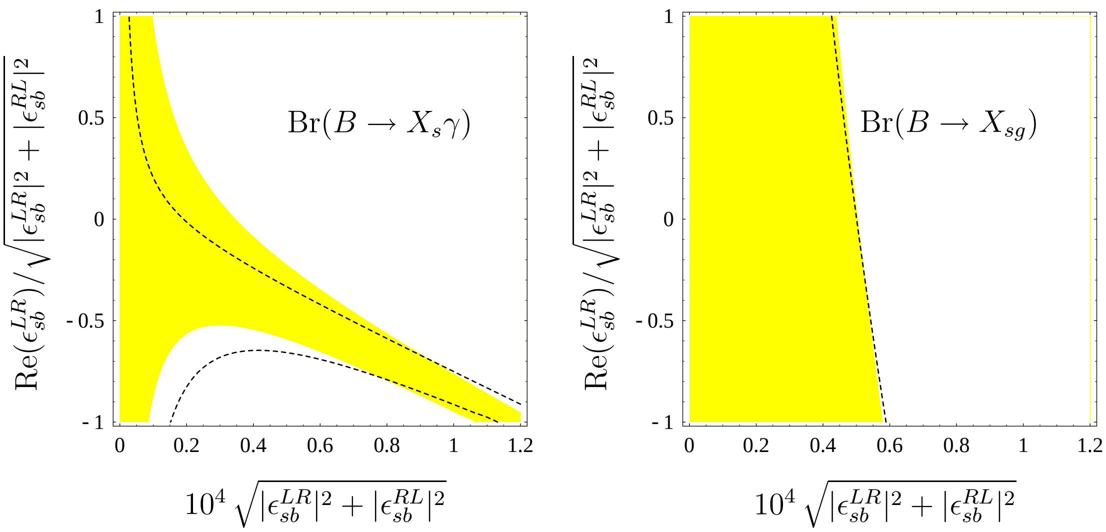


Figure 3: Allowed regions (at 95% c.l.) for the SUSY flavor-changing parameters obtained from the CLEO measurements of the $B \to X_s\gamma$ (left) and $B \to X_{sg}$ (right) branching ratios, using central values for all theory input parameters. The shaded regions correspond to the SUSY model with positive mixing angle θ, the dashed lines refer to negative θ.

$m_c(\mu)/m_b(\mu)$, where the running masses are obtained from $m_c(m_c) = (1.25 \pm 0.10)$ GeV and $m_b(m_b) = (4.20 \pm 0.05)$ GeV.

In the left-hand plot in Figure 3, we confront our theoretical result for the $B \to X_s\gamma$ branching ratio with the CLEO measurement $\mathrm{Br}(B \to X_s\gamma) = (3.06 \pm 0.41 \pm 0.26)\cdot 10^{-4}$ obtained for $E_\gamma > 2$ GeV [21]. (This result actually corresponds to the sum of $B \to X_s\gamma$ and $B \to X_d\gamma$ decays. However, the suppression of the exclusive $B \to \rho\gamma$ decay with respect to the $B \to K^*\gamma$ mode implies that the dominant contribution to the inclusive decay must come from $b \to s\gamma$ transitions. In the context of our model, it follows that the couplings ϵ_{db}^{AB} must obey even tighter constraints than the ϵ_{sb}^{AB}.) It follows that the maximum allowed values of the parameters ϵ_{sb}^{AB} are 10^{-4}, as is already obvious from the magnitude of the coefficients $A_{1,2}$ in Table 2. However, values larger than $5\cdot 10^{-5}$ would require a fine-tuning of the phase of ϵ_{sb}^{LR} and are thus somewhat unnatural. Note that the ratio shown on the vertical axis in the plot is bound to lie between 1 and -1, and in the limit $\epsilon_{sb}^{RL} = 0$ corresponds to $\cos\vartheta_{LR}$, where ϑ_{LR} denotes the CP-violating phase of ϵ_{sb}^{LR}.

The right-hand plot in the figure shows a similar constraint arising from the inclusive charmless decay $B \to X_{sg}$. At leading order, the decay rate for this process is obtained from (16) by the replacements $\alpha \to \frac{4}{3}\,\alpha_s(\mu_b)$ and $C_7^{AB} \to C_8^{AB}$. The allowed region corresponds to the CLEO upper bound of 8.2% (at 95% c.l.) for the $B \to X_{sg}$ branching ratio [22]. There are, however, potentially large theoretical uncertainties in this result, because we neglect NLO corrections to the branching ratio. We therefore refrain from combining the two plots to reduce the allowed parameter space.

In the SM, the direct CP asymmetry in the inclusive decay $B \to X_s\gamma$ is very small, below 1% in magnitude [23]. In the SUSY scenario, on the other hand, the phase of the coupling ϵ_{sb}^{LR} could lead to a large asymmetry. In the approximation where one neglects the SUSY contributions to the CP-averaged decay rate in (16), which is justified in view of the good agreement of the SM prediction with the data, the formulae in [23] yield the prediction $A_{\rm CP} \approx -50\% \times 10^4\,{\rm Im}(\epsilon_{sb}^{LR})$, where we have neglected the small contribution from the charm-quark loops and the yet smaller contribution from $\tilde{b}$-squark loops. (We use the standard phase convention where $\lambda_t = V_{ts}^* V_{tb}$ is real. In general, the CP asymmetry depends on ${\rm Im}(\epsilon_{sb}^{LR}/\lambda_t)$.) It follows that even within the very restrictive bounds shown in Figure 3 there can be a potentially large contribution to the CP asymmetry, which would provide a striking manifestation of physics beyond the SM. In fact, the CLEO bounds $-27\% < A_{\rm CP} < +10\%$ (at 90% c.l.) [24] imply that

$$-2\cdot 10^{-5} < {\rm Im}(\epsilon_{sb}^{LR}) < 5\cdot 10^{-5}\,, \tag{19}$$

placing another tight constraint on the flavor-changing coupling ϵ_{sb}^{LR}.

4 Conclusions

New supersymmetric contributions to b-quark production at hadron colliders can account for the long-standing discrepancy between the measured cross sections and QCD predictions if there is a light $\tilde{b}$ squark with mass in the range 2–5.5 GeV, accompanied by a somewhat heavier gluino [1]. In this Letter, we have explored the phenomenology of rare B decays in such a scenario and have found tight bounds on the flavor-changing parameters controlling supersymmetric contributions to $b \to s$ and $b \to d$ FCNC transitions. The most restrictive constraints arise from virtual effects of light $\tilde{b}$ squarks in $B \to X_s\gamma$ decays. We have analysed this process by constructing a low-energy effective Hamiltonian, in which the gluinos are integrated out, while the $\tilde{b}$ squarks remain as dynamical degrees of freedom. We find that the flavor-changing couplings ϵ_{sb}^{RL} and ϵ_{sb}^{LR} must be of order few times 10^{-5} or less. (Even tighter constraints hold for the analogous $b \to d$ couplings.) This implies that certain off-diagonal entries of the down-squark mass matrix must be suppressed by a similar factor compared to the generic squark-mass squared.

Even with such tight constraints on the couplings, this model allows for interesting and novel New Physics effects in weak decays of B mesons and beauty baryons. As an example, we have discussed the direct CP asymmetry in $B \to X_s\gamma$ decays, which could be enhanced with respect to its Standard Model value by an order of magnitude. Other possible effects include an enhanced $B \to X_{sg}$ decay rate. We have not considered here the possibility of $\tilde{b}$-squark pair-production, which would be kinematically allowed for very light squark masses. The new decay modes $b \to s\tilde{b}\tilde{b}^*$ and $b \to \bar{s}\tilde{b}\tilde{b}$ would affect the decay widths of B mesons and Λ_b baryons differently, and hence might explain the anomaly of the low Λ_b lifetime. We will report on this interesting possibility elsewhere.

Acknowledgments: S.B. and M.N. are supported by the National Science Foundation under Grant PHY-0098631. T.B. is supported by the Department of Energy under Grant DE-AC03-76SF00515, and A.K. under Grant DE-FG02-84ER40153.

References

[1] E. L. Berger et al., Phys. Rev. Lett. **86**, 4231 (2001) [hep-ph/0012001].

[2] A. Dedes and H. K. Dreiner, JHEP **0106**, 006 (2001) [hep-ph/0009001].

[3] M. Carena, S. Heinemeyer, C. E. Wagner and G. Weiglein, Phys. Rev. Lett. **86**, 4463 (2001) [hep-ph/0008023].

[4] J.-j. Cao, Z.-h. Xiong and J. M. Yang, Phys. Rev. Lett. **88**, 111802 (2002) [hep-ph/0111144].

[5] G. C. Cho, preprint hep-ph/0204348.

[6] S.-w. Baek, Phys. Lett. B **541**, 161 (2002) [hep-ph/0205013].

[7] V. Savinov *et al.* [CLEO Collaboration], Phys. Rev. D **63**, 051101 (2001) [hep-ex/0010047].

[8] A. K. Leibovich and D. Rainwater, Phys. Rev. Lett. **88**, 221801 (2002) [hep-ph/0202174].

[9] E. L. Berger and L. Clavelli, Phys. Lett. B **512** (2001) 115 [hep-ph/0105147].

[10] E. L. Berger and J. Lee, Phys. Rev. D **65**, 114003 (2002) [hep-ph/0203092].

[11] M. Ciuchini et al., Phys. Lett. B **316**, 127 (1993) [hep-ph/9307364].

[12] K. Chetyrkin, M. Misiak and M. Münz, Nucl. Phys. B **518**, 473 (1998) [hep-ph/9711266].

[13] C. Greub, T. Hurth and D. Wyler, Phys. Rev. D **54**, 3350 (1996) [hep-ph/9603404].

[14] K. Chetyrkin, M. Misiak and M. Münz, Phys. Lett. B **400**, 206 (1997) [Erratum: *ibid.* B **425**, 414 (1998)] [hep-ph/9612313].

[15] A. L. Kagan and M. Neubert, Eur. Phys. J. C **7**, 5 (1999) [hep-ph/9805303].

[16] P. Gambino and M. Misiak, Nucl. Phys. B **611**, 338 (2001) [hep-ph/0104034].

[17] P. Abreu *et al.* [DELPHI Collaboration], Phys. Lett. B **418**, 430 (1998).

[18] A. H. Hoang, Z. Ligeti and A. V. Manohar, Phys. Rev. Lett. **82**, 277 (1999) [hep-ph/9809423].

[19] M. Neubert and T. Becher, Phys. Lett. B **535**, 127 (2002) [hep-ph/0105217].

[20] Specifically, the expression for the quantity $k_{77}(\delta, \mu_b)$ in eq. (13) of [15] must be replaced by

$$k_{77}(\delta, \mu_b) = S(\delta) \left\{ 1 + \frac{\alpha_s(\mu_b)}{2\pi} \left(r_7 + \gamma_{77} \ln \frac{m_b}{\mu_b} \right) + \frac{2}{3}\, \alpha_s^2(\mu_b) + \frac{\lambda_1 - 9\lambda_2}{2(m_b^{1S})^2} \right\} + \frac{\alpha_s(\mu_b)}{\pi}\, f_{77}(\delta)\,,$$

where $-\lambda_1 \approx (0.25 \pm 0.15)\,\mathrm{GeV}^2$ and $\lambda_2 \approx 0.12\,\mathrm{GeV}^2$ are hadronic parameters.

[21] S. Chen *et al.* [CLEO Collaboration], Phys. Rev. Lett. **87**, 251807 (2001) [hep-ex/0108032].

[22] T. E. Coan *et al.* [CLEO Collaboration], Phys. Rev. Lett. **80**, 1150 (1998) [hep-ex/9710028].

[23] A. L. Kagan and M. Neubert, Phys. Rev. D **58**, 094012 (1998) [hep-ph/9803368].

[24] T. E. Coan *et al.* [CLEO Collaboration], Phys. Rev. Lett. **86**, 5661 (2001) [hep-ex/0010075].