

Neutrino Mass and Dark Energy from Weak Lensing

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Weak gravitational lensing of background galaxies by intervening matter directly probes the mass distribution in the universe. This distribution, and its evolution at late times, is sensitive to both the dark energy, a negative pressure energy density component, and neutrino mass. We examine the potential of lensing experiments to measure features of both simultaneously. Focusing on the radial information contained in a future deep 4000 square degree survey, we find that the expected ($1\text{-}\sigma$) error on a neutrino mass is 0.1 eV, if the dark energy parameters are allowed to vary. The constraints on dark energy parameters are similarly restrictive, with errors on w of 0.09. Much of the restrictive power on the dark energy comes not from the evolution of the gravitational potential but rather from how distances vary as a function of redshift in different cosmologies.

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Introduction. Rapid advances in astronomical observations have enabled us to learn about the *dark sector* in the universe, both dark matter that dominates over ordinary baryonic matter and dark energy which apparently pervades the universe. Some of the dark matter is in the form of massive neutrinos. We know that neutrinos have mass because we have seen evidence for the transformation of one species into another [1–3], a transformation that is impossible for massless neutrinos. The standard cosmology [4] predicts a definite relation between the cosmic neutrino abundance and the cosmic photon abundance. Since the latter is well-measured, a non-zero neutrino mass translates into an unambiguous prediction for the energy density contributed by massive neutrinos. The evidence for the existence of dark energy is twofold. First, distant Type Ia supernovae are fainter than they would be if the universe were decelerating [5], and acceleration can take place only with negative-pressure dark energy. Second, observations of anisotropies in the cosmic microwave background (CMB) [6, 7] confirm that the universe is flat (so that the total density is equal to the critical density), while many independent measurements [8] place the matter density at one third of the critical density: the remaining two-thirds is called dark energy.

There is still a great deal of uncertainty regarding both neutrino masses and dark energy. Both phenomena await a convincing theoretical interpretation in the context of particle physics models. Beyond this theoretical uncertainty, several relevant parameters have not been well measured. There is a wide range of allowed neutrino masses: neutrinos might contribute as much as 20% of the matter density in the universe [9] or as little as 0.3%. The pressure of the dark energy is constrained to be negative, but how negative is still unknown. In particular, a cosmological constant has $w = -1$ where w is the ratio of pressure to energy density. Many dynamical models however predict much different values of w . Better mea-

surements of each of these fundamental parameters, the energy density in massive neutrinos and the equation of state of the dark energy, are clearly needed.

Remarkably both of these new pieces of physics—neutrino masses and dark energy—leave similar signatures in the matter distribution in the universe. In particular, how rapidly structures grow is determined by the energy content of the universe. Since neutrinos are somewhat relativistic even at late times, they inhibit the growth of structure. Similarly, since the dark energy does not cluster, growth in the dark matter slows in a dark-energy dominated universe. If one could measure how rapidly the gravitational potential was evolving in time, one could learn about this new physics.

Weak gravitational lensing offers the promise of making measurements of precisely this evolution. In 2000, four groups [10] announced detection of the distortion of the shapes of distant galaxies due to intervening large scale structure. Since then, the observations have steadily improved, prompting a number of proposals for larger and deeper surveys. Here we explore the potential that these surveys have for measuring neutrino masses and properties of the dark energy [11, 12]. Although the angular correlations of the galaxy ellipticities contain useful cosmological information, we focus on the radial information: the change in the shear field for background galaxies in different redshift bins. Radial tomography has recently been applied to measure the properties of the shear field of a cluster of galaxies [13].

Lensing of Galaxies at Fixed Redshift. Hu [14, 15] pointed out that, by breaking up the background galaxies in a wide, deep survey into redshift bins, one could essentially do tomography. The deeper bins probe an integral of the 3D gravitational potential over distances farther away from us than the nearer bins.

With many background galaxies at a fixed redshift z_s , one can hope to recover the lensing convergence κ in dif-

ferent angular pixels. The convergence in any one of these bins is sensitive to the matter distribution between us and the sources at z_s . In particular,

$$\kappa(z_s, \vec{\theta}) = \int_0^{z_s} dz P(z, z_s) \delta(z, \vec{\theta}), \quad (1)$$

where δ is the fractional deviation of the density from its average value and P is the kernel relating this deviation to the convergence. If we discretize Eq. (1) so that $\kappa_i = [P_{\kappa\Delta}]_{ij} \delta_j$ (where i, j label different redshift bins), then the projection operator is

$$[P_{\kappa\Delta}]_{ij} = \begin{cases} \frac{3}{2} \Omega_m H_0^2 \delta \chi_j \frac{(\chi_{i+1} - \chi_j) \chi_j}{\chi_{i+1}} & \chi_{i+1} > \chi_j \\ 0 & \chi_{i+1} \leq \chi_j \end{cases} \quad (2)$$

where χ_i is the comoving distance out to redshift z_i and $\delta\chi_j$ is the comoving width of the j th redshift bin. We take bins of width $\delta z = 0.1$ and angular bin size of 1 deg^2 in all that follows. Comoving distance depends on the energy density in the universe at various times; it can be expressed as $\chi_i = \int_0^{z_i} dz/H(z)$, where H is the expansion rate of the universe. The bottom panel of Figure 1 shows the projection operator for background galaxies at redshift two. The convergence signal is most sensitive in this case to the matter distribution at redshift 0.5. As the background galaxies move closer to us, they probe the structure at lower redshifts, later times.

It is possible to invert Eq. (1) and extract an estimate of δ from the measurements of convergence [16]. On large scales, δ is drawn from a Gaussian distribution with mean zero and a variance which evolves with time. How this evolution takes place depends upon the underlying cosmology; this dependence is expressed in the *growth function*, $D(a = (1+z)^{-1})$, which is governed by [4]

$$D'' + \left(\frac{H'}{H} + \frac{3}{a} \right) D' - \frac{3}{2} \frac{\Omega_m H_0^2}{a^5 H^2} D = 0, \quad (3)$$

where Ω_m is the matter density today in units of the critical density. When neutrino masses are introduced, the growth function becomes more complicated, and varies for modes with different wavelengths. To a rough approximation, a nonzero neutrino mass produces a fractional decrease in the power on scales probed by lensing surveys equal to $12f_\nu$, where $f_\nu \equiv \Omega_\nu/\Omega_m$, the ratio of the massive neutrino energy density to that in matter. More accurate fitting formulae for the growth function and power spectrum were calculated including nonzero neutrino mass in Ref. [17]. In the standard cosmology, Ω_ν due to one massive neutrino species is equal to $m_\nu/(94h^2 \text{ eV})$ where h parametrizes the Hubble constant. For a flat universe, then, $f_\nu = 0.081 (m_\nu/1 \text{ eV}) (0.13/\Omega_m h^2)$. The combination $\Omega_m h^2$ is well-determined by CMB experiments [18, 19]; it is currently measured to be 0.13 ± 0.01 [20].

The convergence then depends on the expansion history of the universe via both the growth function and the projection operator. The Friedman equation expresses

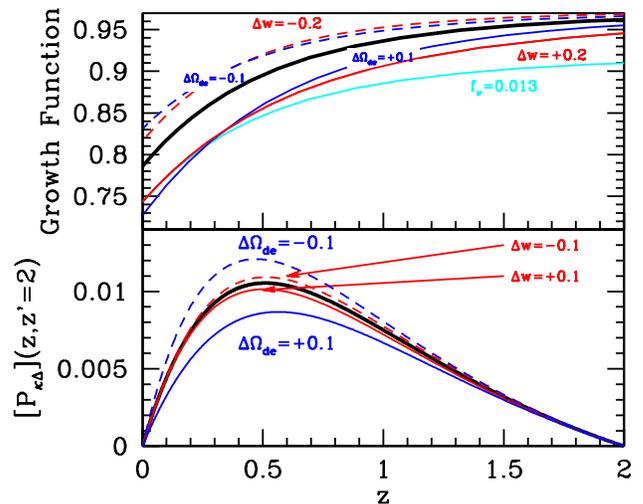


FIG. 1: *Top panel.* Growth function vs. redshift for different choices of cosmological parameters. The base model in both panels (dark solid curve) has $\Omega_{\text{DE}} = 0.65$, $w = -1$ and $f_\nu = 0.005$. Negative changes in the parameters are indicated by dashed curves. *Bottom panel.* Projection kernel as a function of redshift for background galaxies at redshift $z' = 2$. The projection is completely independent of neutrino mass. Weak lensing convergence is the convolution of the two panels.

the expansion rate in terms of energy density

$$\frac{H(z)}{H_0} = \left[(1 - \Omega_{\text{DE}})(1+z)^3 + \Omega_{\text{DE}}(1+z)^{3(1+w)} \right]^{1/2}, \quad (4)$$

where Ω_{DE} is the dark energy density, and w is the dark energy equation of state. In principle, we can hope to measure Ω_{DE} , w , and f_ν from a deep, wide lensing survey.

The top panel in Figure 1 shows the growth function for the base model we use throughout, one in which $\Omega_{\text{DE}} = 0.65$, $w = -1$, $\Omega_m h^2 = 0.13$, and $f_\nu = 0.005$. Also shown are the slight changes in the growth function as these parameters vary. Note that the changes induced by Ω_{DE} and w are very similar to each other. The bottom panel illustrates that the projection operator depends almost exclusively on the dark energy density.

Results. As mentioned above, there are two effects measured in a tomographic weak lensing survey. The first is the evolution of the power spectrum, or the growth function, while the second is the projection of physical distances onto redshift space. Figure 2 shows the relative constraining power of these two effects in a 4000 square degree survey which measures ellipticities and (photometric) redshifts for a hundred galaxies per square arcminute. We use a galaxy redshift distribution of $dN/dz \propto d\chi/dz \exp[-(\chi/\chi_*)^4]$, such that χ_* gives a mean redshift of one, but the results are insensitive to this choice [21]. We assume a constant effective equation of state w , and an intrinsic r.m.s. galaxy ellipticity of 0.3. The inner region in Figure 2 includes both effects; the outer ignores projection effects. The growth function depends on only a combination of Ω_{DE} and w : a change in

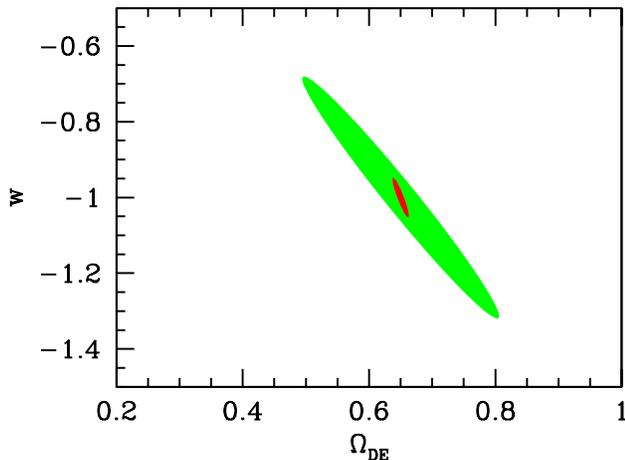


FIG. 2: Errors from a 4000 deg² survey with one hundred galaxies per square arcminute with (inner region) and without (outer region) projection effects. Here the neutrino mass has been fixed. The outer region uses only information from the growth function; this information is much less constraining than including the combination of projection.

one can be offset by a change in the other. Thus, without projection, weak lensing strongly constrains $\Omega_{\text{DE}} + w$ but not either parameter separately. Projection effects break this degeneracy, because (bottom panel of Figure 1) projection is so much more sensitive to Ω_{DE} than to w .

We have assumed here that other cosmological parameters – the amplitude and slope of the primordial spectrum, $\Omega_m h^2$, and $\Omega_b h^2$ – are fixed. If future observations constrain these parameters tightly enough that the resulting uncertainty in the power spectrum is small, then this assumption is valid. Figure 1 indicates that the effects we have considered here induce of order ten percent changes in the power spectrum (which scales as D^2). Uncertainties in the power spectrum from ambiguity in the other parameters are projected to be smaller than one percent after Planck [15].

How do the constraints change when the uncertainty in the neutrino mass is included? Projection is completely independent of neutrino mass, so the constraint on Ω_{DE} remains unchanged. But the limits on the dark energy equation of state are compromised.

The top panel of Figure 3 indicates that the effect of the neutrino mass is very correlated with the dark energy equation of state. When we lack knowledge about the neutrino mass, we therefore lose discriminatory power over the equation of state. But, if the absolute neutrino mass or its uncertainty were constrained by a laboratory experiment (tritium endpoint [22] or neutrinoless double-beta decay [23]) to be less than 0.1 eV, then the resulting constraint also appears in the top panel of Figure 3. Future measurements or limits on neutrino mass will then contribute to precision measurements of the cosmological dark energy density and evolution.

Also of interest are the constraints on the neutrino mass, shown in the bottom panels of Figure 3. There

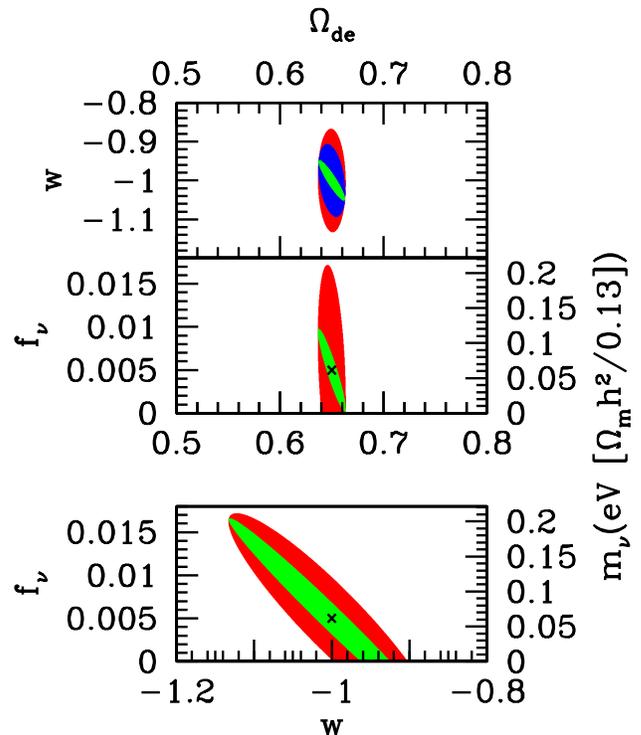


FIG. 3: Projected errors (all 1- σ) on the dark energy density, equation of state, and neutrino mass from 4000 square degree weak lensing survey. In each case, the innermost region is the constraint arising from fixing the parameter not shown (e.g., neutrino mass in the upper panel), while the outermost constraint comes from marginalizing over the third parameter. The middle region in the top panel emerges if the laboratory constraint on the neutrino mass is 0.1 eV.

must be a neutrino mass as least as large as the square root of the atmospheric δm^2 , i.e., 0.05 eV. This corresponds to $f_\nu = 4.1 \times 10^{-3}$ for the current best value of $\Omega_m h^2$. Figure 3 shows that this limit is barely within reach of a comprehensive weak lensing survey if we can assume that the dark energy is a cosmological constant ($w = -1$) and that Ω_{DE} will be determined by other means to within a few percent. On the other hand, if we allow for freedom in the dark energy equation of state, then the mass limit gets worse by a factor of order five. It is conceivable that both Ω_{DE} and w will be determined through other means, e.g., type Ia supernovae, cluster abundances, or the CMB. If so, then the neutrino mass will be detectable with weak lensing.

These projected limits on the neutrino mass are even more powerful than they appear. If there are only three light neutrinos, then the solar and atmospheric neutrino measurements constrain two mass squared differences, as illustrated in Figure 4 (adapted from Ref. [24]). The cosmological limit discussed here is on the sum of all neutrino masses. More stringent limits on the neutrino mass fraction must incorporate the texture of neutrino masses, i.e., if the neutrino masses are a standard or inverted hierarchy. If we assume that the probability distribution

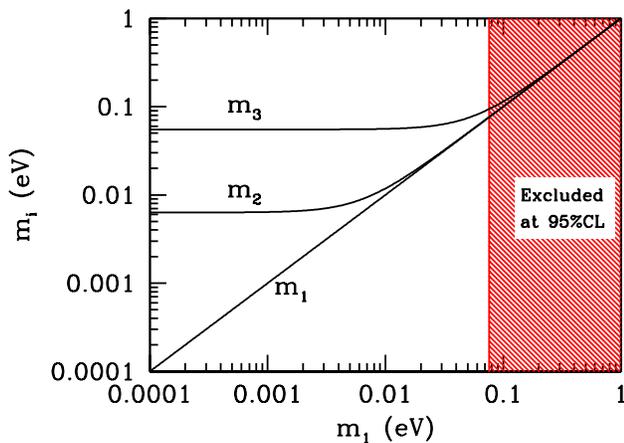


FIG. 4: Masses of the three neutrino species as a function of the lightest neutrino mass in a hierarchical mass scheme [24]. At large m_1 , all three must be nearly degenerate to account for the solar and atmospheric oscillations. The projected cosmological upper limit excludes the shaded region at 95% CL.

for f_ν is Gaussian with mean $f_\nu = 0.005$ and rms error corresponding to the marginalized case $\Delta f_\nu = 0.008$, then the 95% confidence limit on f_ν would be 0.02, corresponding to an upper limit on the sum of the masses of 0.25 eV. Figure 4 shows that the region in which all three neutrino masses are degenerate can be ruled out then by this cosmological measurement, again even accounting for the uncertainty in the dark energy sector.

In this study, we have not included any angular information, which would further pin down the power spectrum of mass extremely well. Overall, the information of the growth of structure from radial tomography of large scale structure via weak lensing will be a powerful method for discovering the nature of both the dark energy and neutrino components of the universe.

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