FERMILAB–PUB–02/104–T
BUHEP-02-25

B-Meson Gateways to Missing Charmonium Levels

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(Dated: June 3, 2002)

We outline a coherent strategy for exploring the four remaining narrow charmonium states [$\eta_c'(2^1\mathrm{S}_0)$, $h_c(1^1\mathrm{P}_1)$, $\eta_{c2}(1^1\mathrm{D}_2)$, and $\psi_2(1^3\mathrm{D}_2)$] expected to lie below charm threshold. Produced in B-meson decays, these levels should be identifiable *now* via striking radiative transitions among charmonium levels and in exclusive final states of kaons and pions. Their production and decay rates will provide much needed new tests for theoretical descriptions of heavy quarkonia.

PACS numbers: 14.40.Nd,14.40.Gx,13.25.Gv

Every new spectroscopy carries the potential to illuminate older spectroscopies in novel—and sometimes unexpected—ways. The decays of charmed mesons have offered new paths to the study of mesons—especially excited states—composed of light quarks, and decays of charmonium ($c\bar{c}$) states have provided new access to glueballs—hadrons composed largely of gluons. The study of semileptonic τ decays has considerably enriched our knowledge of a_1 properties. B-meson decays into charmonium states are an indispensable tool for the exploration of CP violation [1, 2]. They can also serve as gateways to the discovery of hitherto unobserved charmonium states. The properties of these missing states can illuminate the interquark force and reveal effects that lie outside the simple quarkonium potential framework, including the influence of virtual decay channels.

Detailed knowledge of the $c\bar{c}$ spectrum first derived from the study of e^+e^- annihilations, which explored the $^3\mathrm{S}_1$ and $^3\mathrm{D}_1$ $J^{PC} = 1^{--}$ states and the $^3\mathrm{P}_J$ or $^1\mathrm{S}_0$ states connected to them by E1 or M1 radiative transitions. Though incisive, the e^+e^- annihilation channel has limitations. Twenty years have passed without a confirmation of the Crystal Ball claim of the $2^1\mathrm{S}_0$ $\eta_c'(3594\pm5)$ [3], and the complementary technique of charmonium formation in $p\bar{p}$ annihilations does not support the $\eta_c'(3594)$ observation [4]. The sighting of the $1^1\mathrm{P}_1$ level near the $^3\mathrm{P}_J$ center of gravity in $p\bar{p} \to h_c(3526) \to \pi^0 J/\psi$ reported by Fermilab Experiment E760 [5] needs confirmation. The $\eta_{c2}(1^1\mathrm{D}_2)$ and $\psi_2(1^3\mathrm{D}_2)$ have also proved elusive, with only an unsubstantiated claim in the literature for a 2^{--} state, $\psi(3836\pm13)$, in $\pi^\pm N \to J/\psi\pi^+\pi^- +$ anything [6].

Help is on the way. The CDF experiment's observation of B decays to known charmonium levels shed light on prompt and secondary production of ($c\bar{c}$) states, sharpening the puzzle of the production mechanism [7]. The $e^+e^- \to \Upsilon(4\mathrm{S}) \to B\bar{B}$ experiments CLEO [8], BaBar [9], and Belle [10], report increasingly detailed observations of the established quarkonium states in B decay. And now Belle has reported the observation of the $2^1\mathrm{S}_0$ (η_c') level at a new mass (3654 MeV) in exclusive $B \to KK_sK^-\pi^+$ decays [11].

In this paper, we give a template for the spectrum of charmonium based on a Coulomb + linear potential, and we present estimates of the principal decay rates for the unobserved states to make quantitative the expectation that four narrow states remain to be studied. We argue that we may expect ample production of the missing charmonium states in B-meson decays, and we suggest experimental strategies for detecting the missing levels. As with the discoveries of the P–states χ_{cJ} and η_c of charmonium, radiative transitions will be of central importance. Finally, we comment on what we will learn by studying the masses and properties of the missing levels.

The importance of radiative decays to the discovery of charmonium levels, including the D-wave states, has been appreciated since the earliest days of charmonium spectroscopy [12]. Recently, Ko, Lee and Song [13] discussed the observation of the narrow D states by photonic and pionic transitions, and Suzuki [14] emphasized that the cascade decay $B \to h_cK^{(*)} \to \gamma\eta_cK^{(*)}$ offers a promising technique to look for h_c.

The spectrum of charmonium. To estimate the positions of the missing charmonium levels, we have adjusted parameters of the classic Cornell potential [15, 16],

$$\begin{gathered} V(r) = -\frac{\kappa}{r} + \frac{r}{a^2}\ , \\ m_c = 1.84\ \mathrm{GeV},\ \ \kappa = 0.61,\ \ a = 2.38\ \mathrm{GeV}^{-1}, \end{gathered} \tag{1}$$

to reproduce the observed centers of gravity of the 1S and 1P states [38]. No one has produced a satisfactory analytic (or potential-based) explanation of the spin splittings of the $1^3\mathrm{P}_J$ levels. Moreover, the 2S, 1D, and 2P levels are certain to be influenced appreciably by coupling to decay channels. Accordingly, we will not estimate the spin splittings of those levels beyond offering the expectation that they will be small. Our expectations for the charmonium spectrum are summarized in Table I.

The $1^1\mathrm{P}_1$ h_c and the $2^1\mathrm{S}_0$ η_c' of course lie below $D\bar{D}$

TABLE I: $c\bar{c}$ spectrum in the Coulomb + linear potential (1).

State			Mass (MeV)	Remarks
$1^1\mathrm{S}_0$	η_c	c.o.g.	$3067.^a$	2979.8 ± 1.8
$1^3\mathrm{S}_1$	J/ψ			3096.87 ± 0.04
$1^1\mathrm{P}_1$	h_c		3526.	
$1^3\mathrm{P}_0$	χ_{c0}	c.o.g.	$3526.^a$	3415.0 ± 0.8
$1^3\mathrm{P}_1$	χ_{c1}			3510.51 ± 0.12
$1^3\mathrm{P}_2$	χ_{c2}			3556.18 ± 0.13
$2^1\mathrm{S}_0$	η_c'	c.o.g.	3678.	$3654. \pm 10.$
$2^3\mathrm{S}_1$	ψ'			3685.96 ± 0.09
$1^1\mathrm{D}_2$	η_{c2}		3815.	$\not\to D\bar{D}$ (parity)
$1^3\mathrm{D}_1$	ψ	c.o.g.	3815.	3769.9 ± 2.5
$1^3\mathrm{D}_2$	ψ_2			$\not\to D\bar{D}$ (parity)
$1^3\mathrm{D}_3$	ψ_3			$\to D\bar{D}$
2P			3968.	
1F			4054.	
3S			4118.	
$D^0\bar{D}^0$			3729.0	threshold
D^+D^-			3738.6	threshold
$D^0\bar{D}^{*0}$ or $D^{*0}\bar{D}^0$			3871.2	threshold
$D^{\pm}D^{*\mp}$			3879.3	threshold
$D_s^+D_s^-$			3973.2	threshold
$D^{*0}\bar{D}^{*0}$			4013.4	threshold
$D^{*+}D^{*-}$			4020.0	threshold
$D_s^+\bar{D}_s^{*-}$ or $D_s^{*+}\bar{D}_s^-$			4099.0	threshold
$D_s^{*+}D_s^{*-}$			4224.8	threshold

[a] Input values.

TABLE II: Hadronic decay widths of charmonium states.

$c\bar{c}$ state	Decay	Partial Width
$1^1\mathrm{S}_0$	$\eta_c \to gg$	17.4 ± 2.8 MeV [21]
$1^3\mathrm{S}_1$	$J/\psi \to ggg$	52.8 ± 5 keV [22]
$1^1\mathrm{P}_1$	$h_c \to ggg$	720 ± 320 keV[a]
$1^3\mathrm{P}_0$	$\chi_{c0} \to gg$	14.3 ± 3.6 MeV[b]
$1^3\mathrm{P}_1$	$\chi_{c1} \to ggg$	0.64 ± 0.10 MeV[b]
$1^3\mathrm{P}_2$	$\chi_{c2} \to gg$	1.71 ± 0.21 MeV[b]
$2^1\mathrm{S}_0$	$\eta_c' \to gg$	8.3 ± 1.3 MeV[c]
	$\eta_c' \to \pi\pi\eta_c$	160 keV[d]
$2^3\mathrm{S}_1$	$\psi' \to ggg$	23 ± 2.6 keV [22]
	$\psi' \to \pi\pi J/\psi$	152 ± 17 keV [22]
	$\psi' \to \eta J/\psi$	6.1 ± 1.1 keV [22]
$1^1\mathrm{D}_2$	$\eta_{c2} \to gg$	110 keV[e]
	$\eta_{c2} \to \pi\pi\eta_c$	≈ 45 keV[d]
$1^3\mathrm{D}_1$	$\psi \to ggg$	216 keV[f]
	$\psi \to \pi\pi J/\psi$	43 ± 15 keV[g]
$1^3\mathrm{D}_2$	$\psi_2 \to ggg$	36 keV[f]
	$\psi_2 \to \pi\pi J/\psi$	≈ 45 keV[d]
$1^3\mathrm{D}_3$	$\psi_3 \to ggg$	102 keV[f]
	$\psi_3 \to \pi\pi J/\psi$	≈ 45 keV[d]

[a] Computed from $^3\mathrm{P}_J$ rates using formalism of [23]; also see [24].
[b] Compilation of data analyzed by Maltoni, Ref. [23].
[c] Scaled from $\Gamma(\eta_c \to gg)$.
[d] Computed using Eqn. (3.5) of Ref. [20].
[e] Computed using Eqn. (3).
[f] Computed using Eqn. (2).
[g] From rates compiled in Table X of Ref. [20].

threshold, and so will be typically narrow charmonium states. In the absence of strong influence from the coupling to decay channels, the $2^3\mathrm{P}_J$ χ_c' and $2^1\mathrm{P}_1$ h_c' states should lie well above the $D\bar{D}$ and $D^*\bar{D}$ thresholds, and so should have uninhibited strong decays. As has long been known, the $J^{PC} = 2^{-+}$ $1^1\mathrm{D}_2$ η_{c2} and $J^{PC} = 2^{--}$ $1^3\mathrm{D}_2$ ψ_2 states constitute an important special case: they lie between the $D\bar{D}$ and $D^*\bar{D}$ thresholds, but are forbidden (because of their unnatural parity) to decay into $D\bar{D}$. It is therefore plausible that they will appear as narrow levels, and we now quantify this suspicion.

Properties of the missing levels. To estimate the decay rates, we shall use the established values for the $\eta_c, J/\psi, \chi_c, \psi'$, and $\psi(3770)$ states, adopt the Belle value for $M_{\eta_c'}$, set $M_{h_c} = 3526$ MeV, and choose $M_{\eta_{c2}} = M_{\psi_2} = 3815$ MeV. We estimate the rates for hadronic and radiative decays in turn.

Among hadronic decays, we consider transitions ($\pi\pi$ emission) and annihilations. To estimate the $\pi^+\pi^- + \pi^0\pi^0$ transition rates, we use the standard multipole expansion of the color gauge field [17, 18, 19] to express the E1-E1 transition rates through the Wigner-Eckart theorem given in Eqn. (3.5) of Ref. [20], with experimental inputs given in Table X of that paper. The results are shown in Table II. For present purposes, the essential lesson is that we do not expect the $\pi\pi$ transition rates to be large for the missing levels of charmonium.

For the annihilations into two or three gluons, we use the standard (lowest-order) perturbative QCD formulas [25] to scale from available measurements for related states. This is a straightforward exercise for the S-wave levels. We use Maltoni's analysis [23] of the $^3\mathrm{P}_J$ annihilation rates to estimate the rate for $h_c \to ggg$. The rates for annihilations of the $^3\mathrm{D}_J$ states into three gluons (via color-singlet operators) are given by [26, 27]

$$\Gamma(^3\mathrm{D}_J \to ggg) = \frac{10\alpha_s^3}{9\pi}\,\mathcal{C}_J\,\frac{|R_{n2}^{(2)}(0)|^2}{m_c^6}\,\ln 4m_c\langle r\rangle\ , \qquad (2)$$

where $R_{n\ell}^{(\ell)} \equiv d^\ell R_{n\ell}(r)/dr^\ell\big|_{r=0}$, $\langle r\rangle = \int_0^\infty dr\, r\, u_{n\ell}^2(r)$, and $\mathcal{C}_J = \frac{76}{9}, 1, 4$ for $J = 3, 2, 1$. A complete analysis (including color-octet operators as well) has too many unknowns to be of use [39]. The strengths of the $J = 3, 2, 1$ annihilations are more generally proportional to $\mathcal{C}_J$, even if color-octet operators dominate [28]. The two-gluon annihilation rate of the $^1\mathrm{D}_2$ state is given by [29]

$$\Gamma(^1\mathrm{D}_2 \to gg) = \frac{2\alpha_s^2}{3}\,\frac{|R_{n2}^{(2)}(0)|^2}{m_c^6}\ . \qquad (3)$$

Our estimates for the annihilation rates are collected in Table II. The expectation for $\Gamma(\eta_c' \to gg)$ is to be compared with the Belle value of 15 ± 24 (stat) MeV [11].

The most prominent radiative decays of charmonium states are the E1 transitions, for which the rate [29, 30] is given by

$$\begin{aligned}\Gamma(\hat{n}\ {}^{2s+1}\ell_J \xrightarrow{\mathrm{E1}} \hat{n}'\ {}^{2s+1}\ell'_{J'}\,\gamma) &= \frac{4\alpha e_c^2}{3}(2J'+1)k^3 \\ &\times|\mathcal{E}_{\hat{n}\ell:\hat{n}'\ell'}|^2\cdot\max(\ell,\ell')\left\{\begin{matrix} J & 1 & J' \\ \ell' & s & \ell\end{matrix}\right\}^2 ,\end{aligned} \tag{4}$$

where $e_c = \frac{2}{3}$ is the charm-quark charge, k is the photon energy, the E1 transition matrix element is $\mathcal{E}_{\hat{n}\ell:\hat{n}'\ell'} = \frac{3}{k}\int_0^\infty dr\, u_{n\ell}(r)u_{n'\ell'}(r)\left[\frac{kr}{2}j_0(\frac{kr}{2}) - j_1(\frac{kr}{2})\right] + O(k/m_c)$, $\hat{n} \equiv n-\ell$ is the radial quantum number, and $\{:::\}$ is a 6-j symbol. For M1 transitions, the rate is given by

$$\Gamma(\hat{n}^{2s+1}\ell_J \xrightarrow{\mathrm{M1}} \hat{n}'^{2s'+1}\ell_{J'}\,\gamma) = \frac{4\alpha e_c^2}{3m_c^2}(2J'+1)k^3|\mathcal{M}_{\hat{n}\ell:\hat{n}'\ell}|^2 , \tag{5}$$

where $\mathcal{M}_{n\ell:n'\ell} = \int_0^\infty dr\, u_{n\ell}(r)u_{n'\ell}(r)j_0(\frac{kr}{2})$.

The calculated rates for the prominent transitions among charmonium states are shown in Table III. Values enclosed in parentheses have been corrected for the effects of coupling to decay channels, following the procedure developed in [16]. The calculated values reproduce the patterns exhibited by measurements, and are in good agreement with other calculations in the literature [31]. We expect them to provide reasonable guidance to the radiative decay rates of the missing charmonium levels.

Integrating all the calculated rates, we note that the radiative decays should be prominent, with branching fractions $B(h_c \to \eta_c\gamma) \approx \frac{2}{5}$, $B(\eta_{c2} \to h_c\gamma) \approx \frac{2}{3}$, and $B(\psi_2 \to \chi_{c1,2}\gamma) \approx \frac{4}{5}$, of which $B(\psi_2 \to \chi_{c1}\gamma) \approx \frac{2}{3}$.

Charmonium production in B decays. Expectations for the fractions of B-meson decays leading to charmonium production are presented in Table IV. To estimate the $B \to {}^1\mathrm{S}_0$ production rates, we appeal to the suggestion [35] that the ratio of spin-singlet to spin-triplet decay rates is relatively insensitive to poorly calculated matrix elements, $\Gamma(B \to n^3\mathrm{S}_1 + X)/\Gamma(B \to n^1\mathrm{S}_0 + X) = 1 + 8m_c^2/m_b^2 \approx 1.5$. The inclusive production of 1P states in B decays can be expressed [40] in terms of color-singlet and color-octet contributions as [36]:

$$\begin{aligned}\Gamma(b \to h_c + X)/\Gamma(b \to \ell^-\bar{\nu}_\ell + X) &\approx 14.7\widetilde{H}_8 \\ \Gamma(b \to \chi_{c0} + X)/\Gamma(b \to \ell^-\bar{\nu}_\ell + X) &\approx 3.2\widetilde{H}_8 \\ \Gamma(b \to \chi_{c1} + X)/\Gamma(b \to \ell^-\bar{\nu}_\ell + X) &\approx 12.4\widetilde{H}_1 + 9.3\widetilde{H}_8 \\ \Gamma(b \to \chi_{c2} + X)/\Gamma(b \to \ell^-\bar{\nu}_\ell + X) &\approx 15.3\widetilde{H}_8 \end{aligned} \tag{6}$$

Using the measured rates for inclusive χ_{c1} and χ_{c2} production summarized in Table IV we extract $\widetilde{H}_8 = (8.95 \pm 3.79) \times 10^{-5}$ and $\widetilde{H}_1 = (2.18 \pm 0.31) \times 10^{-4}$, which determine the inclusive branching fractions for χ_{c0} and h_c. No measurements exist to guide our expectations for the production of 1D states in B decays, so we must rely for the moment on theoretical calculations [34] that suggest production rates roughly comparable to those for other charmonium states.

TABLE III: Calculated and observed rates for radiative transitions among charmonium levels in the potential (1).

Transition	γ energy k (MeV)	Partial width (keV) Computed	Measured[a]
$\psi \xrightarrow{\mathrm{M1}} \eta_c\gamma$	115	1.92	1.13±0.41
$\chi_{c0} \xrightarrow{\mathrm{E1}} J/\psi\gamma$	303	120 (105)[b]	98±43
$\chi_{c1} \xrightarrow{\mathrm{E1}} J/\psi\gamma$	390	242 (215)[b]	240±51
$\chi_{c2} \xrightarrow{\mathrm{E1}} J/\psi\gamma$	429	315 (289)[b]	270±46
$h_c \xrightarrow{\mathrm{E1}} \eta_c\gamma$	504	482	
$\eta'_c \xrightarrow{\mathrm{E1}} h_c\gamma$	126	51	
$\psi' \xrightarrow{\mathrm{E1}} \chi_{c2}\gamma$	128	29 (25)[b]	22±5
$\psi' \xrightarrow{\mathrm{E1}} \chi_{c1}\gamma$	171	41 (31)[b]	24±5
$\psi' \xrightarrow{\mathrm{E1}} \chi_{c0}\gamma$	261	46 (38)[b]	26±5
$\psi' \xrightarrow{\mathrm{M1}} \eta'_c\gamma$	32	0.04	
$\psi' \xrightarrow{\mathrm{M1}} \eta_c\gamma$	638	0.91	0.75±0.25
$\psi(3770) \xrightarrow{\mathrm{E1}} \chi_{c2}\gamma$	208	3.7	
$\psi(3770) \xrightarrow{\mathrm{E1}} \chi_{c1}\gamma$	250	94	
$\psi(3770) \xrightarrow{\mathrm{E1}} \chi_{c0}\gamma$	338	287	
$\eta_{c2} \xrightarrow{\mathrm{E1}} \psi(3770)\gamma$	45	0.34	
$\eta_{c2} \xrightarrow{\mathrm{E1}} h_c\gamma$	278	303	
$\psi_2 \xrightarrow{\mathrm{E1}} \chi_{c2}\gamma$	250	56	
$\psi_2 \xrightarrow{\mathrm{E1}} \chi_{c1}\gamma$	292	260	

[a] Derived from Ref. [22].

[b] Corrected for coupling to decay channels as in Ref. [16].

Observing the missing narrow states. Radiative transitions among charmonium levels are the key to discovering the remaining narrow states. Approximately 90K $B \to K\eta_c$ events are produced in the Belle experiment's data sample. Using the production rates of Table IV, and making the plausible assumption that $B(B \to K^{(*)} + (c\bar{c}))/B(B \to X + (c\bar{c}))$ is universal, we estimate that 70K $K\eta_c$ events are directly produced, 8.1K events arise in the cascade $B \to K\eta_{c2} \to K\gamma(280\text{ MeV})h_c \to K\gamma(280\text{ MeV})\gamma(500\text{ MeV})\eta_c$, and 11.7K events arise from $B \to Kh_c \to K\gamma(500\text{ MeV})\eta_c$, and that the sample that yielded the 39 ± 11 η'_c discovery events was about 30K events. Likewise, the large radiative branching ratios of ψ_2 to $\chi_{c1,2}$ and of $\chi_{c1,2}$ to J/ψ provide another striking double-gamma transition with $B(B \to X\psi_2)\sum_{J=1,2}B(\psi_2 \to \chi_{cJ}\gamma)B(\chi_{cJ} \to J/\psi\gamma)/B(B \to XJ/\psi) \simeq 0.12$. The signal $B \to K\eta_{c2} \to K\gamma(280\text{ MeV})h_c \to K\gamma(280\text{ MeV}) + \text{hadrons}$ may also provide a simultaneous observation of η_{c2} and h_c.

We close with a few examples of the insights to be expected from the discovery and investigation of the missing charmonium levels. The displacement of the $1^1\mathrm{P}_1$ h_c from the $1^3\mathrm{P}_J$ centroid is sensitive to Lorentz structure of the interquark potential. The ψ'-η'_c splitting is sensitive to a number of influences beyond simple potential

TABLE IV: Measured and estimated branching fractions for B decays to quarkonium levels.

$c\bar{c}$ state		$\Gamma(B \to (c\bar{c}) + X)/\Gamma(B \to \text{all})$ (%)
1^1S_0	η_c	$\approx 0.53^a$
1^3S_1	J/ψ	$0.789 \pm 0.010 \pm 0.034^{b\,c}$
1^1P_1	h_c	0.132 ± 0.060^d
1^3P_0	χ_{c0}	0.029 ± 0.012^d
1^3P_1	χ_{c1}	$0.353 \pm 0.034 \pm 0.024^{b\,e}$
1^3P_2	χ_{c2}	$0.137 \pm 0.058 \pm 0.012^b$
2^1S_0	η_c'	$\approx 0.18^a$
2^3S_1	ψ'	$0.275 \pm 0.020 \pm 0.029^b$
1^1D_2	η_{c2}	0.23^f
1^3D_1	ψ	0.28^f
1^3D_2	ψ_2	0.46^f
1^3D_3	ψ_3	0.65^f

[a] Scaled from $^3\text{S}_1$ rate.
[b] Data from [32] and [33].
[c] Known feed-down from 2S state removed.
[d] Scaled from $^3\text{P}_{1,2}$ rates using Eqn. (6).
[e] Known feed-down from 2S and 1P states removed.
[f] Computed; see [34].

models, including the effect of virtual decay channels [37]. The positions of η_{c2} and ψ_2 will further constrain analytic calculations of spin-dependent forces. When compared with $\psi(3770)$, they will provide another test of the influence of decay channels in the charm threshold region. Observation of $\psi(3770)$ in $B \to K^{(*)}D\bar{D}$ will help to calibrate expectations for the production of the narrow states. The same final state might yield evidence for ψ_3 $^3\text{D}_3$. The details of $\psi(3770)$ decays are sensitive to S-D mixing [27].

Outlook: The discovery of η_c' as a product of B decays realizes a long-held hope and raises new possibilities for filling out the charmonium spectrum. The CP violation experiments will enrich our knowledge of $B \to (c\bar{c}) + X$, aiding our ability to estimate the production of unknown states.

We thank Steve Olsen for posing the question that stimulated this work. KL's research was supported in part by a Fermilab Frontier Fellowship and by the Department of Energy under Grant No. DE–FG02–91ER40676. Fermilab is operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the U.S. Department of Energy.

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[38] For quarks bound in a central potential, it is convenient to separate the Schrödinger wave function into radial and

angular pieces as $\Psi_{n\ell m}(\vec{r}) = R_{n\ell}(r) Y_{\ell m}(\theta, \varphi)$, where n is the principal quantum number, ℓ and m are the orbital angular momentum and its projection, $R_{n\ell}(r)$ is the radial wave function, and $Y_{\ell m}(\theta, \varphi)$ is a spherical harmonic. It is also useful to introduce the reduced radial wave function, $u_{n\ell}(r) = r R_{n\ell}(r)$.

[39] A theoretical (lattice) calculation of the color-octet matrix elements is needed.

[40] In writing Eqn. (6), we have absorbed coefficient functions ($C_\pm$) and m_B dependence into our $\widetilde{H}$.