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The New Cosmology: Mid-term Report Card for Inflation

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ABSTRACT

Inflation has been the driving idea in cosmology for two decades and is a pillar of the New Cosmology. The inflationary paradigm has now passed its first round of significant tests, with two of its three basics predictions confirmed at about the 10% level (spatially flat Universe and density perturbations produced from quantum fluctuations with $|n-1| \sim \mathcal{O}(0.1)$). *The Inflationary Paradigm has some of the truth.* Over the next decade the precision of these tests, most of which involve measurements of CMB anisotropy and polarization, will improve 30 fold or more(!), testing inflation more sharply and possibly elucidating the underlying cause. Especially important in this regard is detecting the inflation-produced gravitational waves, either directly or through their CMB polarization signature. While inflation has by no means been verified, its successes have raised the bar for competitor theories: Any alternative must feature the two hallmarks of inflation: superluminal expansion and entropy production.

1 Introduction

Today, cosmology has a comprehensive and self-consistent mathematical model that accounts for all the observed features of the Universe. However, unlike its predecessor, the hot big-bang model or standard cosmology, there is no standard name. For now, I will refer to it as the New Cosmology [1].

The New Cosmology incorporates the hot big-bang model (every successor theory eats its predecessor whole!), as well as an early inflationary epoch and the present stage of accelerated expansion. The New Cosmology includes a full accounting of the shape and composition of the Universe today: spatially flat, with the critical density distributed as follows: baryons ($4 \pm 1\%$), nonbaryonic dark matter ($29 \pm 4\%$) and dark energy ($67 \pm 6\%$) [2].

While we can now say that massive neutrinos account for between 0.1% and 5% [3] of the nonbaryonic dark matter – comparable to the 0.5% contributed by stars – most of the dark matter is thought to be slowly moving elementary particles (cold dark matter), with the leading candidates being the axion and neutralino [5].

Dark energy is the name I use for the mysterious "energy stuff" that dominates the mass-energy budget and whose large negative pressure ($p < -\rho/2$) is responsible for the accelerated expansion [4]. Dark energy could be as "mundane" as the the energy of the quantum vacuum, or something so exotic that it has thus far eluded the minds of the most creative theorists [6]. Dark energy is truly one of the great mysteries in all of science.

The standard cosmology can properly claim to give a reliable and tested description of the Universe from a fraction of a second onward. While the New Cosmology cannot yet make a similar claim for extending our understanding back to an early, inflationary epoch, inflation is nonetheless a pillar of the New Cosmology, and, as I will describe, precision CMB observations are beginning to test its basic predictions (so far, so good). This is a remarkable development that belies the prediction made by some astronomers (and the fear of many inflationary theorists) that inflation would never be tested.

2 The Inflationary Paradigm

Few ideas in theoretical physics have had as much impact as inflation. Introduced by Alan Guth in a 1980 paper [7] that explained the cosmological virtues of exponential expansion as well as why his version of it (based upon a symmetry-breaking phase transition) did *not* work(!), inflation has been the driving idea in cosmology since.

The virtues of inflation trace to its ability to lessen the dependence of the present state of the Universe upon initial conditions (though quantifying how successful it is at achieving this is difficult [8]). In addition to the predictions discussed below, it explains the high degree of homogeneity and isotropy observed in our Hubble volume, the heat of the big bang, and the absence of superheavy magnetic monopoles.

In a flurry of activity during the early 1980s the basic inflation paradigm [9] was worked out [10, 11, 12, 13]. In brief, essentially all models of inflation can be described in terms of the classical evolution of a single scalar field (dubbed the inflaton) initially displaced from the minimum of its potential [$V(\phi)$]:

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0 \qquad (1)$$

$$N \equiv \int H dt = \frac{8\pi}{{m_{\rm Pl}}^2} \int \frac{V(\phi) d\phi}{V'(\phi)} \qquad (2)$$

where N is the number of e-folds of inflation, prime denotes $d/d\phi$ and dot denotes d/dt. Models of inflation differ only in the form of their scalar potential and their motivations.

Inflation occurs while the inflaton is slowly rolling: during the "slow roll," the *nearly* constant potential energy density associated with the inflaton drives a nearly exponential expansion ("superluminal expansion"). At the end of the slow roll, the inflaton is left oscillating about its potential energy minimum; these oscillations correspond to a condensate of zero-momentum ϕ particles. Their decay (by coherent and/or incoherent processes) produces lighter particles which thermalize; this "reheats" the Universe and exponentially increases its entropy (per comoving volume). (Note: The Universe need not be hot before inflation.) The standard hot big bang phase commences thereafter, with the tremendous entropy release accounting its heat.

Quantum fluctuations in ϕ ($\Delta\phi \sim H/2\pi$) give rise to energy density fluctuations ($\delta\rho = \Delta\phi V'$), which ultimately result in inflation's signature adiabatic density perturbations. They are of astrophysical interest because of their physical size is stretched from the subatomic to the astrophysical during the period of exponential expansion. Similarly, quantum fluctuations in the metric itself ($h \sim \delta g \sim H/m_{\rm Pl}$) give rise to a spectrum of gravitational waves with wavelengths of astrophysical interest (fluctuations in other light scalar fields can result in particle production or isocurvature perturbations).

Inflation is a paradigm and not a model because there is no agreed upon identity for the inflaton field. Many models exist, with the inflaton playing a variety of roles, from inducing electroweak symmetry breaking to breaking supersymmetry to the compactification of extra dimension(s), and the energy scale of inflation ranging from 1 TeV to 10^{16} GeV [14]. *While there is no standard model, each model makes its own set of precise predictions.*

That being said, the inflationary paradigm does make a set of generic predictions that can be used to test – and even falsify – it. The basic predictions of inflation are:

- Spatially flat Universe
- Not quite scale-invariant, almost power-law spectrum of Gaussian, adiabatic density perturbations
- Not quite scale-invariant, almost power-law spectrum of gravitational waves

The first two of these predictions are now being tested by precision measurements of cosmic microwave background (CMB) anisotropy on sub-degree angular scales, with early results consistent with inflation. The third prediction, which may hold the key to definitively testing inflation and shedding light upon the underlying cause, is inspiring a new generation of very challenging experiments.

3 Specific models make specific predictions [15]

While all models of inflation are based upon speculative physics that goes beyond (usually well beyond) the standard model of particle physics, each model makes predictions that sharpen the basic predictions of the paradigm. The reason for this is simple: The physics is speculative but the rules are well defined – the semi-classical evolution of a scalar field.

While models have been constructed where two or more fields evolve during inflation, or where the kinetic term for the inflaton is not canonical, I will discuss the predictions for single-field inflation, assuming that the kinetic term is canonical. More complicated models also make definite predictions, though the relationship of observable quantities to the potential(s) can be different.

The prediction of a flat Universe is not tied to the form of the potential; only that inflation lasts sufficiently long to explain the homogeneity and isotropy. Generally speaking, the duration of inflation far exceeds that needed to produce a flat Universe (although models of inflation have been tuned to give Ω_0 less than 1). The inflationary prediction of a flat Universe corresponds to density parameter $\Omega_0 = 1$, where Ω_0 is the ratio of the total matter/energy density to the critical density.

The other two predictions of inflation involve the metric perturbations that arise from the quantum fluctuations associated with deSitter space: adiabatic density (or scalar) perturbations from fluctuations in the inflaton potential energy and gravity waves (tensor perturbations) from fluctuations in the metric itself. Their amplitude and variation with scale do depend upon the properties of the scalar-field potential, and this is the basis for the belief that observations may someday pin down the underlying model of inflation [16]

Both the scalar and tensor perturbations have an approximately – but not exactly – scale-invariant spectrum. This fact fundamentally traces to the approximate deSitter space associated with the inflationary phase. In physical terms, that means that the dimensionless strain amplitude of gravity waves when they re-enter the horizon after inflation is almost independent of scale: $h_{\rm HOR} \sim H/m_{\rm Pl} \sim V^{1/2}/{m_{\rm Pl}}^2$. For density perturbations, it is the amplitude of the density perturbation at horizon crossing that is almost independent of scale: $(\delta\rho/\rho)_{\rm HOR} \sim H^2/\dot{\phi} \sim V^{3/2}/{m_{\rm Pl}}^3 V'$. Because the post-horizon-crossing evolution of both density perturbations and gravity waves depends upon the time elapsed since horizon crossing – which is longer for longer wavelength perturbations – the spectra of density perturbations and gravity waves is not scale invariant today. The Fourier components of both scalar and tensor perturbations are approximately power-law in wavenumber k of the fluctuations; at horizon crossing:

$$
\begin{aligned}
(\delta\rho/\rho)_{\rm HOR} &\propto k^{(n-1)/2} \\
h_{\rm HOR} &\propto k^{n_T/2}
\end{aligned}
$$

where n (n_T) are the power-law indices for scalar (tensor) perturbations.

Both density perturbations and gravity waves lead to fluctuations in the temperature and polarization of the CMB across the sky. Predictions for observable quantities can be expressed in terms of the inflationary potential. For instance, the scalar (S) and the tensor contributions (T) to the CMB quadrupole anisotropy are

$$
S \equiv \frac{5C_2^S}{4\pi} \simeq 2.9 \, \frac{V/{m_{\rm Pl}}^4}{(m_{\rm Pl}V'/V)^2} \tag{3}
$$

$$
T \equiv \frac{5C_2^T}{4\pi} \simeq 0.56(V/{m_{\rm Pl}}^4) \tag{4}
$$

where C_2^S and C_2^T are the contribution of scalar and tensor perturbations to the variance of the $l = 2$ multipole amplitude ($\langle|a_{2m}|^2\rangle = C_2^S + C_2^T$) and V is the value of the inflationary potential when the scale $k = H_0$ (present horizon scale) crossed the Hubble radius during inflation. The numerical coefficients in these expressions depend slightly upon the composition of the Universe; the numbers shown are for $\Omega_M = 0.35$ and $\Omega_\Lambda = 0.65$ [17].

The power-law indices that characterize the scalar and gravity-wave spectra can be expressed in terms of the inflationary potential and its derivatives:

$$n - 1 = -\frac{1}{8\pi}\left(\frac{m_{\rm Pl}V'}{V}\right)^2 + \frac{m_{\rm Pl}}{4\pi}\left(\frac{m_{\rm Pl}V'}{V}\right)' \tag{5}$$

$$n_T = -\frac{1}{8\pi}\left(\frac{m_{\rm Pl}V'}{V}\right)^2 \tag{6}$$

For typical inflationary potentials the deviations from scale invariance are expected to be of order 10%: $|n-1| \sim \mathcal{O}(0.1)$ and $n_T \sim -\mathcal{O}(0.1)$ [18].

Note that the ratio between the gravity-wave contribution to the CMB quadrupole anisotropy and the density-perturbation contribution to the CMB quadrupole anisotropy provides a consistency test if the tensor spectral index can be measured:

$$T/S = -5n_T \tag{7}$$

for $\Omega_M = 0.35$ and $\Omega_\Lambda = 0.65$ [17].

The fluctuation spectra are not exact power laws (except in the case of power-law inflation); variations in the power-law indices with k may be expressed in terms of higher derivatives of $V(\phi)$ [19].

$$\begin{aligned} \frac{dn}{d\ln k} &= -\frac{{m_{\rm Pl}}^2}{8\pi}\left(\frac{V'}{V}\right)\frac{dn}{d\phi} \\ &= -\frac{1}{32\pi^2}\left(\frac{{m_{\rm Pl}}^3 V_*'''}{V_*}\right)\left(\frac{m_{\rm Pl}V_*'}{V_*}\right) \\ &\qquad + \frac{1}{8\pi^2}\left(\frac{{m_{\rm Pl}}^2 V_*''}{V_*}\right)\left(\frac{m_{\rm Pl}V_*'}{V_*}\right)^2 - \frac{3}{32\pi^2}\left(m_{\rm Pl}\frac{V_*'}{V_*}\right)^4 \qquad (8) \\ \frac{dn_T}{d\ln k} &= -\frac{{m_{\rm Pl}}^2}{8\pi}\left(\frac{V'}{V}\right)\frac{dn_T}{d\phi} \\ &= \frac{1}{32\pi^2}\left(\frac{{m_{\rm Pl}}^2 V''}{V}\right)\left(\frac{m_{\rm Pl}V'}{V}\right)^2 - \frac{1}{32\pi^2}\left(\frac{m_{\rm Pl}V'}{V}\right)^4 \qquad (9) \end{aligned}$$

Finally, once the form of the density perturbations and the composition of the Universe are specified, the initial conditions for the formation of structure in the Universe are set. Inflation specifies the form of the density perturbations and the composition of the Universe is known – 29% cold dark matter (with a dash of it in massive neutrinos), 4% baryons and 66% dark energy. This means that the large (and growing) body of observational data that probes the formation of structure in the Universe provides an additional test of the inflationary paradigm. In particular, they have significant potential to determine n and test the Gaussian nature of the underlying density perturbations.

4 Mid-term report card and future expectations

The three basic predictions of inflation blossom into a series of 9 testable consequences, most of which can also probe the underlying inflationary model.

The cosmic microwave background will play the leading in testing these predictions because its anisotropy and polarization are such a clean probe of the gravity waves and density perturbations produced by inflation. The predictions of inflationary models can be stated in terms of the predicted variances of the multipole amplitudes that characterize CMB anisotropy and polarization [20]; because there are only $2\ell + 1$ multipole amplitudes for multipole ℓ, even an ideal experiment is limited by sample size (often referred to as cosmic variance) in estimating the true variance.

The discovery of CMB anisotropy on angular scales of order 10 degrees by the COBE Differential Microwave Radiometer in 1992 [21] opened the door for testing inflation. The measurement of CMB anisotropy on sub-degree angular scales by balloon-borne and ground-based experiments allowed the serious testing to begin (most of the features and probative power lies in the anisotropy on sub-degree scales) [20]. Presently, it is the results of the BOOMERanG, DASI, CBI, Maxima, CAT, and Archeops experiments that define the state-of-the-art in our knowledge of the CMB angular power spectrum. Soon, the MAP all-sky satellite-based experiment will report its first results and really clean up the anisotropy power spectrum out to $\ell \sim 900$. The ESA/NASA Planck satellite is scheduled for launch in 2007; it should provide the definitive power spectrum out to $\ell \sim 3000$ as well as significant results on polarization.

The following are the nine predictions, the present status report and prospects for the future. A Table summary is given at the end of this section.

1. Spatial flatness
This prediction is the most straightforward; it simply implies $\Omega_0 = 1.0 \pm 0.00001$. The '$\pm 0.00001$' arises because fluctuations on the current Hubble scale will led to the knowable part of the Universe appearing slightly open (underdensity) or slightly closed (overdensity).
Now: Measurements of the position of the first acoustic peak in the CMB power spectrum indicate that $\Omega_0 = 1.03 \pm 0.03$, consistent with spatial flatness [22]. Further, a direct accounting of the amount of matter and energy leads to an independent, though less precise, determination of the total mass/energy that is reassuringly consistent with spatial flatness, $\Omega_0 = 1 \pm 0.25$.
Future: The precision testing of this prediction lies with MAP, Planck and other future CMB experiments that will probe the smallest angular scales and fix the positions of the acoustic peaks with high precision. Since the positions of the acoustic peaks also depend upon the composition of the Universe, information from large-scale structure measurements that constrains the matter density is also critical. The ultimate precision to which Ω_0 can be probed will likely be in the range $\sigma_{\Omega_0} \sim 0.005 - 0.001$ and will depend upon how well the composition can be fixed by other independent methods [23, 24].

2. Density Perturbations from Quantum Fluctuations
The most striking prediction of inflation may be that the density perturbations that seeded structure on scales of millions of light years and larger arose from quantum fluctuations on subatomic scales; if this is true, the CMB is a picture of quantum noise(!). This prediction breaks down into 5 separate testable consequences.

a. Acoustic peaks: Since the density perturbations are impressed at very early times ($\ll$ sec), by the time of last scattering ($t \sim 400,000\,\mathrm{yrs}$) all perturbations are purely "growing mode," leading to a synchronizing of the perturbations on different scales. Some modes

were caught at maximum compression or rarefaction, leading to the prediction of a series of "acoustic peaks" in the angular (multipole) power spectrum [20].
Now: Current CMB experiments have probed the angular power spectrum out to $\ell \sim 2000$; at least three and perhaps as many five peaks have been resolved [22]. There is no question that the acoustic peaks associated adiabatic perturbations have been seen.
Future: Planck and other future CMB experiments that probe the smallest angular scales should resolve six or more acoustic peaks (the damping of anisotropy on very small angular scales due to the finite thickness of the last scattering surface exponentially diminishes the amplitude of successive peaks). These experiments will also be able to separate out any small admixture of isocurvature perturbations (which could arise during inflation or later on).

b. Gaussianity: The inflation-produced density perturbations arise from quantum fluctuations in a very weakly coupled (essentially free) scalar field and hence should be Gaussian to a high degree of precision. The CMB has the greatest power to test this prediction since it probes the density perturbations when they were linear (nonGaussianity automatically develops when gravity drives the amplitude of the perturbations into the nonlinear regime).
Now: There is no evidence for nonGaussianity.
Future: The all-sky CMB mapping experiments (MAP and Planck) which are designed to control systematics have the greatest the potential to test this prediction. Important to quantifying how well the Gaussianity prediction is faring is the construction of a realistic model with nonGaussianity to compare with.

c. Almost scale-invariant spectrum: Inflation predicts an almost, but not quite scale invariant spectrum [10]. Typically, the deviations from scale invariance are of order 10%: $|n-1| \sim \mathcal{O}(0.1)$, with indeterminate sign [18]. This is not only a key test of inflation, but a window to the underlying physics.
Now: Measurements of CMB anisotropy at small angular scales can probe this prediction most sharply. Current observations are beginning to significantly constrain n: $n = 1.05 \pm 0.09$ [22], consistent with

inflation but not precise enough to test the inflationary prediction sharply.

Future: The Planck Mission should be able to determine n to a precision of ± 0.008 [23]. An ideal CMB experiment could achieve a precision of almost ten times better [24]; whether or not a future ground-based and space-based experiment with such capability is carried out remains to be seen.

d. Almost power-law: Inflation predicts that the power-law index varies with scale, with $dn/d\ln k \sim \pm 10^{-3}$ for many models and ten times larger for some [19].

Now: Current CMB measurements, $dn/d\ln k = -0.02 \pm 0.04$ [25], are consistent with the inflationary prediction, but lack the precision to measure a variation.

Future: It is likely that the CMB offers is the most powerful probe, with Planck projected to do a factor of ten better than the current limit [23]; an ideal CMB experiment might reach the expected level of variation in n.

e. CDM scenario for structure formation: Nearly scale-invariant, Gaussian adiabatic density perturbations is one of the two pillars of the highly successful CDM paradigm for structure formation (the other being slowly moving, weakly interacting dark matter particles). As such, tests of the CDM paradigm are tests of inflation. In particular, the study of large-scale can constrain n and the abundance of rare objects, such as clusters, can be used to test the Gaussianity prediction.

3. Gravity Waves from Quantum Fluctuations

a. Amplitude: The amplitude of the gravity waves is directly proportional to the energy scale of inflation and does not depend upon the shape of the potential. Unfortunately, theory gives very little advice about the amplitude, and unlike density perturbations which are de rigueur (to seed structure formation) the Universe can live just fine without gravity waves. Huterer and I have explored what can be said about T/S without regard to specific potentials by reformulating the equations of motion for inflation. We concluded that if $n > 0.9$ and the potential has no unnatural features (the key here is the definition of unnatural), T/S must exceed 10^{-3} [26].

Now: There is no evidence for inflation-produced gravity waves; current limits from the CMB imply $T/S < \mathcal{O}(1)$ [27].
Future: There are two basic ways to get at the gravity waves. The first is to use their signature in CMB anisotropy and polarization. Unlike the series of acoustic peaks associated with density perturbations that extend out to $\ell \sim 3000$, gravity waves produce a rather featureless angular power spectrum that dies off at around $\ell \sim 100$. Because the precision of even a perfect CMB experiment is limited by sampling variance, CMB anisotropy alone can separate the gravity-wave signature only if $T/S > 0.1$ [28]. However, the polarization signature is not so limited: gravity waves excite a mode of polarization (B-mode or curl) that density perturbations cannot [29]. The only limitations to detecting gravity waves through their polarization signature are sensitivity and foregrounds. It appears that a dedicated polarization experiment might be able to achieve a sensitivity to T/S as good as 10^{-3} [30] (recall, that if $n > 0.9$ and the potential is "natural", T/S is expected to be this large). By comparison, Planck is expected to achieve $T/S \sim 0.02$ [23].

The CMB is only sensitive to the longest wavelength gravity waves, $\lambda \sim 10^{26}\,$cm to $10^{28}\,$cm; however, the spectrum extends to wavelengths as short as 1 km. Direct detection of inflation-produced gravity waves will be very challenging as $\Omega_{\rm GW}$ is at most 10^{-15} at the mHz-kHz frequencies where the planned detectors will operate: the projected sensitivities of LIGO and LISA to a stochastic background of gravitational waves are 10^{-10} and 10^{-12} respectively [31], far short of what is expected.

b. Spectrum: The predicted spectral index for one-field models is related to the amplitude, $n_T = -\frac{1}{5}\frac{T}{S}$, which allows a consistency test if n_T can be measured. The combination of direct and CMB detections could measure n_T accurately: owing to the long lever-arm between the Hz frequencies of gravity-wave detectors and the very long wavelength of gravity waves that affect the CMB, a factor of 2 precision in each measurement would result in a few percent measurement of n_T.

c. Consistency: For single-field inflation models $n_T = -\frac{1}{5}\frac{T}{S}$, as noted above.

Table 1: **TESTING INFLATION: YEAR 2002 SUMMARY**

Prediction	Mid-Term	Expectation
1. Flatness		
a. $\Omega_0 = 1$	$\Omega_0 = 1.03 \pm 0.03$	± 0.001
2. Density perturbations		
a. Adiabatic: acoustic peaks	at least 3	6 or 7
b. Gaussian	no evidence against	??
c. $\|n-1\| \sim \mathcal{O}(0.1)$	$n = 1.05 \pm 0.09$	± 0.001
d. $dn/d\ln k \sim \mathcal{O}(10^{-3})$	$dn/d\ln k = -0.02 \pm 0.04$	$\pm 10^{-3}$
e. CDM Paradigm	many successes	
3. Gravity waves		
a. Amplitude	$T/S < \mathcal{O}(1)$	$T/S > 10^{-3}$
b. $n_T \sim \mathcal{O}(-0.1)$	–	± 0.03 ??
c. Consistency: $T/S = -5n_T$	–	–

5 How much of the truth does inflation have and who is ϕ?

A key to testing inflation and getting at the underlying physics is the detection of gravitational waves. Not only is the amplitude of the gravitational waves directly proportional to the energy scale of inflation, $T \propto V/m_{\rm Pl}{}^4$, but this third prediction of inflation is an undeniable smoking gun for inflation.

Let me elaborate and comment. Some would claim that the first two predictions of inflation – flat Universe and scale-invariant density perturbations – have long been considered features of any sensible cosmological model. Certainly both were discussed well before inflation (e.g., flatness by Peebles and Dicke [32] and scale-invariant density perturbations by Harrison and Zel'dovich [33]). Thus, there is some truth to this point of view. However, because inflation provides a mechanism for actually producing a flat Universe with almost scale-invariant density perturbations it also makes a prediction about how close to scale invariant they should be. As I have emphasized, an important test of inflation is its prediction that $|n-1| \sim \mathcal{O}(0.1)$ and not $n = 1$.

In addition to providing a smoking gun for inflation, the detection of gravity waves instantly reveals the epoch and energy scale of inflation:

$$
\begin{aligned}
H_I^{-1} &\simeq \frac{2 \times 10^{-39}\,\mathrm{sec}}{\sqrt{T/S}} \\
V^{1/4} &= 3 \times 10^{-3} m_{\mathrm{Pl}}\,(T/S)^{1/4} \simeq 3 \times 10^{16}\,\mathrm{GeV}(T/S)^{1/4}
\end{aligned}
$$

Further, the values of T/S and $n-1$ can be used to solve for the inflationary potential and its first two derivatives:

$$V = 1.8 T\, {m_{\mathrm{Pl}}}^4 \tag{10}$$

$$V' = \pm\sqrt{\frac{8\pi}{5}\frac{T}{S}}\,V/m_{\mathrm{Pl}}, \tag{11}$$

$$V'' = 4\pi\left[(n-1) + \frac{3}{5}\frac{T}{S}\right] V/{m_{\mathrm{Pl}}}^2 \tag{12}$$

where the numerical factors depend upon the composition of the Universe and are given for $\Omega_M = 0.35$ and $\Omega_\Lambda = 0.65$ [17]. Measurements of T, S, and $(n-1)$ can be used to shed light on the underlying inflaton potential.

6 If it smells like a rose, is it a rose?

Inflation has some of the truth, but it has by no means been verified to the degree that we can safely include it as part of "a new standard cosmology." Further, while inflation seemed like a bold and expansive step forward twenty years ago, by today's standards it seems more modest: It only explains the isotropy and homogeneity on a temporary basis (albeit an exponentially long temporary basis); It does not address the question of the initial singularity (or why there is a universe at all); and It still stands disconnected from string theory or any other fundamental theory of elementary particle physics. For all of these reasons we should be open to new ideas. At the moment there are several intriguing ideas – e.g., ekpyrotic/cyclic model [34] and variable speed of light theories [35] – but none have reached the point of making sharp predictions like inflation.

The cosmological data that we have seen to date have not only provided the first tests of inflation, but they have also raised the bar for its alternatives. In fact, a die-hard inflationist might well argue that any theory that is able to account for the observable facts without appealing to initial conditions will have to look very much like inflation.

To be specific, the existence of adiabatic density perturbations on superhorizon scales at the time of last scattering and the enormous heat of the big bang (quantified by the entropy of 10^{90} within our current Hubble volume) require any scenario that does not simply appeal to initial conditions to incorporate superluminal expansion (accelerated expansion or accelerated contraction) and entropy production, the two hallmarks of inflation [36, 37].

The necessity of each is easy to explain. First, to causally create a density perturbation in an expanding Universe requires that a sub-hubble-scale sized region at very early times ($l \lesssim H^{-1}$) must grow to a size much, much greater than the Hubble length at last scattering. This translates into a kinematic requirement: There must be an epoch where the scale factor grows faster than t. (More precisely and more generally, the condition is that $\dot{R}$ and $\ddot{R}$ have the same sign, which is called superluminal expansion.)

Today's Hubble volume contains an entropy of about 10^{90} (largely in the form of photons and neutrinos). At early times, the entropy within a Hubble volume is at most $(m_{\rm Pl}/T)^3$ (equality pertaining for a radiation dominated phase). In the absence of massive entropy production the superluminal phase will produce very large, but very empty (low entropy) density perturbations. Entropy production is needed to create enough photons and eventually enough matter to account for the 10^{68} baryons (and even more dark matter particles) per galaxy. (This same argument could have been worded in terms of producing a smooth region of the Universe corresponding to our present Hubble volume.)

To summarize, an alternative to inflation must involve superluminal expansion and massive entropy production; in addition, if it can be described in 4-dimensions by a the evolution of a single degree of freedom (which could be called a scalar field), an inflation hegemonist might be tempted to call it a realization of inflation, rather than a fundamentally new paradigm. She might have a good case, since inflation is still a paradigm in search of a model.

7 Concluding remarks

The chance that any major new idea in theoretical physics has something to do with the truth is very small; in cosmology the odds are even longer. Nonetheless, inflation has passed its first round of major tests and it does appear that inflation has some of the truth.

Precision measurements of CMB anisotropy on sub-degree angular scales by ground-based and balloon-borne experiments have led the way in testing inflation. Over the next decade or so, results from satellite-borne experiments with even greater precision and control of systematics – MAP, Planck and possibly a new satellite mission dedicated to polarization – will sharpen the tests by more than 30 fold and the crucial gravity-wave signature of inflation may be detected.

The challenge and importance of verifying the third prediction of inflation cannot be overstated. Discovery of inflation-produced gravity waves is a smoking gun signature of inflation, immediately identifies the epoch and scale of inflation and reveals information about the inflationary potential. If its spectral index can be measured, the consistency of the single-field inflation paradigm can be checked.

While it is too early to bring on the champagne – plenty could still go wrong – the bar has been raised for any new competitor to inflation. Accounting for the superhorizon-sized adiabatic density perturbations whose existence has been confirmed by detection of acoustic peaks in the CMB angular power spectrum requires both superluminal expansion and entropy, the two hallmarks of inflation.

While the successes of inflation are gratifying, we are very far from understanding the underlying cause. Single-field models require very weakly coupled scalar fields (dimensionless couplings of order 10^{-14} or so) and the potentials seem hopelessly contrived. It could be that our 4-dimensional scalar-field formulation of inflation makes things look unnatural, and that when viewed in higher dimensions or when the scalar field is properly interpreted, the inflaton and its potential will make perfectly good sense.

In addition to the “who is ϕ question”, there are other questions: the details of reheating, the possibility of quantum fluctuations during inflation excite isocurvature modes or produce particle relics, the search for a connection between planck scale physics and CMB anisotropy (the modes seen on the CMB sky today were subPlanckian in size during inflation), and the multiverse.

Perhaps the most daunting challenge facing inflation is convincing even ourselves that inflation *really* happened. Even if inflation passes all of its tests and its gravity waves are detected both by their CMB signature and directly by Hz-frequency laser interferometers, will we be able to say, with the same confidence that use in discussing big-bang nucleosynthesis or the quark/hadron transition, that the Universe really did inflate (rather than inflation provides a consistent way of describing what we see today)? Our confidence in big-bang nucleosynthesis derives from laboratory-based nuclear physics and in the quark/hadron transition from computer simulations and accelerator experiments. Will we be to find a laboratory crosscheck for inflation that will gives us similar confidence?

Finally, there is a downside to inflation: If inflation does succeed in making the present state of the Universe insensitive to its initial state, it will place a veil between us the beginning of the Universe.

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