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A Model of CPT Violation for Neutrinos

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Abstract

Any local relativistic quantum field theory of Dirac-Weyl fermions conserves CPT. Here we examine whether a simple nonlocal field theory can violate CPT. We construct a new relativistic field theory of fermions, which we call “homeotic”, which is nonlocal but causal and Lorentz invariant. The free homeotic theory is in fact equivalent to free Dirac theory. We show that a homeotic theory with a suitable nonlocal four-fermion interaction is causal and as a result has a well-defined perturbative S-matrix. By coupling a right-handed homeotic fermion to a left-handed Dirac-Weyl fermion, we obtain a causal theory of CPT-violating neutrino oscillations.

1 Introduction

CPT violating neutrino masses allow the possibility [1]-[3] of reconciling the LSND [4], atmospheric [5], and solar oscillation [6, 7] data without resorting to sterile neutrinos. As argued in our previous work [2], there are good reasons to imagine that CPT violating dynamics couples directly to the neutrino sector, but not to other Standard Model degrees of freedom. For CPT violating Dirac mass splittings Δm on the order of 1 eV or less, the feed-through to other Standard Model processes is completely negligible, suppressed by $G_F(\Delta m)^2$ times loop factors.

CPT violation in the neutrino sector would be a very exciting development, and raises important theoretical issues. CPT is conserved quite generally in local relativistic quantum field theory [8], and even in large classes of nonlocal effective field theories [9]. While it is straightforward to construct field theories which break CPT via spontaneous violation of Lorentz invariance [10], this would seem to entail other drastic consequences [11], unless restricted to the context of cosmology. As far as is known, CPT is not gauged in string theory, and thus there is no general argument that it should be respected at high energies [12]. However this begs the question of how CPT violation will appear in the effective low energy theory.

If CPT violation is observed in the neutrino sector, must we give up either on-shell Lorentz invariance or effective quantum field theory in our description of the low energy world? To answer this question, we have constructed what we believe is the minimal relativistic quantum field theory model of explicitly CPT violating neutrinos. The CPT violation arises from a Dirac type local bilinear coupling $\bar{\nu}_L N_R + \bar{N}_R \nu_L$, where $\nu_L(x)$ is the usual left-handed Dirac-Weyl neutrino that occupies an electroweak doublet with a Standard Model charged lepton. The other field, $N_R(x)$, is a Standard Model singlet fermion with novel properties, which we call *homeotic*.

In the next section we develop the relativistic quantum field theory of homeotic fermions, in analogy to that of Dirac and Dirac-Weyl fermions. Homeotic fermions obey standard Fermi statistics, and we exhibit interacting lagrangian field theories for homeotic fermions which are both unitary and causal. Not surprisingly, the theory of free homeotic fermions is physically equivalent to the usual field theory of free Dirac fermions: it has the same degrees of freedom, the same hamiltonian and the same symmetries. However the homeotic free field theory has a nonlocal lagrangian, and the homeotic fields have noncanonical anticommutation relations.

By adding nonlocal relativistic four-fermion interactions to the free homeotic theory, we obtain a causal interacting field theory with a well-defined unitary S-matrix. Indeed this interacting homeotic theory appears to have the same S-matrix as Dirac theory with a local four-fermion interaction. The homeotic theory with only local four-fermion interactions, on the other hand, is acausal and does not have a well-defined S-matrix. Naively this theory would violate crossing symmetry at tree level.

The homeotic field theories are CPT invariant, but CPT is of necessity realized by a different symmetry operator than for Dirac theory. The bilinear mass term discussed above, which couples a left-handed Dirac-Weyl fermion to a right-handed homeotic fermion, thus violates CPT.

The resulting field theory provides the simplest relativistic quantum field theory model for CPT violating neutrino oscillations. There are still issues in understanding the off-shell description of neutrino propagation, as discussed in the work of Greenberg [13]. But there is great virtue in having a simple explicit model.

2 Free homeotic fermions

In this section we develop the field theory of free homeotic fermions, in analogy with free Dirac theory. We employ the notation and conventions of Peskin and Schroeder [14]. Recall that in Dirac theory we introduce positive and negative frequency solutions of the Dirac equation:

$$\begin{aligned} \psi_+(x) &= u_+(p)\mathrm{e}^{-ip\cdot x}\,, \quad p^2 = m^2, \quad p_0 > 0; \\ \psi_-(x) &= u_-(p)\mathrm{e}^{-ip\cdot x}\,, \quad p^2 = m^2, \quad p_0 < 0, \end{aligned} \tag{2.1}$$

and the 4-component spinors $u_+(p)$, $u_-(p)$ satisfy

$$(\not{p} - m)u_\pm(p) = 0. \tag{2.2}$$

Note that $u_+(p)$, $u_-(p)$ are usually written as $u(p)$, $v(-p)$, respectively, where $u(p)$, $v(p)$ obey the equations

$$\begin{aligned}(\not{p} - m)u(p) &= 0,\\ (-\not{p} - m)v(p) &= 0.\end{aligned} \tag{2.3}$$

The homeotic theory is built from spinors $u_+(p)$, $u_-(p)$ which satisfy

$$(\not{p} - m\epsilon(p_0))u_\pm(p) = 0, \tag{2.4}$$

where $\epsilon(p_0)$ is the sign function of the standard delta calculus. Note that solutions of Eq. (2.4) automatically satisfy the Klein-Gordon equation and thus the usual on-shell dispersion relation $p^2 = m^2$. This in turn implies that the equation of motion Eq. (2.4) is Lorentz covariant, since $\epsilon(p_0)$ is Lorentz invariant for timelike 4-momenta.

Rewriting $u_+(p)$, $u_-(p)$ as $u(p)$, $\tilde{u}(-p)$, respectively, we see that homeotic fermions are built out two sets of u-spinors, both satisfying the usual positive frequency relation

$$(\not{p} - m)u(p) = (\not{p} - m)\tilde{u}(p) = 0. \tag{2.5}$$

On-shell fields of the homeotic theory are assembled from the obvious plane wave expansions:

$$\begin{aligned}\psi(x) &= \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\mathbf{p}}}}\sum_s \left[a^s_{\mathbf{p}} u^s(p) \mathrm{e}^{-ip\cdot x} + b^{s\dagger}_{\mathbf{p}} \tilde{u}^s(p)\mathrm{e}^{ip\cdot x}\right],\\ \psi^\dagger(x) &= \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\mathbf{p}}}}\sum_s \left[b^s_{\mathbf{p}} \tilde{u}^{s\dagger}(p) \mathrm{e}^{-ip\cdot x} + a^{s\dagger}_{\mathbf{p}} u^{s\dagger}(p)\mathrm{e}^{ip\cdot x}\right],\end{aligned} \tag{2.6}$$

where $\mathbf{p}$ denotes 3-momenta, and $s = 1,\, 2$, is a spin label. The homeotic field $\psi(x)$ is a solution of the equation of motion

$$i\partial\!\!\!/\psi(t,\mathbf{x}) = -\frac{im}{\pi}\,\mathbf{P}\int dt'\,\frac{1}{t-t'}\,\psi(t',\mathbf{x})\,, \tag{2.7}$$

where $\mathbf{P}$ denotes the principal value integral, which we will now assume throughout.

This equation of motion can be obtained from variation of the following nonlocal action:

$$\mathbf{S} = \int d^4x\,\bar{\psi}(x)i\partial\!\!\!/\psi(x) + \frac{im}{\pi}\int d^3x\int dtdt'\,\bar{\psi}(t,\mathbf{x})\,\frac{1}{t-t'}\,\psi(t',\mathbf{x}). \tag{2.8}$$

This action is Lorentz invariant, as can be seen by rewriting the mass term as

$$\frac{im}{\pi}\int d^3xd^3x'\int dtdt'\,\bar{\psi}(t,\mathbf{x})\,\frac{\delta^{(3)}(\mathbf{x}-\mathbf{x}')}{t-t'}\,\psi(t',\mathbf{x}'), \tag{2.9}$$

and observing that the integral

$$\int d^3xdt\,\frac{\delta^{(3)}(\mathbf{x})}{t}f(t,\mathbf{x}) \tag{2.10}$$

with an arbitrary scalar function $f(t,\mathbf{x})$, is invariant under boosts.

To quantize this theory, we treat $a^s_{\mathbf{p}}$ and $b^s_{\mathbf{p}}$ as anticommuting Fock operators obeying

$$\{a^r_{\mathbf{p}}, a^{s\dagger}_{\mathbf{q}}\} = \{b^r_{\mathbf{p}}, b^{s\dagger}_{\mathbf{q}}\} = (2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{q})\delta^{rs}\,. \tag{2.11}$$

These relations guarantee conventional Fermi statistics. The Hilbert space built from the corresponding Fock vacuum supports a standard representation of the Lorentz group.

The homeotic fields $\psi(x)$ do not have canonical anticommutation relations:

$$\begin{aligned}\left\{\psi^\dagger_a(x),\psi_b(y)\right\}_{x^0=y^0} &= \int\frac{d^3p}{(2\pi)^3}\sum_s\frac{1}{2E_{\mathbf{p}}}\left[u^{s\dagger}_a(p)u^s_b(p)\mathrm{e}^{ip\cdot(x-y)} + \tilde{u}^{s\dagger}_a(p)\tilde{u}^s_b(p)\mathrm{e}^{-ip\cdot(x-y)}\right]_{x^0=y^0}\\ &= \delta^{(3)}(\mathbf{x}-\mathbf{y})\delta_{ab} + m(\gamma^0)_{ab}\left[D(x-y)+D(y-x)\right]\,,\end{aligned} \tag{2.12}$$

where $D(x-y)$ is the familiar function invariant under proper orthochronous Lorentz transformations:

$$D(x-y) = \int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\mathbf{p}}}\mathrm{e}^{-ip\cdot(x-y)}\,. \tag{2.13}$$

The general anticommutator function [15] of the homeotic theory is given by

$$\left\{\bar{\psi}_a(x), \psi_b(y)\right\} = (i\partial\!\!\!/_x + m)_{ab}\, D(x-y) + (i\partial\!\!\!/_y + m)_{ab}\, D(y-x)\ , \tag{2.14}$$

which differs from the Dirac result by the sign of the first term proportional to m.

An important feature of Eq. (2.12) is that the noncanonical term occurs only for anti-commuting homeotic fermion components of opposite chiralities:

$$\begin{aligned}
\left\{\psi^\dagger_{La}(x), \psi_{Lb}(y)\right\}_{x^0=y^0} &= \delta^{(3)}(\mathbf{x}-\mathbf{y})\delta_{ab}\ , \\
\left\{\psi^\dagger_{Ra}(x), \psi_{Rb}(y)\right\}_{x^0=y^0} &= \delta^{(3)}(\mathbf{x}-\mathbf{y})\delta_{ab}\ , \\
\left\{\psi^\dagger_{La}(x), \psi_{Rb}(y)\right\}_{x^0=y^0} &= m\delta_{ab}\left[D(x-y) + D(y-x)\right]\ .
\end{aligned} \tag{2.15}$$

We stress again that homeotic fermions obey conventional Fermi statistics; this was guaranteed from the start by Eqs. (2.11).

Note that in the strict nonrelativistic limit ($|\mathbf{p}| \to 0$ with m fixed) the equal-time anti-commutators reduce to

$$\begin{aligned}
\left\{\psi^\dagger_{La}(x), \psi_{Lb}(y)\right\}_{x^0=y^0} &= \delta^{(3)}(\mathbf{x}-\mathbf{y})\delta_{ab}\ , \\
\left\{\psi^\dagger_{Ra}(x), \psi_{Rb}(y)\right\}_{x^0=y^0} &= \delta^{(3)}(\mathbf{x}-\mathbf{y})\delta_{ab}\ , \\
\left\{\psi^\dagger_{La}(x), \psi_{Rb}(y)\right\}_{x^0=y^0} &= \delta^{(3)}(\mathbf{x}-\mathbf{y})\delta_{ab}\ ,
\end{aligned} \tag{2.16}$$

where the third anticommutator would vanish if these were Dirac fermions. Evidently one of the zero momentum modes, $\psi_L + \psi_R$, is canonically quantized, while the other one, $\psi_L - \psi_R$, has a vanishing anticommutator. Thus zero mode quantization requires special care in homeotic theory.

The symmetrized stress-energy tensor derived from the action (2.8) has the same form as in Dirac theory and is conserved on-shell:

$$\Theta^{\mu\nu}(x) = \frac{i}{4}\left[\bar{\psi}\gamma^\mu\partial^\nu\psi - \partial^\nu\bar{\psi}\gamma^\mu\psi + (\mu \leftrightarrow \nu)\right]. \tag{2.17}$$

From this expression we obtain the hamiltonian of the free homeotic theory:

$$\mathbf{H} \ = \int d^3x\, \mathcal{H}(x) = \int d^3x\, \Theta^{00}(x) = \int d^3x\, \tfrac{i}{2}\left[\psi^\dagger\dot{\psi} - \dot{\psi}^\dagger\psi\right]$$

$$= \int \frac{d^3p}{(2\pi)^3} \sum_s E_{\mathbf{p}} \left[a^{s\dagger}_{\mathbf{p}} a^s_{\mathbf{p}} + b^{s\dagger}_{\mathbf{p}} b^s_{\mathbf{p}} \right] , \tag{2.18}$$

where we have dropped the normal ordering constant. Similarly, we obtain the momentum operator:

$$\mathbf{P^i} = \int d^3x \, \Theta^{0i}(x) = \int \frac{d^3p}{(2\pi)^3} \sum_s \mathbf{p^i} \left[a^{s\dagger}_{\mathbf{p}} a^s_{\mathbf{p}} + b^{s\dagger}_{\mathbf{p}} b^s_{\mathbf{p}} \right] . \tag{2.19}$$

The boost operator is defined by

$$\mathbf{K^i} = \int d^3x \left(x^i \Theta^{00} - x^0 \Theta^{0i} \right) = -t\mathbf{P^i} + \int d^3x \, \mathbf{x^i} \mathcal{H}(x) . \tag{2.20}$$

As a straightforward but nontrivial check of Lorentz invariance, one can verify that the definitions (2.18-2.20) together with the anticommutation relations (2.14) produce the on-shell relation

$$\left[\mathbf{K^i}, \mathbf{H} \right] = i\mathbf{P^i} . \tag{2.21}$$

The action (2.8) also has a global $U(1)$ symmetry under phase rotations of $\psi(x)$. The conserved charge is

$$Q = \int \frac{d^3p}{(2\pi)^3} \sum_s \left[a^{s\dagger}_{\mathbf{p}} a^s_{\mathbf{p}} - b^{s\dagger}_{\mathbf{p}} b^s_{\mathbf{p}} \right] . \tag{2.22}$$

These symmetry operators (2.18-2.22) are the same as in Dirac theory. The matrix elements of the free homeotic theory are in one-to-one equivalence with matrix elements of free Dirac theory.

3 Interacting homeotic fermions

In the free homeotic theory, the conserved charge Q of Eq. (2.22) is not given by the spatial integral of a local charge density. Instead (following a trick of Pauli's [16]) we find the expression

$$\begin{aligned} Q &= \int d^3x \, \bar{\psi} \gamma^0 \psi \\ &+ \frac{m}{\pi} \int_{-\infty}^{t} dt_1 \int_{-\infty}^{\infty} dt_2 \int d^3x \, \frac{1}{t_1 - t_2} \left[\bar{\psi}(t_1, \mathbf{x}) \psi(t_2, \mathbf{x}) + \bar{\psi}(t_2, \mathbf{x}) \psi(t_1, \mathbf{x}) \right] . \end{aligned} \tag{3.1}$$

The corresponding conserved current is:

$$J^\mu(x) = J^\mu_D(x) + \delta^{\mu 0}\frac{m}{\pi}\int_{-\infty}^{t} dt_1 \int_{-\infty}^{\infty} dt_2\, \frac{1}{t_1 - t_2}\left[\bar\psi(t_1,\mathbf{x})\psi(t_2,\mathbf{x}) + \bar\psi(t_2,\mathbf{x})\psi(t_1,\mathbf{x})\right] , \quad (3.2)$$

where $J^\mu_D(x) = \bar\psi\gamma^\mu\psi$ is the conserved current of Dirac theory.

As with any conserved abelian current in the canonical formalism, the following current algebra relations hold:

$$\begin{aligned} \left[J^0(x), J^0(y)\right]_{x^0=y^0} &= 0 , \\ \left[J^0(x), J^i(y)\right]_{x^0=y^0} &= 0 , \\ \left[J^i(x), J^j(y)\right]_{x^0=y^0} &\propto \delta^{(3)}(\mathbf{x}-\mathbf{y}) , \end{aligned} \quad (3.3)$$

where it is understood that in the canonical formalism we fail to pick up the expected Schwinger term in the second and third expression [17].

3.1 A causal interacting theory

We can construct a model of interacting homeotic fermions by introducing a simple current-current four-fermion interaction:

$$H_I(t) = \int d^3x \mathcal{H}_I(x) = \int d^3x J^\mu(x) J_\mu(x) . \quad (3.4)$$

Since the homeotic fields do not obey canonical anticommutation relations, we must carefully define what we mean by a relativistic quantum field theory. The obvious approach is to go to the interaction picture and attempt to construct a perturbative S-matrix. One can easily verify that the homeotic free hamiltonian H_0 generates conventional interaction picture time evolution:

$$H_I(t) = \mathrm{e}^{iH_0 t}\, H_I\, \mathrm{e}^{-iH_0 t} , \quad (3.5)$$

where it is understood that the mass parameter which appears in H_0, and thus in the anticommutation relations (2.12), is the physical mass.

The perturbative S-matrix is defined as the matix elements of the Dyson series:

$$\mathcal{S} = 1 + \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \cdots d^4x_n \; T\left\{\mathcal{H}_I(x_1)\cdots\mathcal{H}_I(x_n)\right\} \;, \tag{3.6}$$

where T denotes time-ordering. The current algbera relations (3.3) imply the causality condition

$$\left[\mathcal{H}_I(x), \mathcal{H}_I(y)\right]_{x^0=y^0} \propto \delta^{(3)}(\mathbf{x}-\mathbf{y}) \;, \tag{3.7}$$

which in turn guarantees the Lorentz invariance of the time-ordering in (3.6). Together with the properties of the free homeotic theory detailed above, this is sufficient to prove that our interacting homeotic theory has a unitary Lorentz invariant S-matrix. This in turn can be regarded as proof that the homeotic theory is a sensible relativistic quantum field theory. Indeed this S-matrix appears equivalent to what we would have obtained from Dirac theory with an analogous current-current interaction.

This interacting homeotic theory is quite a *rara avis*: despite the fact that the lagrangian is highly nonlocal the field theory is unitary, causal, and Lorentz invariant. It also exhibits crossing symmetry and conserves CPT. However, CPT is not realized in the standard way. The CPT operator on Dirac spinors is γ^5, but on homeotic spinors it is $\gamma^2\gamma^5$. This difference comes from the charge conjugation operation (time reversal and parity are realized in the standard way) and is the direct consequence of the lack of $v(p)$ spinors in the homeotic theory. Whereas in Dirac theory charge conjugation relates a $u(p)$ spinor to $\gamma^2 v^*(p)$, in homeotic theory it merely exchanges $u(p) \leftrightarrow \tilde{u}(p)$. Note that the nonstandard CPT properties of homeotic theory are essential to reconciling CPT conservation with the existence of the conserved current (3.2): both of the terms in (3.2) are even under CPT, in contrast to the Dirac case where $\bar{\psi}\gamma^\mu\psi$ is CPT odd.

The perturbative Feynman rules of this theory are easily derived. The propagator is defined unambiguously following the discussion of Weinberg [18]:

$$-i\Delta^h_{ab}(x,y) = \theta(x^0-y^0)\left\{\psi^+_a(x), \psi^{+\dagger}_b(y)\right\} - \theta(y^0-x^0)\left\{\psi^{-\dagger}_b(y), \psi^-_a(x)\right\} \;, \tag{3.8}$$

where $\psi^\pm(x)$ denote the positive/negative frequency contributions to $\psi(x)$. A simple calculation gives

$$\Delta^h_{ab}(x,y) = \int \frac{d^4p}{(2\pi)^4} \frac{i\left[\not{p} + m\epsilon(p_0)\right]_{ab}}{p^2 - m^2 + i\epsilon} \, \mathrm{e}^{-ip\cdot(x-y)} \;. \tag{3.9}$$

This propagator is not equivalent to the usual Feynman propagator and does not give causal propagation. However there is a conspiracy in this theory: acausal propagation combined with nonlocal interactions yield a causal theory, as exemplified by the S-matrix.

3.2 An acausal interacting theory

Suppose that instead of (3.4) we had attempted to define an interacting homeotic theory from a *local* four-fermi interaction:

$$H_I(t) = \int d^3x \mathcal{H}_I(x) = \int d^3x \, (\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) \; . \tag{3.10}$$

Despite the fact that this theory has a simple local interaction, it exhibits strange properties. This theory preserves the global $U(1)$ invariance, as demonstrated by the fact that the local "current" $J_D^\mu(x) = \bar{\psi}\gamma^\mu\psi$ commutes with the charge Q defined in (3.1). However this local current is not conserved, and does not obey the current algebra relations (3.3). We find instead

$$[J^\mu(x), J^\nu(y)]_{x^0=y^0} = -im \left[D(x-y) + D(y-x)\right] \left(\bar{\psi}(x)\sigma^{\mu\nu}\psi(y) + \bar{\psi}(y)\sigma^{\mu\nu}\psi(x)\right) \; . \tag{3.11}$$

As a result, the causality condition Eq.(3.7) fails to hold, and the S-matrix is not well-defined. Put another way, if we compute tree-level $2 \to 2$ scattering of these homeotic fermions using naive Feynman rules, we find that the amplitudes violate crossing symmetry. Thus if the S-matrix were well-defined, we would be exhibiting a simple local interaction that violates crossing symmetry, which would be quite remarkable.

4 CPT violation and neutrinos

The left-handed neutrinos of the Standard Model are Dirac-Weyl fermion; they could not be represented as homeotic fermions unless we drastically altered the gauge interactions of the Standard Model to somehow make them compatible with the nonlocality of the free homeotic lagrangian.

Because neutrinos have mass, it is natural to suppose that the left-handed neutrinos $\nu_L^i(x)$, i = 1,2,3, have bilinear couplings to right-handed neutrinos $N_R^i(x)$. These right-handed neutrinos are Standard Model singlets, and might not carry any conserved local charges. This fact is usually used as motivation for assuming that the N_R^i neutrinos have Majorana masses; instead we will use it as motivation for assuming that the N_R neutrinos are homeotic.

Consider the simplest possible model, where the N_R^i neutrinos are the right-handed components of free homeotic fermions N^i. We assume the bilinear coupling

$$m \int d^4x \, \left(\bar{\nu}_L(x) N_R(x) + \bar{N}_R(x) \nu_L(x)\right) \, , \tag{4.1}$$

where we are now suppressing flavor indices for simplicity. This coupling is both local and Lorentz invariant.

We can assume that m is parametrically small compared to other contributions to the neutrino mass matrix, in which case it is sensible to treat (4.1) perturbatively, *i.e.*, to consider

$$m \int d^3x \, \left(\bar{\nu}_L(x) N_R(x) + \bar{N}_R(x) \nu_L(x)\right) \tag{4.2}$$

as part of the interaction hamiltonian density $\mathcal{H}_I(x)$.

Remarkably, the hybrid Dirac-homeotic theory thus defined obeys the causality condition Eq.(3.7). This can be worked out explicitly, but it is nearly obvious from the anticommutation relations (2.15) for homeotic fermions. We see there that right-handed homeotic fermions have canonical anticommutation relations with other right-handed homeotic fermions. Since the interaction hamiltonian density constains only right-handed homeotic fermion fields (and Dirac-Weyl fermion fields), the causality condition is obeyed just as it is for an ordinary Dirac mass term. This simple hybrid theory thus has a well-defined unitary Lorentz invariant S-matrix. The lagrangian is nonlocal, but the nonlocality appears only in the kinetic terms of the homeotic fermions.

This theory also violates CPT. This is apparent from the fact that the CPT symmetry operator is of necessity different for Dirac theory and homeotic theory. We can see the CPT violation explicitly by evaluating (4.2) on-shell. For simplicity, suppose that (4.2) is a perturbation on a theory of a Dirac neutrino (ν_L, ν_R) and a homeotic neutrino (N_L, N_R) with equal masses. Then on-shell (4.2) becomes:

$$\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\mathbf{p}}}\sum_s\left[m\left(a^{s\dagger}_{\mathbf{p}D}a^s_{\mathbf{p}h}-b^{s\dagger}_{\mathbf{p}h}b^s_{\mathbf{p}D}\right)+E_{\mathbf{p}}\left(b^s_{\mathbf{p}D}a^s_{-\mathbf{p}h}\mathrm{e}^{-2iE_{\mathbf{p}}t}+a^{s\dagger}_{\mathbf{p}D}b^{s\dagger}_{-\mathbf{p}h}\mathrm{e}^{2iE_{\mathbf{p}}t}\right)+h.c.\right], \tag{4.3}$$

where $a^{s\dagger}_{\mathbf{p}D}$, $b^{s\dagger}_{\mathbf{p}D}$ are Dirac creation operators, and $a^{s\dagger}_{\mathbf{p}h}$, $b^{s\dagger}_{\mathbf{p}h}$ are homeotic creation operators. Since CPT exchanges $a^{s\dagger}_{\mathbf{p}D}\leftrightarrow b^{s\dagger}_{\mathbf{p}D}$ and exchanges $a^{s\dagger}_{\mathbf{p}h}\leftrightarrow b^{s\dagger}_{\mathbf{p}h}$, the expression (4.3) is CPT odd.

Applied to neutrinos, this model produces precisely the CPT violating neutrino mass spectra postulated in our phenomenological work. Here we have obtained it from a simple unitary causal framework.

5 Discussion

In this paper we have introduced what is certainly the simplest Lorentz invariant field theory model for CPT violating neutrino oscillations. This model can be used to study a number of important issues, such as CPT violating baryogenesis, and off-shell neutrino propagation.

Homeotic fermions are interesting in their own right. They are in some sense a unique alternative to Dirac fermions. They allow interacting field theories which are nonlocal but causal. There appears to be a deep connection – a kind of duality – between non-locality in homeotic theory and "negative frequency spinors" in Dirac theory. Homeotic supersymmetry is likely to have novel properties, which could be of phenomenological importance.

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