

Two-Loop Results for M_W in the Standard Model and the MSSM

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Recent higher-order results for the prediction of the W-boson mass, M_W , within the Standard Model are reviewed and an estimate of the remaining theoretical uncertainties of the electroweak precision observables is given. An updated version of a simple numerical parameterisation of the result for M_W is presented. Furthermore, leading electroweak two-loop contributions to the precision observables within the MSSM are discussed.

1. INTRODUCTION

The comparison of electroweak precision measurements with the theoretical predictions allows to test the electroweak theory at the quantum level. In this way indirect constraints on unknown parameters of the theory can be obtained, in particular constraints on the Higgs-boson mass, M_H , within the Standard Model (SM) and constraints on the parameters of the Higgs and scalar top and bottom sector within the Minimal Supersymmetric extension of the SM (MSSM).

Fig. 1 shows the result of a global fit to all data within the SM [1,2]. The theoretical predictions are affected by two kinds of uncertainties: the uncertainties from unknown higher-order corrections, indicated by a “blue band” in Fig. 1, and uncertainties from experimental errors of the input parameters, indicated in Fig. 1 by two fit curves corresponding to two different values of $\Delta\alpha_{\text{had}}^{(5)}$, the hadronic contribution to the shift in the fine structure constant (the experimental error of the top-quark mass, m_t , is directly included in the fit). The upper plot in Fig. 1 shows the result based on the most recent data (summer 2002 [1]), and the currently best estimate of the theoretical uncertainties from unknown higher-order corrections, while the lower plot shows the fit result based on the previous estimate of the theoretical uncertainties and the winter 2001 data [2]. The comparison in Fig. 1

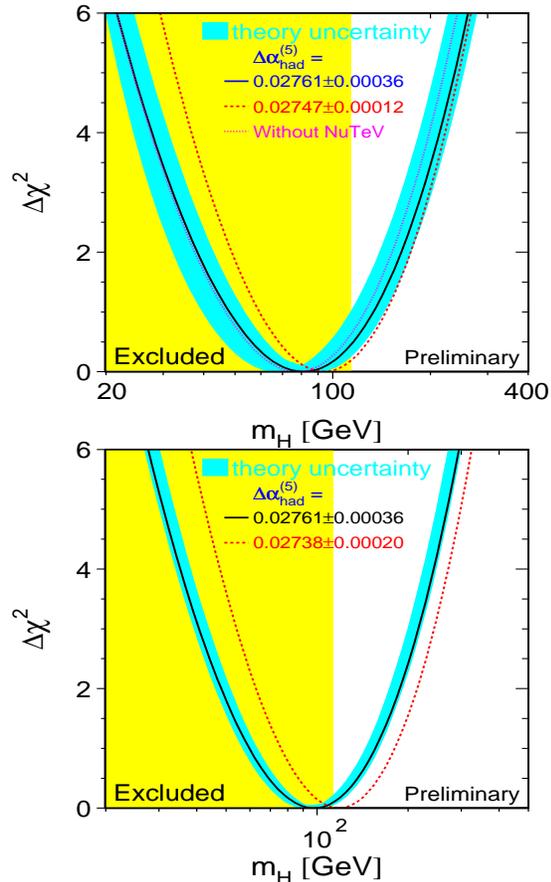


Figure 1. Global fit to all data in the SM: comparison of the present fit (upper plot) with the one of the winter 2001 conferences (lower plot).

*Talk presented by G. Weiglein.

shows that the present estimate of the theoretical uncertainties from unknown higher-order corrections yields a *larger* value than the previous estimate. This was triggered by the recently obtained result for the complete fermionic two-loop corrections to the W-boson mass, M_W , in the SM [3], leading to an improved estimate of the remaining theoretical uncertainties in the prediction for the leptonic effective weak mixing angle, $\sin^2 \theta_{\text{eff}}$ [4].

In the following section the present status of the prediction for M_W in the SM is reviewed and an estimate of the remaining theoretical uncertainties of the electroweak precision observables is given. Sect. 3 summarises the impact of new electroweak two-loop contributions on the precision observables within the MSSM [5].

2. PREDICTION FOR M_W IN THE SM

The prediction for M_W is obtained from relating the result for the muon lifetime within the SM (and analogously for the MSSM) to the definition of the Fermi constant, G_μ (by convention, the QED corrections within the Fermi Model, which are known up to two-loop order [6], are split off in the defining equation for G_μ). This leads to the relation

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi\alpha}{\sqrt{2}G_\mu} (1 + \Delta r), \quad (1)$$

where the radiative corrections are summarised in the quantity Δr . The one-loop result for Δr [7] has first been improved by resummations of the leading one-loop contributions from fermion loops [8]. Concerning irreducible two-loop contributions, the $\mathcal{O}(\alpha\alpha_s)$ [9] corrections are known for some time, while in the electroweak sector results have been restricted until recently to asymptotic expansions for large Higgs [10] and top-quark masses [11].

Complete results for the fermion-loop contributions at two-loop order were first obtained for the Higgs-mass dependence of M_W in Ref. [12]. The full result for the fermion-loop contributions at two-loop order was derived in Refs. [3,4]. Fig. 2 shows the relative importance of the different contributions with one closed fermion loop to Δr at two-loop order, whose sum is denoted

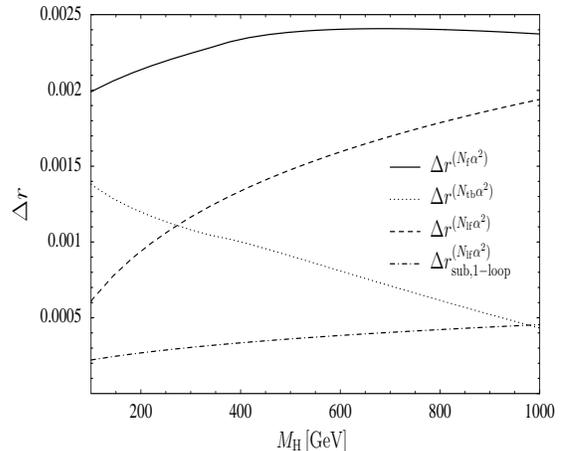


Figure 2. Relative importance of different two-loop contributions to Δr with one closed fermion loop as a function of the Higgs-boson mass, see text.

by $\Delta r^{(N_f \alpha^2)}$. It can be seen that both corrections with a top-/bottom-loop, $\Delta r^{(N_t b \alpha^2)}$, and with a light-fermion loop, $\Delta r^{(N_l \alpha^2)}$, yield important contributions. It should be noted, however, that the light-fermion contribution contains the numerically relatively large term $2\Delta\alpha\Delta r_{\text{bos}}^{(\alpha)}$, which can easily be separated from the genuine two-loop contribution of the light fermions. In order to investigate the numerical relevance of the latter, in Fig. 2 also the difference $\Delta r_{\text{sub,1-loop}}^{(N_l \alpha^2)} = \Delta r^{(N_l \alpha^2)} - 2\Delta\alpha\Delta r_{\text{bos}}^{(\alpha)}$ is shown. While these genuine light-fermion two-loop contributions do not exceed the top-/bottom contributions for any value of the Higgs-boson mass below 1 TeV, they nevertheless amount up to 3.3×10^{-4} , which corresponds to a shift in M_W of more than 5 MeV.

Recently also the Higgs-mass dependence of the purely bosonic two-loop corrections became available [4]. Finally, the full result for the purely bosonic two-loop corrections has been obtained in Ref. [13], completing in this way the calculation of muon decay at the two-loop level. The numerical effect of the purely bosonic two-loop corrections turned out to be relatively small, giving rise to a shift in M_W of less than ± 1 MeV for $M_H \leq 1$ TeV. From the higher-order contributions to Δr (for a discussion, see e.g. Ref. [4])

only the top-bottom contributions at $\mathcal{O}(\alpha_s^2)$ [14] were found to be non-negligible in view of the present experimental accuracies.

Below a simple parameterisation of the result for M_W is given, being based on taking into account the following contributions to Δr ,

$$\begin{aligned} \Delta r = & \Delta r^{(\alpha)} + \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} \\ & + \Delta r^{(N_f\alpha^2)} + \Delta r^{(N_f^2\alpha^2)} + \Delta r^{(\alpha^2,\text{bos})}, \end{aligned} \quad (2)$$

where $\Delta r^{(\alpha)}$ is the one-loop result, $\Delta r^{(\alpha\alpha_s)}$ and $\Delta r^{(\alpha\alpha_s^2)}$ are the two-loop and three-loop QCD corrections, $\Delta r^{(N_f\alpha^2)}$ and $\Delta r^{(N_f^2\alpha^2)}$ are the electroweak two-loop contributions with one and two fermion loops, respectively, and $\Delta r^{(\alpha^2,\text{bos})}$ is the purely bosonic two-loop contribution according to the expression given in Ref. [15]. The numerically rather small electroweak higher-order corrections have been neglected here. The parameterisation of the result for M_W reads

$$\begin{aligned} M_W = & M_W^0 - d_1 \text{dH} - d_2 \text{dH}^2 + d_3 \text{dH}^4 \\ & - d_4 \text{d}\alpha + d_5 \text{d}t - d_6 \text{d}t^2 - d_7 \text{dH dt} \\ & - d_8 \text{d}\alpha_s + d_9 \text{d}Z, \end{aligned} \quad (3)$$

where the dependence on the variables M_H , m_t , α , α_s and M_Z is expressed by $\text{dH} = \ln(M_H/(100 \text{ GeV}))$, $\text{d}t = (m_t/(174.3 \text{ GeV}))^2 - 1$, $\text{d}\alpha = \Delta\alpha/0.05924 - 1$, $\text{d}\alpha_s = \alpha_s(M_Z)/0.119 - 1$, and $\text{d}Z = M_Z/(91.1875 \text{ GeV}) - 1$. The coefficients d_1, \dots, d_9 have the following values (in GeV)

$$\begin{aligned} M_W^0 = & 80.3757, & d_5 = & 0.5236, \\ d_1 = & 0.05515, & d_6 = & 0.0727, \\ d_2 = & 0.009803, & d_7 = & 0.00541, \\ d_3 = & 0.0006078, & d_8 = & 0.0765, \\ d_4 = & 1.078, & d_9 = & 115.0. \end{aligned} \quad (4)$$

Employing these coefficients, the simple parameterisation of eq. (3) approximates the full result for M_W based on the contributions given in eq. (2) with an accuracy of better than 0.3 MeV for $65 \text{ GeV} \leq M_H \leq 1 \text{ TeV}$ and 2σ variations of all other experimental input values. This formula, which includes the recently obtained result

for $\Delta r^{(\alpha^2,\text{bos})}$ [13,15], updates the parameterisation given in Ref. [3]. As discussed above, the corresponding shift in M_W lies within $\pm 1 \text{ MeV}$.

The remaining theoretical uncertainties of the electroweak precision observables from unknown higher-order corrections, taking into account all known contributions, can be estimated with the methods described in Refs. [4,16] as:

$$\delta M_W^{\text{th}} \approx \pm 6 \text{ MeV}, \quad \delta \sin^2 \theta_{\text{eff}}^{\text{th}} \approx \pm 7 \times 10^{-5}. \quad (5)$$

They are smaller at present than the parametric uncertainties from the experimental errors of the input parameters m_t and $\Delta\alpha_{\text{had}}$. The experimental errors of $\delta m_t = \pm 5.1 \text{ GeV}$ [1] and $\delta(\Delta\alpha_{\text{had}}) = 36 \times 10^{-5}$ [1] induce theoretical uncertainties of

$$\begin{aligned} \delta M_W^{\text{th}} \approx & \pm 31 \text{ MeV}, \quad \delta \sin^2 \theta_{\text{eff}}^{\text{th}} \approx \pm 16 \times 10^{-5}, \\ \delta M_W^{\text{th}} \approx & \pm 6.5 \text{ MeV}, \quad \delta \sin^2 \theta_{\text{eff}}^{\text{th}} \approx \pm 13 \times 10^{-5}, \end{aligned} \quad (6)$$

respectively. For comparison, the present experimental errors of M_W and $\sin^2 \theta_{\text{eff}}$ are [1]

$$\delta M_W^{\text{exp}} \approx \pm 34 \text{ MeV}, \quad \delta \sin^2 \theta_{\text{eff}}^{\text{exp}} \approx \pm 17 \times 10^{-5}.$$

At the next generation of colliders, i.e. RunII of the Tevatron, the LHC and an e^+e^- Linear Collider running at the Z-boson resonance and the WW-threshold, these experimental errors will be reduced to about (see Ref. [16] and references therein)

$$\delta M_W^{\text{exp}} \approx \pm 6-7 \text{ MeV}, \quad \delta \sin^2 \theta_{\text{eff}}^{\text{exp}} \approx \pm 1 \times 10^{-5}. \quad (7)$$

At the same time, improved measurements will also reduce the parametric uncertainty from the experimental errors of the input parameters to about [16,17].

$$\delta M_W^{\text{th}} \approx \pm 2 \text{ MeV}, \quad \delta \sin^2 \theta_{\text{eff}}^{\text{th}} \approx \pm 2 \times 10^{-5}. \quad (8)$$

Further work on higher-order corrections will clearly be needed in order to reduce the uncertainties from unknown higher-order corrections below the level of eqs. (7), (8).

3. LEADING ELECTROWEAK 2-LOOP CORRECTIONS IN THE MSSM

The situation concerning theoretical uncertainties of the electroweak precision observables M_W and $\sin^2 \theta_{\text{eff}}$ from unknown higher-order corrections within the MSSM is significantly worse than in the SM. Comparing the available results for higher-order corrections in both models, the uncertainties from unknown higher-order corrections within the MSSM can be estimated to be at least a factor of 2 larger than the ones in the SM as given in eq. (5).

The leading higher-order corrections from quark and squark loops enter via the quantity $\Delta\rho$,

$$\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}, \quad (9)$$

where $\Sigma_{Z,W}(0)$ denote the transverse parts of the unrenormalised Z- and W-boson self-energies at zero momentum transfer, respectively. Within the MSSM, the two-loop corrections of $\mathcal{O}(\alpha\alpha_s)$ to $\Delta\rho$ [18] as well as the gluonic two-loop corrections to Δr [19] have been obtained. Concerning electroweak two-loop corrections, in the limit of a large SUSY scale, $M_{\text{SUSY}} \gg M_Z$, where the SUSY particles decouple, the contributions in the MSSM reduce to those of a Two-Higgs-Doublet model with MSSM restrictions. As a first result in this context, the $\mathcal{O}(\alpha_t^2)$ corrections in the limit where the lightest \mathcal{CP} -even Higgs boson mass vanishes, i.e. $m_h \rightarrow 0$, have been obtained in Ref. [20]. Recently the $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_t\alpha_b)$ and $\mathcal{O}(\alpha_b^2)$ contributions to $\Delta\rho$ for $M_{\text{SUSY}} \gg M_Z$ have been evaluated for arbitrary values of m_h [5]. As in the case of the SM, the numerical effect of going to non-vanishing values of the Higgs-boson mass turned out to be sizable.

In Fig. 3 the numerical effect of the $\mathcal{O}(\alpha_t^2)$ corrections on M_W is analysed. In addition to the MSSM $\mathcal{O}(\alpha_t^2)$ correction to δM_W also the ‘effective’ change from the SM result (where the value of the SM Higgs boson mass has been set to m_h) to the new MSSM result is shown. The parameters in Fig. 3 are chosen according to the m_h^{max} benchmark scenario [21], i.e. $M_{\text{SUSY}} = 1 \text{ TeV}$, $X_t = 2M_{\text{SUSY}}$, where $m_t X_t$ is the off-diagonal entry in the \tilde{t} mass matrix. The other

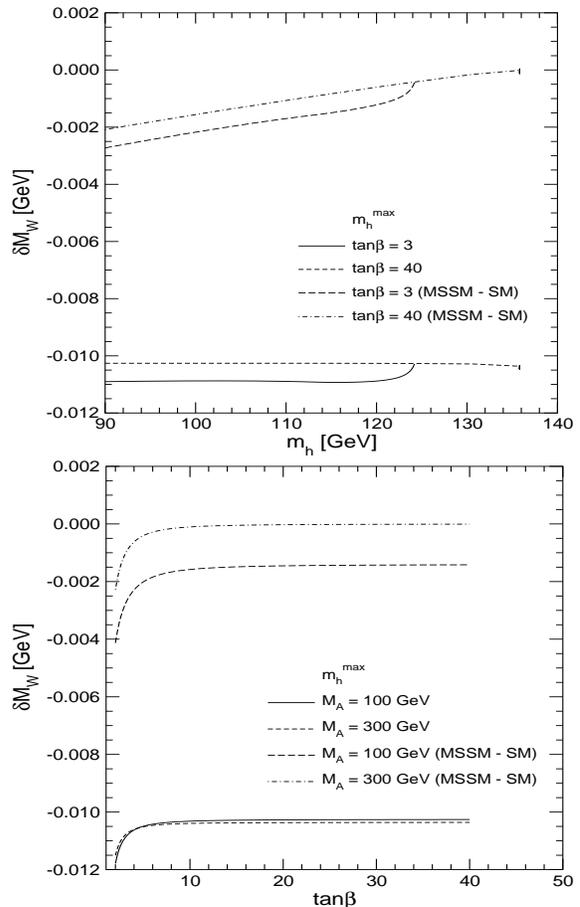


Figure 3. Contribution of the $\mathcal{O}(\alpha_t^2)$ MSSM corrections to M_W as a function of m_h (upper plot) and $\tan\beta$ (lower plot).

parameters are $\mu = 200 \text{ GeV}$, $A_b = A_t$. The Higgs-boson mass m_h is obtained in the upper plot from varying M_A from 50 GeV to 1000 GeV, while keeping $\tan\beta$ fixed at $\tan\beta = 3, 40$. In the lower plot, $\tan\beta$ is varied from 2 to 40, M_A is kept fixed at $M_A = 100, 300 \text{ GeV}$. The calculation of m_h from the other MSSM parameters contains corrections up to two-loop order, as implemented in the program *FeynHiggs* [22].

The effect of the $\mathcal{O}(\alpha_t^2)$ MSSM contributions on δM_W amounts up to -12 MeV . For large $\tan\beta$ it saturates at about -10 MeV . The ‘effective’ change in M_W in comparison with the corresponding SM result with the same value of

the Higgs-boson mass is significantly smaller. It amounts up to -3 MeV and goes to zero for large M_A as expected from the decoupling behaviour. For a small \mathcal{CP} -odd Higgs boson mass, $M_A = 100$ GeV, a shift of -2 MeV in M_W remains also in the limit of large $\tan\beta$, since the two Higgs doublet sector does not decouple from the MSSM. For large M_A , $M_A = 300$ GeV, for nearly all $\tan\beta$ values the effective change in M_W is small.

The absolute contribution for $\delta \sin^2 \theta_{\text{eff}}$ (which is not shown here) is around $+6 \times 10^{-5}$. The effective change ranges between $+3 \times 10^{-5}$ for small $\tan\beta$ and small M_A and approximately zero for large $\tan\beta$ and large M_A .

Acknowledgements: G.W. thanks the organisers of “RADCOR 2002 – Loops & Legs 2002” for the invitation and the pleasant atmosphere at the meeting.

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