



## Measuring the Higgs Boson Self Coupling at the LHC and Finite Top Mass Matrix Elements

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Inclusive Standard Model Higgs boson pair production and subsequent decay to same-sign dileptons via weak gauge  $W^\pm$  bosons at the CERN Large Hadron Collider has the capability to determine the Higgs boson self-coupling,  $\lambda$ . The large top quark mass limit is found not to be a good approximation for the signal if one wishes to utilize differential distributions in the analysis. We find that it should be possible at the LHC with design luminosity to establish that the Standard Model Higgs boson has a non-zero self-coupling and that  $\lambda/\lambda_{SM}$  can be restricted to a range of 0–3.7 at 95% confidence level if its mass is between 150 and 200 GeV.

The CERN Large Hadron Collider (LHC) is widely regarded as capable of directly observing the agent responsible for electroweak symmetry breaking and fermion mass generation. This is generally believed to be a light Higgs boson with mass  $M_H < 200$  GeV [1]. Furthermore, the LHC promises complete coverage of Higgs decay scenarios [2], including general parameterizations in the Minimal Supersymmetric Standard Model [2,3], invisible Higgs decays [4], and possibly even Higgs boson decays to muons [5]. This broad capability was made possible largely by the addition of the weak boson fusion production channel to the search strategies [3,6]. Observation of a Higgs boson in this channel also contains additional information in the angular distributions of the scattered quarks which reveal the fundamental tensor structure of the  $VVH$  vertex [7]. With mild theoretical assumptions, the Higgs boson total width,  $\Gamma_H$ , can be determined via combination of all available channels, which in turn yields the gauge and various Yukawa couplings [8,9]. The weak boson fusion channels have received considerable attention in the LHC experimental collaborations, and a number of detailed detector simulation studies on them have already been completed, with very encouraging results [10].

While these studies have shown that the LHC promises broad and significant capability to measure various properties of the Higgs sector, what remains is to determine the actual Higgs potential. This appears in the Lagrangian as

$$V(\Phi) = -\lambda v^2 (\Phi^\dagger \Phi) + \lambda (\Phi^\dagger \Phi)^2,$$

where  $\Phi$  is the Higgs field,  $v = (\sqrt{2}G_F)^{-1/2}$  is the vacuum expectation value, and  $G_F$  is the Fermi constant. In the Standard Model (SM),  $\lambda = \lambda_{SM} = M_H^2/(2v^2)$ . Regarding the SM as an effective theory,  $\lambda$  is *per se* a free parameter. Its upper limit can be determined using unitarity arguments, assuming the model's validity to high energy scales [11]. To measure  $\lambda$ , and thus determine the Higgs potential, at a minimum ex-

periments must observe Higgs boson pair production; while this has been shown to be possible for a light Higgs boson at a future Linear Collider [12], no study has yet been presented which suggests this is possible at the LHC for the SM Higgs boson.

We simulate the signal process, pair production of two SM Higgs bosons in gluon fusion, at the parton level for  $pp$  collisions at  $\sqrt{s} = 14$  TeV. Both Higgs bosons are decayed to  $W$  boson pairs, which subsequently are decayed to four jets and two same-sign leptons<sup>1 2</sup>:

$$gg \rightarrow HH \rightarrow (W^+W^-)(W^+W^-) \rightarrow (jj\ell^\pm\nu)(jj\ell'^\pm\nu),$$

where  $\ell, \ell'$  are any combination of electrons or muons. The intermediate Higgs and  $W$  bosons are treated off-shell using finite widths in the double pole approximation. We calculate the signal using two methods: exact loop matrix elements [14] and the infinite top quark mass limit. The latter, which is commonly used in place of exact matrix elements to speed up the calculation, reproduces the correct total cross section for  $HH$  production to within 10% to 30% for Higgs masses between 140 GeV and 200 GeV. However, it produces completely incorrect kinematic distributions.

Signal results are computed consistently to leading order QCD with the top quark mass set to  $m_t = 175$  GeV and SM top Yukawa coupling, and the renormalization and factorization scales are taken to be the Higgs boson mass [14]. The effects of next-to-leading (NLO) order QCD corrections are included by multiplying the differential cross section by an overall factor  $K = 1.65$  ( $K$ -factor), as suggested by Ref. [15] where the QCD corrections for  $gg \rightarrow HH$  have been computed in the large  $m_t$  limit. The multiplicative effects of NLO QCD corrections are not expected to depend on whether the signal is calculated with exact matrix elements or in the infinite top quark mass limit.

The SM backgrounds of interest are those that produce two same-sign leptons and four well-separated jets which reconstruct in two pairs to a window around the  $W$  boson mass. The

<sup>1</sup>While our study was in progress, Ref. [13] appeared. It includes a brief discussion of Higgs boson pair production at an upgraded LHC, which would gather 20 times the amount of data expected in the first run.

<sup>2</sup>Unfortunately the search in this channel cannot be generalized to the supersymmetric case, since the branching fraction to  $W$  bosons is suppressed for the light Higgs scalar close to the decoupling regime.

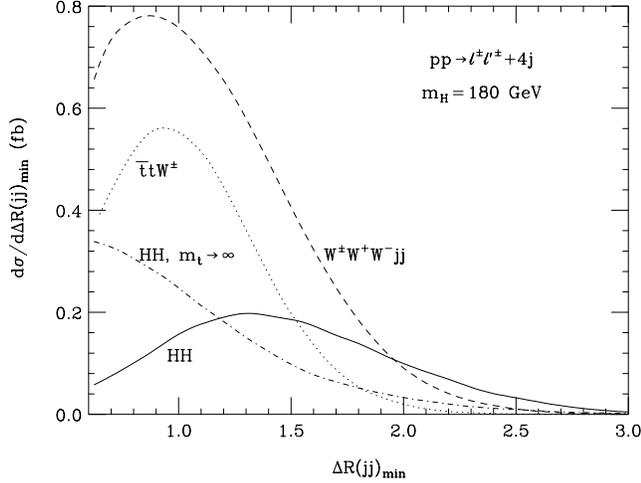


Figure 1. Minimum separation between jets,  $\Delta R(jj)_{\min}$ , for the  $M_H = 180$  GeV, signal with exact matrix elements (solid line) and in the large  $m_t$  limit (dot-dashed line), and the  $WWWjj$  (dashed line) and  $t\bar{t}W$  backgrounds (dotted line). Qualitatively similar results are obtained for other values of the Higgs boson mass in the range  $150 \text{ GeV} \leq M_H \leq 200 \text{ GeV}$ .

largest contribution originates from  $W^\pm W^+ W^- jj$  production, followed by  $t\bar{t}W^\pm$  where one top quark decays leptonically, the other hadronically, and neither  $b$  quark jet is tagged. Other backgrounds, which in sum contribute at the  $< 5\%$  level [13], are:  $t\bar{t}t\bar{t}$  production, where none of the  $b$  quark jets are tagged, and additional jets or leptons are not observed;  $W^\pm Z jjjj$  production with leptonic  $Z$  decay (including off-shell photon interference) where one lepton is not observed; and  $t\bar{t}j$  events where one  $b$  quark decays semileptonically with good hadronic isolation, and the other is not tagged. For this letter we consider only the two dominant backgrounds; since the others enter in sum at less than 5% of the total contribution, they do not change our results noticeably.

We simulate both leading backgrounds at the parton level using exact matrix elements generated with MADGRAPH [16]. For  $WWWjj$  production we evaluate the strong coupling constant  $\alpha_s$  and the parton distribution functions at a scale  $\mu$  given by  $\mu^2 = \sum p_T^2$ , where the sum extends over all final state particles; for  $t\bar{t}W$  production we take  $\mu = m_t + M_W/2$ . We use a value for the strong coupling constant of  $\alpha_s(M_Z) = 0.1185$ . QCD corrections are not taken into account in our calculation of  $WWWjj$  and  $t\bar{t}W$  production. The top quarks are generated on shell (narrow width approximation), while all  $W$  bosons in both processes are allowed to be off shell. Assuming a  $b$  quark tagging efficiency of 50%, only 1/4 of the  $t\bar{t}W$  rate contributes to the background; events with one or two tagged  $b$  quarks are rejected. All signal and background cross sections are calculated using CTEQ4L [17] parton distribution functions.

The kinematic acceptance cuts for both signal and backgrounds are:

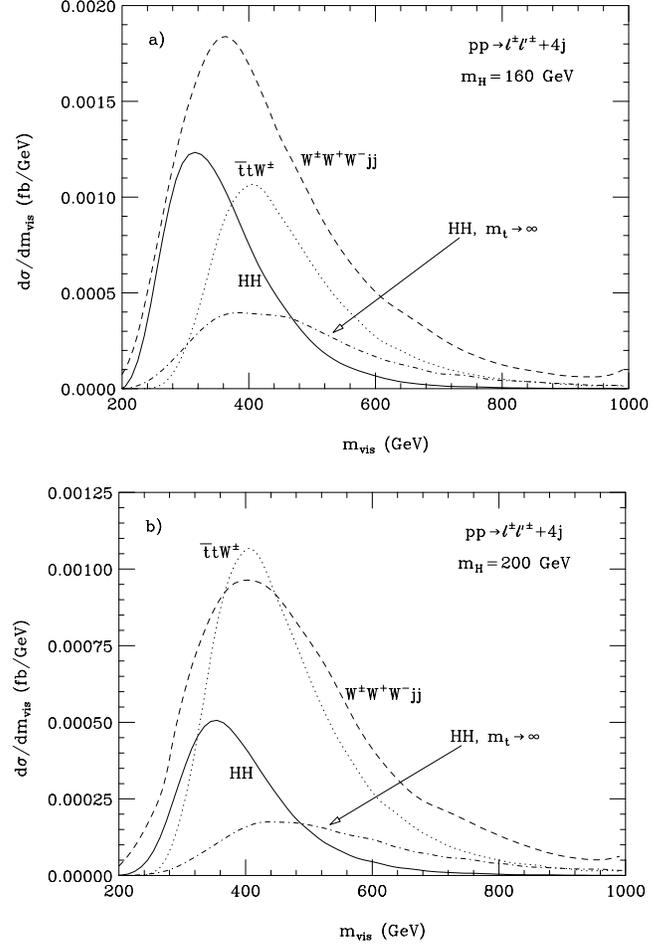


Figure 2. Distribution of the invariant mass of all observable final state particles,  $m_{\text{vis}}$ , after all cuts, for the signal with a)  $M_H = 160$  GeV and b)  $M_H = 200$  GeV, and the dominant backgrounds. The  $m_{\text{vis}}$  distribution of the signal evaluated in the large  $m_t$  limit is also shown.

$$\begin{aligned}
 p_T(j) &> 30, 30, 20, 20 \text{ GeV}, & p_T(\ell) &> 15, 10 \text{ GeV}, \\
 |\eta(j)| &< 3.0, & |\eta(\ell)| &< 2.5, \\
 \Delta R(jj) &> 0.6, & \Delta R(j\ell) &> 0.4, & \Delta R(\ell\ell) &> 0.2,
 \end{aligned}$$

where  $\Delta R = [(\Delta\phi)^2 + (\Delta\eta)^2]^{1/2}$  is the separation in the pseudorapidity – azimuthal angle plane. In addition we require the four jets to combine into two pseudo- $W$  pairs with invariant masses between 50 and 110 GeV and assume that this captures 100% of the signal and backgrounds. We do not impose a missing transverse momentum cut which would remove a considerable fraction of the signal events.

Both backgrounds are multi body production processes, and one therefore expects that the distribution of the invariant mass,  $\sqrt{s}$ , of the system peaks at values significantly above threshold. In contrast, the signal is a two-body production process for which the  $\sqrt{s}$  distribution will exhibit a sharper threshold behavior. Since the Higgs bosons are produced

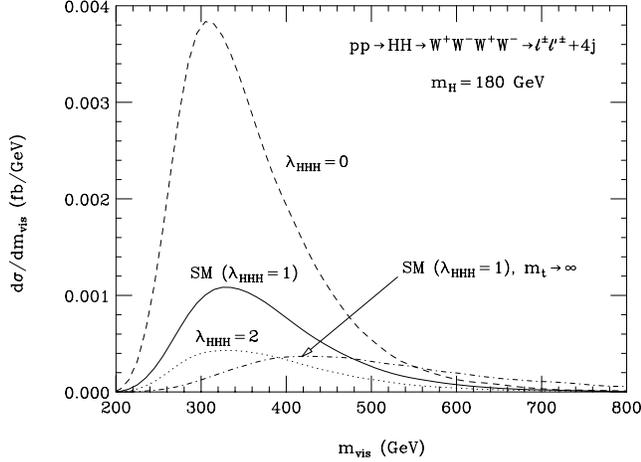


Figure 3. The  $m_{vis}$  distribution of the signal for  $M_H = 180$  GeV in the SM (solid curve), for  $\lambda_{HHH} = \lambda/\lambda_{SM} = 0$  (dashed line) and for  $\lambda_{HHH} = 2$  (dotted line). The dot-dashed line shows the SM cross section in the large  $m_t$  limit. Qualitatively similar results are obtained for other values of  $M_H$ .

almost at rest, the final state particles are distributed fairly isotropically, resulting in a distribution of the minimum jet-jet separation,  $\Delta R(jj)_{\min}$ , which is peaked at  $\Delta R(jj)_{\min} \approx 1.3$  (see Fig. 1). In contrast, the  $\Delta R(jj)_{\min}$  distribution for the background processes peaks at a lower value ( $\Delta R(jj)_{\min} \approx 0.9$ ). In the following, we therefore impose a more restrictive jet-jet separation cut of  $\Delta R(jj) > 1.0$ , which results in a  $\approx 45\%$  reduction of the background cross sections while reducing the signal only by about 7 – 8%. Note that in the large  $m_t$  limit the  $\Delta R(jj)_{\min}$  distribution of the signal peaks at  $\Delta R(jj)_{\min} = 0$  and drastically differs in shape from that calculated using the exact loop matrix elements. If one were to calculate the signal cross section in the large  $m_t$  limit, a  $\Delta R(jj)$  cut would not result in a reduction of the background.

Unfortunately, with two neutrinos present in the final state,  $\sqrt{s}$  cannot be reconstructed. However, we anticipate that the invariant mass of all observed final state leptons and jets,  $m_{vis}$ , will retain most of the expected behavior of the different production processes. Figure 2 clearly demonstrates that this is the case: the signal peaks at lower values of  $m_{vis}$  than either background, especially for lower Higgs boson masses. However, the  $WWWjj$  background has a significant contribution from  $WH(\rightarrow W^+W^-)jj$  production, resulting in a  $m_{vis}$  distribution which is similar in shape to that of the  $HH$  signal. Whereas the signal is concentrated in the region  $m_{vis} < 500$  GeV, the background processes have a significant tail extending to  $m_{vis} = 1$  TeV. This makes it possible to normalize the background using data from the  $m_{vis} > 500$  GeV region. Using exact loop matrix elements, the signal displays a pronounced peak which gradually moves to higher values of  $m_{vis}$  with increasing Higgs boson mass. In contrast, in the large  $m_t$  limit, the  $m_{vis}$  distribution of the signal is extremely broad.

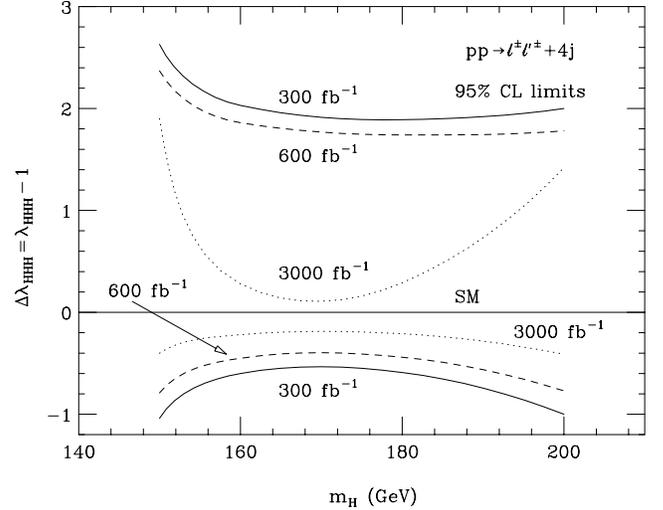


Figure 4. Limits achievable at 95% CL for  $\Delta\lambda_{HHH} = \lambda_{HHH} - 1$  ( $\lambda_{HHH} = \lambda/\lambda_{SM}$ ) in  $pp \rightarrow \ell^\pm \ell'^\pm + 4j$  at the LHC. Bounds are shown for integrated luminosities of  $300 \text{ fb}^{-1}$  (solid lines),  $600 \text{ fb}^{-1}$  (dashed lines) and  $3000 \text{ fb}^{-1}$  (dotted lines). The allowed region is between the two lines of equal texture. The Higgs boson self-coupling vanishes for  $\Delta\lambda_{HHH} = -1$ .

The Feynman diagrams contributing to  $gg \rightarrow HH$  in the SM consist of fermion triangle and box diagrams [14]. Non-standard Higgs boson self-couplings only affect the triangle diagrams with a Higgs boson exchanged in the  $s$ -channel. They only contribute to the  $J = 0$  partial wave, and thus impact the  $m_{vis}$  distribution mostly at small values. This is illustrated in Fig. 3 for  $M_H = 180$  GeV and two values of  $\lambda_{HHH} = \lambda/\lambda_{SM}$ . Since box and triangle diagrams interfere destructively, the  $gg \rightarrow HH$  cross section for  $1 < \lambda_{HHH} < 2.7$  is smaller than in the SM. The absence of a Higgs boson self-coupling ( $\lambda_{HHH} = 0$ ) results in a Higgs pair production cross section which is about a factor 3 larger than the SM result.

The shape change of the  $m_{vis}$  distribution induced by non-standard values of  $\lambda_{HHH}$  can be used to derive quantitative sensitivity bounds on the Higgs boson self-coupling. Figure 4 shows the 95% confidence level (CL) limits for  $\Delta\lambda_{HHH} = \lambda_{HHH} - 1$  which are obtained from a  $\chi^2$  test of the  $m_{vis}$  distribution. The allowed region is between the two lines of equal texture. In deriving the bounds displayed, we combine channels with electrons and muons in the final state, conservatively assuming a common lepton identification efficiency of  $\epsilon = 0.85$  for each lepton. In order to approximately take into account the (small) contributions to the background from  $t\bar{t}\bar{t}$ ,  $WZ + 4j$  and  $t\bar{t}j$  production which we have ignored in our analysis, we scale the background differential cross section by a factor 1.1. As mentioned before, our calculation of the background processes does not include QCD corrections which are expected to modify the relevant cross sections by 20 – 40%. In order to derive realistic limits, we therefore allow for a normal-

ization uncertainty of 30% of the SM cross section. Since the background cross section can be directly determined from the high  $m_{vis}$  region with a statistical precision of 15% or better for the assumed integrated luminosities, the bounds we derive are conservative.

We derive sensitivity limits for integrated luminosities of  $300 \text{ fb}^{-1}$ ,  $600 \text{ fb}^{-1}$  and  $3000 \text{ fb}^{-1}$ , and Higgs boson masses in the range  $150 \text{ GeV} \leq M_H \leq 200 \text{ GeV}$ . Outside this range, the number of signal events is too small to yield meaningful bounds. For  $M_H < 150 \text{ GeV}$ , this is due to the small  $H \rightarrow W^*W$  branching ratio. For  $M_H > 200 \text{ GeV}$ , the  $gg \rightarrow HH$  cross section is too small. An integrated luminosity of  $300 \text{ fb}^{-1}$  ( $600 \text{ fb}^{-1}$ ) corresponds to 3 years of running at the LHC design luminosity with one (two) detectors. The larger value of  $3000 \text{ fb}^{-1}$  can be achieved in about 3 years of running if the planned luminosity upgrade to  $\mathcal{L} = 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$  [13] is realized. Figure 4 demonstrates that, for  $300 \text{ fb}^{-1}$ , a vanishing Higgs boson self-coupling ( $\Delta\lambda_{HHH} = -1$ ) is excluded at the 95% CL or better, and that  $\lambda$  can be determined with a precision of up to 60%. Doubling the integrated luminosity to  $600 \text{ fb}^{-1}$  improves the sensitivity by 10 – 25%. For  $300 \text{ fb}^{-1}$  and  $600 \text{ fb}^{-1}$ , the bounds for positive values of  $\Delta\lambda_{HHH}$  are significantly weaker than those for  $\Delta\lambda_{HHH} < 0$ , due to the limited number of signal events in this region of parameter space. For  $3000 \text{ fb}^{-1}$ , the Higgs boson self-coupling can be determined with an accuracy of 20 – 30% for  $160 \text{ GeV} \leq M_H \leq 180 \text{ GeV}$ .

In summary, inclusive pair production of Higgs bosons at the LHC, with decays to a same-sign lepton pair and four jets via four  $W$  bosons, will make it possible to perform a first, albeit not very precise, measurement of the Higgs boson self-coupling  $\lambda$ . The non-vanishing of  $\lambda$  can be established at 95% CL or better for  $150 \text{ GeV} \leq M_H \leq 200 \text{ GeV}$ . The bounds on  $\lambda$  derived here should be viewed as approximate. They can probably be strengthened by including other final states such as  $3\ell + 2j$ , or by using more powerful statistical tools than the simple  $\chi^2$  test we performed. More details of our analysis, along with inclusion of additional channels, will be presented elsewhere [18].

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