Supernovae and Neutrinos

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A long-standing problem in supernova physics is how to measure the total energy and temperature of \(\nu_\mu\), \(\nu_\tau\), \(\bar{\nu}_\mu\), and \(\bar{\nu}_\tau\). While of the highest importance, this is very difficult because these flavors only have neutral-current detector interactions. We propose that neutrino-proton elastic scattering, \(\nu + p \rightarrow \nu + p\), can be used for the detection of supernova neutrinos in scintillator detectors. It should be emphasized immediately that the dominant signal is on free protons. Though the proton recoil kinetic energy spectrum is soft, with \(T_p \approx 2E_\nu^2/M_p\), and the scintillation light output from slow, heavily ionizing protons is quenched, the yield above a realistic threshold is nearly as large as that from \(\bar{\nu}_e + p \rightarrow e^+ + n\). In addition, the measured proton spectrum is related to the incident neutrino spectrum. The ability to detect this signal would give detectors like KamLAND and Borexino a crucial and unique role in the quest to detect supernova neutrinos. These results are now published: J. F. Beacom, W. M. Farr and P. Vogel, Phys. Rev. D 66, 033001 (2002) [arXiv:astro-ph/0205120]; the details are given there.

1. Introduction

When the next Galactic supernova occurs, approximately \(10^4\) detected neutrino events are expected among the several detectors around the world. It is widely believed that these \(10^4\) events will provide important clues to the astrophysics of the supernova as well as the properties of the neutrinos themselves. Interestingly, recent breakthroughs in understanding solar and atmospheric neutrinos each occurred when the accumulated samples of detected events first exceeded \(10^4\).

But will we have enough information to study the supernova neutrino signal in detail? Almost all of the detected events will be charged-current \(\bar{\nu}_e + p \rightarrow e^+ + n\), which will be well-measured, both because of the large yield and because the measured positron spectrum is closely related to the neutrino spectrum. Because of the charged-lepton thresholds, the flavors \(\nu_\mu\), \(\nu_\tau\), \(\bar{\nu}_\mu\), and \(\bar{\nu}_\tau\) can only be detected in neutral-current reactions, of which the total yield is expected to be approximately \(10^3\) events. However, in general one \textit{cannot} measure the neutrino energy in neutral-current reactions. This talk presents an exception. These four flavors are expected to carry away about \(2/3\) of the supernova binding energy, and are expected to have a higher temperature than \(\nu_e\) or \(\bar{\nu}_e\). However, there is no experimental basis for these statements, and present numerical models of supernovae cannot definitively address these issues either. If there is no spectral signature for the neutral-current detection reactions, then neither the total energy carried by these flavors nor their temperature can be separately determined from the detected number of events.

But it is crucial that these quantities be measured. Both are needed for comparison to numerical supernova models. The total energy is needed to determine the mass of the neutron star, and the temperature is needed for studies of neutrino oscillations. At present, such studies would suffer from the need to make model-dependent assumptions. This problem has long been known, but perhaps not widely enough appreciated. In this talk, I clarify this problem, and provide a realistic solution that can be implemented in two detectors, KamLAND (already operating) and Borexino (to be operating soon). The solution is based on neutrino-proton elastic scattering, which has never before been shown to be a realistic detection channel for low-energy neutrinos.

In this talk, I will focus on just the problem of measuring the temperature and total energy of \(\nu_\mu\), \(\nu_\tau\), \(\bar{\nu}_\mu\), and \(\bar{\nu}_\tau\), since everything else in understanding supernova neutrinos depends on it.
2. Cross Section

The cross section for neutrino-proton elastic scattering is an important prediction \( [8] \) of the Standard Model, and it has been confirmed by extensive measurements at GeV energies (see, e.g., Ref. \([9]\)). At the energies considered here, the full cross section formula \( [8] \) can be greatly simplified. The differential cross section as a function of neutrino energy \( E_\nu \) and struck proton recoil kinetic energy \( T_p \) (and mass \( M_p \)) is

\[
\frac{d\sigma}{dT_p} = \frac{G_F^2 M_p}{\pi} \left( 1 - \frac{M_p T_p}{2E_\nu} \right)c_v^2 + \left( 1 + \frac{M_p T_p}{2E_\nu} \right)c_A^2 \times \left( 1 - \frac{M_p T_p}{2E_\nu} \right)
\]

(1)

In this equation, we have taken \( (E_\nu - T_p)^2 \sim E_\nu^2 \) (i.e., keeping only the lowest order in \( E_\nu/M_p \), a very good approximation); the full expression was used in the calculations below. The neutral-current coupling constants between the exchanged \( Z^\ast \) and the proton are

\[
c_v = \frac{1 - 4\sin^2 \theta_W}{2} = 0.04, \quad (2)
\]

\[
c_A = \frac{1.27}{2}, \quad (3)
\]

where the factor 1.27 is determined by neutron beta decay. The cross section for antineutrinos is obtained by the substitution \( c_A \to -c_A \). As will be emphasized below, our results are totally independent of oscillations among active flavors, as this is a neutral-current reaction.

We consider only free proton targets; the small yield from bound protons creates a small background signal, as discussed in Ref. \([6]\). We use the struck proton kinetic energy in the laboratory frame as our kinematic variable, as appropriate to scintillator detectors. For a neutrino energy \( E_\nu \), \( T_p \) ranges between 0 and \( T_p^{\text{max}} \), where

\[
T_p^{\text{max}} = \frac{2E_\nu^2}{M_p + 2E_\nu} \approx \frac{2E_\nu^2}{M_p}. \quad (4)
\]

The maximum is obtained when the neutrino recoils backwards with its original momentum \( E_\nu \), and thus the proton goes forward with momentum \( 2E_\nu \). Since \( c_A \gg c_v \), the largest proton recoils are favored, which is optimal for detection.

3. Supernova Neutrinos

In this talk, I characterize the supernova neutrino signal in a very simple way, though consistently with numerical supernova models \([10]\). The change in gravitational binding energy between the initial stellar core and the final proto-neutron star is about \( 3 \times 10^{53} \) ergs, about \( 99\% \) of which is carried off by all flavors of neutrinos and antineutrinos over about 10 s. The emission time is much longer than the light-crossing time of the proto-neutron star because the neutrinos are trapped and must diffuse out, eventually escaping with approximately Fermi-Dirac spectra characteristic of the surface of last scattering. In the usual model, \( \nu_\mu \), \( \nu_e \) and their antiparticles are emitted with temperature \( T \sim 8 \) MeV, \( \bar{\nu}_e \) has \( T \approx 5 \) MeV, and \( \nu_\tau \) has \( T \approx 3.5 \) MeV. The temperatures differ from each other because \( \bar{\nu}_e \) and \( \nu_e \) have charged-current opacities (in addition to the neutral-current opacities common to all flavors), and because the proto-neutron star has more neutrons than protons. It is generally assumed that each of the six types of neutrino and antineutrino carries away about \( 1/6 \) of the total binding energy, though this has an uncertainty of at least \( 50\% \) \([11]\). The supernova rate in our Galaxy is estimated to be \( (3 \pm 1) \) per century (this is reviewed in Ref. \([12]\)).

The expected number of events (assuming a hydrogen to carbon ratio of \( 2:1 \)) is

\[
N = 70.8 \left[ \frac{E}{10^{53} \text{ erg}} \right] \left[ \frac{1 \text{ MeV}}{T} \right] \\
\times \left[ \frac{10 \text{ kpc}^2}{D} \right] \left[ \frac{M_D}{1 \text{ kton}} \right] \left[ \frac{\langle \sigma \rangle}{10^{-42} \text{ cm}^2} \right]. \quad (5)
\]

(Though written slightly differently, this is equivalent to the similar expression in Ref. \([11]\).) We assume \( D = 10 \) kpc, and a detector fiducial mass of 1 kton for KamLAND. As written, Eq. \((5)\) is for the yield per flavor, assuming that each carries away a portion \( E \) of the total binding energy (nominally, \( E_B = 3 \times 10^{53} \) ergs, and \( E = E_B/6 \)). The thermally-averaged cross section (the integral of the cross section with normalized Fermi-Dirac distribution) is defined for each \( \text{CH}_2 \) “molecule”, and a factor of 2 must be included for electron or free proton targets.
Prior to Ref. 11, the largest expected yield in any oil or water detector was from $\bar{\nu}_e + p \rightarrow e^+ + n$. The total cross sections for charged-current $\bar{\nu}_e + p \rightarrow e^+ + n$ and neutral-current $\nu + p \rightarrow \nu + p$ have similar forms, though the latter is about 4 times smaller. However, this is compensated in the yield by the contributions of all six flavors, as well as the higher temperature assumed for $\bar{\nu}_\mu$ and $\nu_e$ ($T = 8$ MeV instead of 5 MeV). Thus, the total yield from $\nu + p \rightarrow \nu + p$ is larger than that from $\bar{\nu}_e + p \rightarrow e^+ + n$, when the detector threshold is neglected.

Taking into account radiative, recoil, and weak magnetism corrections, the thermally-averaged cross section for $\bar{\nu}_e + p \rightarrow e^+ + n$ at $T = 5$ MeV is $44 \times 10^{-42}$ cm$^2$ (for 2 protons) 12. These corrections reduce the thermally-averaged cross section by about 20%, and also correct the relation $E_e = E_{\nu} - 1.3$ MeV. The total expected yield from this reaction is thus about 310 events in 1 kton.

Since the struck protons in $\nu + p \rightarrow \nu + p$ have a relatively low-energy recoil spectrum, and since realistic detectors have thresholds, it is crucial to consider the proton spectrum in detail, and not just the total yield of neutrinos that interact.

4. Proton Recoil Spectrum

The elastically-scattered protons will have kinetic energies of a few MeV. Obviously, these non-relativistic protons will be completely invisible in any Čerenkov detector like Super-Kamiokande. However, such small energy depositions can be readily detected in scintillator detectors such as KamLAND and Borexino. We first consider the true proton spectrum, and then how this spectrum would appear in a realistic detector.

The proton spectrum (for one flavor of neutrino) is given by

$$
\frac{dN}{dT_p}(T_p) = C \int_{(E_\nu)_{min}}^{\infty} dE_\nu f(E_\nu) \frac{dE_\nu}{dT_p}(E_\nu, T_p),
$$

where $f(E_\nu)$ is a normalized Fermi-Dirac spectrum and the differential cross section is given by Eq. (24). For a given $T_p$, the minimum required neutrino energy is

$$
(E_\nu)_{min} = T_p + \sqrt{T_p (T_p + 2M_p)} \approx \sqrt{\frac{M_p T_p}{2}},
$$

The normalization constant $C$ is determined by Eq. (24), as the integral of Eq. (24) over all $T_p$ without the $C$ factor is $\langle \sigma \rangle$.

Throughout, I refer to the $\nu_e$ ($T = 3.5$ MeV), $\bar{\nu}_e$ ($T = 5$ MeV), and the combined $\nu_\mu$, $\nu_\tau$, $\bar{\nu}_\mu$, and $\bar{\nu}_\tau$ ($T = 8$ MeV) flavors. Since we know that there are neutrino oscillations, this language is somewhat incorrect. However, our results are totally insensitive to any oscillations among active neutrinos or antineutrinos (since this is a neutral-current cross section), and also to oscillations between active neutrinos and antineutrinos (since the cross section is dominated by the $c^2_{\alpha\beta}$ terms). Thus when we refer to the $\nu_e$ flavor, we mean “those neutrinos emitted with a temperature $T = 3.5$ MeV, whatever their flavor composition now,” etc. The true proton spectra corresponding to the various flavors are shown in Fig. 1. As seen in the figure, the contributions

![Figure 1. The true proton spectrum in KamLAND, for a standard supernova at 10 kpc. In order of increasing maximum kinetic energy, the contributions from $\nu_\mu$, $\nu_\tau$, and the sum of $\nu_\mu$, $\nu_\tau$, $\bar{\nu}_\mu$, and $\bar{\nu}_\tau$ are shown with dashed lines. The solid line is the sum spectrum for all flavors. Taking the detector properties into account substantially modifies these results, as shown below.](image)
of $\nu_e$ and $\bar{\nu}_e$ are quite suppressed relative to the sum of $\nu_\mu$, $\nu_\tau$, $\bar{\nu}_\mu$, and $\bar{\nu}_\tau$.

5. Quenching

For highly ionizing particles like low-energy protons, the light output is reduced or “quenched” relative to the light output for an electron depositing the same amount of energy. The observable light output $E_{\text{equiv}}$ (i.e., equivalent to an electron of energy $E_{\text{equiv}}$) can be calculated by integrating Birk’s Law with tables of $dE/dx$ for protons in the KamLAND oil-scintillator mixture $[^{149}]$. The observed energy in terms of the proton kinetic energy is shown in Fig. 2. Thus the proton quenching factor ($E_{\text{equiv}}/T_p$) is thus roughly 1/2 at 10 MeV, 1/3 at 6 MeV, 1/4 at 3 MeV, and so on.

Using the quenching function shown in Fig. 2, we can transform the true proton spectrum shown in Fig. 1 into the expected measured proton spectrum, shown in Fig. 3. If the quenching factor were a constant, it would simply change the units of the $T_p$ axis. However, it is nonlinear, and reduces the light output of the lowest recoils the most. It also reduces the number of events above threshold. The anticipated threshold in KamLAND is 0.2 MeV electron equivalent energy. With the expected proton quenching, this corresponds to a threshold on the true proton kinetic energy of 1.2 MeV. The number of events above this threshold for each flavor appears in Table 1. The measured proton spectrum will primarily reflect the shape of the underlying Fermi-Dirac spectrum for the sum of $\nu_\mu$, $\nu_\tau$, $\bar{\nu}_\mu$, and $\bar{\nu}_\tau$. This has been convolved with both the differential cross section (which gives a range of $T_p$ for a given $E_\nu$), and also the effects of quenching. However, as we will show, the properties of the initial neutrino spectrum can still be reliably deduced.

Background considerations, while important, are a small correction, and so we ignore them here; see Ref. 3 for a complete discussion.

6. Proton Spectrum Fits

The measured proton spectrum can be used to separately determine the total energy of the $\nu_\mu$, $\nu_\tau$, $\bar{\nu}_\mu$, and $\bar{\nu}_\tau$ neutrinos and their time-averaged temperature. The total number of de-
Table 1

<table>
<thead>
<tr>
<th>Neutrino Spectrum</th>
<th>$E_{\nu_{e}} = 0$</th>
<th>$0.2\text{ MeV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu: T = 3.5\text{ MeV}$</td>
<td>51</td>
<td>3</td>
</tr>
<tr>
<td>$\bar{\nu}: T = 5\text{ MeV}$</td>
<td>80</td>
<td>17</td>
</tr>
<tr>
<td>$2\nu: T = 8\text{ MeV}$</td>
<td>244</td>
<td>127</td>
</tr>
<tr>
<td>$2\bar{\nu}: T = 8\text{ MeV}$</td>
<td>243</td>
<td>126</td>
</tr>
<tr>
<td>All</td>
<td>624</td>
<td>273</td>
</tr>
</tbody>
</table>

The number of events in KamLAND (1 kton mass assumed) above the noted thresholds for a standard supernova at 10 kpc, for the separate flavors or their equivalents after oscillations. Oscillations do not change the number of neutrinos at a given energy, and the neutral-current yields are insensitive to the neutrino flavor. Equilibration among the six flavors is assumed (see the text for discussion). The thresholds are in electron equivalent energy, and correspond to minimum true proton kinetic energies of 0 and 1.2 MeV.

Depend only on nuclear level splittings.

We performed quantitative tests of how well the parameters $E_{\nu}^{\text{st}}$ and $T$ can be determined from the measured proton spectrum. (We did also investigate the effects of a chemical potential in the Fermi-Dirac distribution, but found that it had little effect. This is simply because the cross section is not rising quickly enough to see the tail of the thermal distribution in detail.) Of course, if the distance to the supernova is not known, then we are effectively fitting for $E_{\nu}^{\text{st}}/D^2$.

We performed Monte Carlo simulations of the supernova signal in KamLAND and made chi-squared fits to determine $E_{\nu}^{\text{st}}$ and $T$ for each fake supernova. To perform the fits, we started with an “ideal” spectrum, as described by the integral:

$$
\left( \frac{dN}{dT_p} \right)_{\text{ideal}} = C \int_0^\infty dT'_p \ G(T'_p; T, \delta T_p) \times \int_{(E_{\nu})_{min}}^{(E_{\nu})_{max}} dE_{\nu} \ f(E_{\nu}) \ \frac{d\sigma}{dT_p}(E_{\nu}, T'_p) \ (9)
$$

where the inner integral is as in Eq. (5), with the addition that quenching corrections are applied to $T'_p$ after convolution with $f(E_{\nu})$. For the Gaussian energy resolution $G(T'_p; T, \delta T_p)$, we used $\delta T_p = 0.1 \sqrt{T_p/(1 \text{ MeV})}$ (5). The normalization constant $C$ is given by comparison to Eq. (5). Example spectra are shown in Fig. 5.

Using $(dN/dT_p)_{\text{ideal}}$, we binned the spectrum by the following integral:

$$
N_i = \int_{(E_{\nu})_{min}}^{(E_{\nu})_{max}} dT_p \ \left( \frac{dN}{dT_p} \right)_{\text{ideal}} \ (10)
$$

where $N_i$ is the number of events in bin $i$, and $(E_{\nu})_{min}$ and $(E_{\nu})_{max}$ are the minimum and maximum energies for bin $i$. Eight bins of variable width were used, chosen to contain roughly the same number of expected events per bin. For a chosen $E_{\nu}^{\text{st}}$ and $T$, this was the starting point of our Monte Carlo (and the bin boundaries were kept fixed). For each fake supernova, we sampled the number of events in each of these bins according to the appropriate Poisson distributions. The resulting spectrum was as one might obtain from a single supernova, given the finite number of events expected. We then varied $E_{\nu}^{\text{st}}$ and $T$ in Eq. (10) until the values that best fit the fake...
Figure 4. Example spectra with different values of $E_{\text{fit}}$ and $T$, all chosen to give the same number of events above an electron equivalent threshold of 0.2 MeV (true proton energy 1.2 MeV) in KamLAND. Though not shown in this figure, the spectrum above 2 MeV is included in our analysis. At the 0.2 MeV point, from left to right these correspond to $(E_{\text{fit}}, T) = (4.2, 6), (2.0, 8), (1.4, 10)$, respectively, with $E_{\text{fit}}$ in $10^{53}$ ergs and $T$ in MeV.

spectral data were determined. For a given set of assumed $E_{\text{fit}}$ and $T$, this procedure was repeated many times. The distributions of the final $E_{\text{fit}}$ and $T$ thus reveal the expected errors on fitting $E_{\text{fit}}$ and $T$ for a single real future supernova. Three examples are shown in Fig. 4, where one can see that $E_{\text{fit}}$ and $T$ can each be determined with roughly 10% error.

7. Discussion and Conclusions

We have shown that neutrino-proton elastic scattering, previously unrecognized as a useful detection reaction for low-energy neutrinos, in fact has a yield for a supernova comparable to $\bar{\nu}_e + p \rightarrow e^+ + n$, even after taking into account the quenching of the proton scintillation light and assuming a realistic detector threshold.

In addition, the measured proton spectrum is related to the incident neutrino spectrum. We have shown explicitly that one can separately measure the total energy and temperature of $\nu_\mu$, $\nu_\tau$, $\bar{\nu}_\mu$, and $\bar{\nu}_\tau$, each with uncertainty of order 10% in KamLAND. This greatly enhances the importance of detectors like KamLAND and Borexino for detecting supernova neutrinos.

For Borexino, the useful volume for supernova neutrinos is 0.3 kton, and the hydrogen to carbon ratio in pure pseudocumene ($C_8H_{12}$) is $1.3 : 1$ \cite{12}, so there are about 4.7 times fewer free proton targets than assumed for KamLAND. However, the quenching is less in pure scintillator (KamLAND is about 20% pseudocumene and 80% paraffin oil \cite{10}), and the errors on $E_{\text{fit}}$ and $T$ scale as $1/\sqrt{N}$, so that the precision in Borexino should be about 20% or better. Other techniques for bolometric measurements of supernova neutrino fluxes have been studied. Detectors for elastic neutral-current neu-
trino scattering on electrons [13] and coherently on whole nuclei [14] have been discussed, but never built. If neutrino oscillations are effective in swapping spectra, then the temperature of the “hot” flavors may be revealed in the measured positron spectrum from \( p + e^- \rightarrow e^+ + n \); two recent studies have shown very good precision (\(< 5\%\)) for measuring the temperatures and the total binding energy [15, 16]. However, they assumed exact energy equipartition among the six neutrino flavors, whereas the uncertainty on equipartition is at least 50% [6]. Nevertheless, under less restrictive assumptions, this technique may play a complementary role. Finally, since for different cross sections, the neutral-current yields depend different from temperature, comparison of the yields may provide some information [17]. However, there are caveats. In neutrino-electron scattering, the neutrino energy is not measured because the neutrino-electron angle is much less than the angular resolution due to multiple scattering. The scattered electrons, even those in a forward cone, sit on a much larger background of \( p + e^- \rightarrow e^+ + n \) events, so it is difficult to measure their spectrum [18]; also, their total yield is only weakly dependent on temperature. At the other extreme (see Fig. 3 of Ref. [17]), the yield of neutral-current events [19] on \(^{13}\)O depends strongly on a possible chemical potential term in the thermal distribution.

It is important to note that the detection of recoil protons from neutron-proton elastic scattering at several MeV has been routinely accomplished in scintillator detectors (see, e.g., Ref. [20]). Since both particles are massive, the proton will typically take half of the neutron energy. This reaction provides protons in the same energy range as those struck in neutrino-proton elastic scattering with \( E_\nu \sim 30 \) MeV. This is a very important proof of concept for all aspects of the detection of low-energy protons.

Though low-energy backgrounds will be challenging, it is also important to note that the background requirements for detecting the supernova signal are approximately 3 orders of magnitude less stringent than those required for detecting solar neutrinos in the same energy range (taking quenching into account for our signal). Borexino has been designed to detect very low-energy solar neutrinos, and KamLAND hopes to do so in a later phase of the experiment.

These measurements would be considered in combination with similar measurements for \( \nu_e \) and \( \bar{\nu}_e \) from charged-current reactions in other detectors. Separate measurements of the total energy and temperature for each flavor will be invaluable for comparing to numerical supernova models [21]. They will also be required to make model-independent studies of the effects of neutrino oscillations [22]. If the total energy release \( E_B \) in all flavors has been measured, then

\[
E_B \simeq \frac{3}{5} \frac{GM_S^2}{R_N},
\]

thus allowing a direct and unique measurement of the newly-formed neutron star properties, principally the mass \( M_{NS} \) [23].

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