Fermilab FERMILAB-Conf-02/212-T September 2002

FERMILAB-Conf-02/212-T
TRINLAT-02/06

Remark on the Theoretical Uncertainty in B^0-$\bar{B}^0$ Mixing

Sinéad M. Ryan[a] and Andreas S. Kronfeld[b]

[a]School of Mathematics, Trinity College, Dublin 2, Ireland

[b]Theoretical Physics Department, Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA

We re-examine the theoretical uncertainty in the Standard Model expression for B^0-$\bar{B}^0$ mixing. We focus on lattice calculations of the ratio ξ, needed to relate the oscillation frequency of B_s^0-$\bar{B}_s^0$ mixing to V_{td}. We replace the usual linear chiral extrapolation with one that includes the logarithm that appears in chiral perturbation theory. We find a significant shift in the ratio ξ, from the conventional 1.15 ± 0.05 to $\xi = 1.32 \pm 0.10$.

1. INTRODUCTION

The Standard Model (SM) expression for $\bar{B}^0$–B^0 mixing,

$$\Delta m_q = \left(\frac{G_F^2 m_W^2 S_0}{16\pi^2 m_{B_q}}\right) |V_{tq}^* V_{tb}|^2 \eta_B \mathcal{M}_q, \tag{1}$$

where $q \in \{d, s\}$, can be used to determine the CKM element $|V_{tq}|$. This result can be compared to other determinations to test the SM. In Eq.(1) the quantities in parenthesis are precisely known and $\Delta m_d = 0.503 \pm 0.006\text{ps}^{-1}$ [1]. A measurement of Δm_s, at the percent level, is expected at the Tevetron [2]. $|V_{tq}|$ is therefore dominated by QCD uncertainties in the hadronic matrix element, $\mathcal{M}_q = \langle \bar{B}_q^0 | [\bar{b}\gamma^\mu(1-\gamma^5)q][\bar{b}\gamma_\mu(1-\gamma^5)q] | B_q^0 \rangle$, which is known to $\approx 20\%$ by lattice QCD [3]. $\mathcal{M}_q$ can be computed directly, or parameterised as $\mathcal{M}_q = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q}$ and reconstructed from separate determinations of f_{B_q} and B_{B_q}. The latter approach is useful when studying the light quark mass dependence. A recent discussion of the methodologies and results is in Ref. [3].

The percent-level accuracies of Δm_s and Δm_d make it instructive to form the ratio

$$\frac{\Delta m_s}{\Delta m_d} = \left|\frac{V_{ts}}{V_{td}}\right|^2 \frac{m_{B_s}}{m_{B_d}} \xi^2 \quad \text{with} \quad \xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}. \tag{2}$$

CKM unitarity implies $|V_{ts}| \approx |V_{cb}|$ and therefore $\delta|V_{td}| = \sqrt{(\delta|V_{cb}|)^2 + (\delta\xi)^2}$. Current uncertainty on $|V_{cb}|$ is 2-4%, so the error in $|V_{td}|$ is largely due to the uncertainty in ξ^2.

Conventional wisdom holds that in the ratio ξ^2 many systemtic uncertainties cancel, implying a precise determination of $|V_{td}|$ is possible. However, typical lattice calculations use $0.5 m_s \le m_q \le m_s$. To reach physical light quark masses the results are chirally extrapolated. The usual linear and quadratic fits, which may be reasonable in the region of lattice data, fail to account for the presence of chiral logarithms predicted by chiral perturbation theory [4]. The effect of these logs was first explored by JLQCD [5] and included in the error budget of ξ in Ref. [6]. We argue that these logarithms change the extrapolation and therefore the value of ξ in the chiral limit. We find a shift in ξ from 1.15 ± 0.05 to 1.32 ± 0.1. Further details of this work are in Ref. [7]. Fig. 1

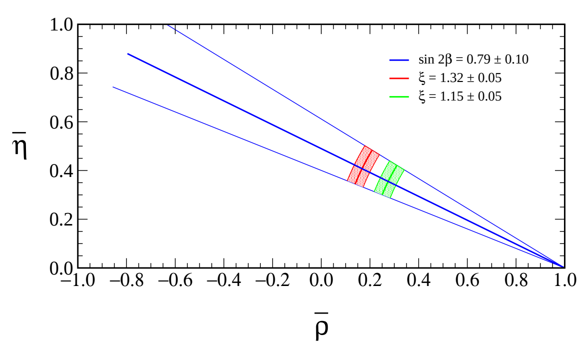

Figure 1. A sketch of the constraints on the apex of the unitarity triangle with $\sin 2\beta = 0.79 \pm 0.10$, $\Delta m_s = 20\ \text{ps}^{-1}$ and $\xi = 1.32 \pm 0.05$ or 1.15 ± 0.05.

shows how $\sin 2\beta$ and $\Delta m_s/\Delta m_d$ combine to constrain the apex of the unitarity triangle. The effect of a shift in ξ is highlighted.

2. CHIRAL LOGS AND ξ

The scales between $1/m_s$ and $1/m_d$ are best described by chiral perturbation theory. Neglecting $1/m_b$ corrections

$$\sqrt{m_{B_q}} f_{B_q} = \Phi\left[1+\Delta f_q\right], \qquad (3)$$

$$B_{B_q} = B\left[1+\Delta B_q\right], \qquad (4)$$

where Δf_q and ΔB_q contain the logarithms and are given in Ref. [4]. The expressions for Δf_q and ΔB_q yield

$$\begin{aligned}\xi_f - 1 &= \Delta f_s - \Delta f_d \\ &= (m_K^2 - m_\pi^2) f_2(\mu) - \frac{1+3g^2}{(4\pi f)^2}\Big(\tfrac{1}{2} m_K^2 \ln\tfrac{m_K^2}{\mu^2} \\ &\quad + \tfrac{1}{4} m_\eta^2 \ln\tfrac{m_\eta^2}{\mu^2} - \tfrac{3}{4} m_\pi^2 \ln\tfrac{m_\pi^2}{\mu^2}\Big), \end{aligned} \qquad (5)$$

$$\begin{aligned}\xi_B^2 - 1 &= \Delta B_s - \Delta B_d \\ &= (m_K^2 - m_\pi^2) B_2(\mu) - \frac{1-3g^2}{(4\pi f)^2}\Big(\tfrac{1}{2} m_\eta^2 \ln\tfrac{m_\eta^2}{\mu^2} \\ &\quad - \tfrac{1}{2} m_\pi^2 \ln\tfrac{m_\pi^2}{\mu^2}\Big). \end{aligned} \qquad (6)$$

$f_2(\mu)$ and $B_2(\mu)$ are low-energy constants describing dynamics at scales $< \mu^{-1}$, f is the pion decay constant and g^2 is the $B^* B\pi$ coupling. This has recently been determined by CLEO from a measurement of the D^* width. They find $g^2_{D^*D\pi} = 0.35$ [8] and by heavy quark symmetry, $g^2_{B^*B\pi} = 0.35$, to a good approximation. Thus, while the effect of the chiral log in B_d is small, since $(1-3g^2) = -0.05$, it may be significant for f_{B_d}, where $(1+3g^2) = 2.05$.

Therefore, we refine our discussion and consider $\xi_f = f_{B_s}/f_{B_d}$. Further details of the chiral log in B_d and the effect of varying g^2 are in Ref. [7]. To study the light quark mass dependence of ξ_f, Eq. (5) is rewritten using the Gell-Mann–Okubo relations, as

$$\begin{aligned}\xi_f(r) - 1 &= m_{ss}^2 (1-r)\Big\{\tfrac{1}{2} f_2(\mu) \\ &\quad - \frac{1+3g^2}{(4\pi f)^2}\left[\frac{5}{12}\ln(\tfrac{m_{ss}^2}{\mu^2}) + l(r)\right]\Big\}, \end{aligned} \qquad (7)$$

where $m_{qq}^2 = r m_{ss}^2$ and

$$\begin{aligned} l(r) &= \frac{1}{1-r}\left[\frac{1+r}{4}\ln\left(\frac{1+r}{2}\right)\right. \\ &\quad \left. + \frac{2+r}{12}\ln\left(\frac{2+r}{3}\right) - \frac{3r}{4}\ln(r)\right]. \end{aligned} \qquad (8)$$

Fig. 2 shows the function $\chi(r) = (1-r)l(r)$ which contains the chiral logarithms. Typical lattice

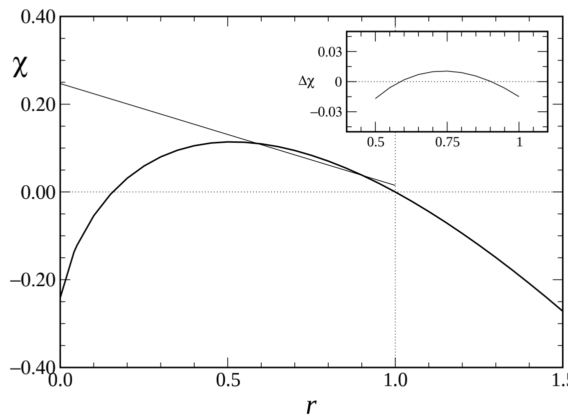

Figure 2. The chiral log $\chi(r)$ varying the mass ratio $r = m_{qq}^2/m_{ss}^2 = m_q/m_s$, compared with a straight line fit for $0.5 \le r \le 1.0$. The difference between the curve and the fit is shown in the inset.

calculations are performed at $0.5 \le r \le 1.0$ and extrapolated to $r_d \approx 1/25$, corresponding to the down quark. Fig. 2 shows that the difference between a linear fit and one including chiral logarithms is easily masked by statistical error in the region where data are available. However, the effect of the logarithms is significant at smaller r values and is not reproduced by a linear fit.

To include these logarithms in the chiral extrapolation of ξ_f (or $\xi_f - 1$) the low-energy constant $f_2(\mu)$ is required. This can be extracted from current lattice data [1]. The usual, linear functional form applied to $\xi_f - 1$ is

$$\xi_f(r) = (1-r) S_f. \qquad (9)$$

[1] Note that the dependence on the scale μ cancels in the total.

Assuming this is sensible for $r = r_0 \sim 1$, as indicated by Fig. 2, and by equating Eqs. (7) and (9)

$$\begin{aligned} \frac{m_{ss}^2}{2} f_2(\mu) &= S_f \\ &+ \quad m_{ss}^2 \frac{1+3g^2}{(4\pi f)^2} \left[\frac{5}{12} \ln(\tfrac{m_{ss}^2}{\mu^2}) + l(r_0) \right], \end{aligned} \quad (10)$$

which plugged into Eq. (7) gives

$$\begin{aligned} \xi_f(r) - 1 = & \ (1-r) \Big\{ S_f \\ & + \quad m_{ss}^2 \frac{1+3g^2}{(4\pi f)^2} \left[l(r_0) - l(r) \right] \Big\}. \end{aligned} \quad (11)$$

Knowing g^2 from experiment and S_f from lattice calculations gives a new handle on ξ_f. To evaluate $\xi_f(r) - 1$, $f = 130$ MeV, $g^2 = 0.35$ and $S_f = 0.15 \pm 0.05$ [6] are used. The parameter r_0 is a remaining uncertainty. Fig. 3 shows the dependence of ξ_f on r_0. Fig. 3 yields a value of

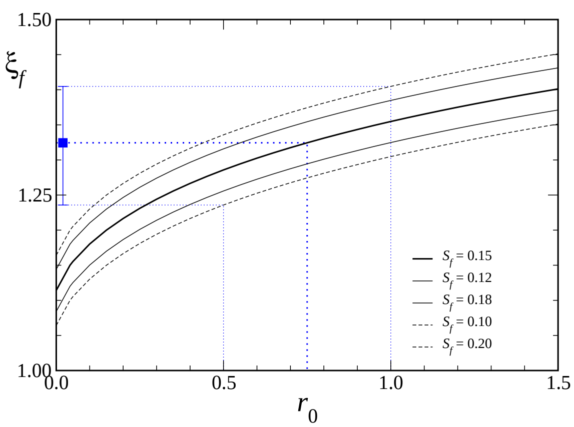

Figure 3. The dependence of ξ_f on r_0. $(1 - r_d)S_f = 0.15 \pm 0.05, m_{ss}^2 = 2(m_K^2 - m_\pi^2), r = r_d = m_\pi^2/m_{ss}^2$.

$\xi_f = 1.32 \pm 0.08$. The conservative error attached to S_f leads to the larger then usual error on ξ_f. A similar analysis of ξ_B is described in Ref. [7], giving $\xi_B = 0.998 \pm 0.025$. The combination leads to

$$\xi = 1.32 \pm 0.10. \quad (12)$$

3. CONCLUSIONS

The importance of reliable chiral extrapolations has become more and more widely appreciated [9]. This is especially true in the quenched approximation where many other systematic errors have been controlled, leaving chiral extrapolation as the major uncertainty [10]. To date, most lattice calculations of ξ have relied on linear (or quadratic) fits. A combination of the new CLEO value of the coupling with lattice data for $f_2(\mu)$ allows a determination of ξ incorporating chiral logarithm effects. Reducing the uncertainty in ξ is possible by designing lattice calculations to determine the low-energy constants.

Ultimately, unquenched lattice calculations at light quark masses should "see" the chiral logarithms. Until then, the central value and error assigned to ξ should reflect the uncertainty.

REFERENCES

1. F. Ronga (Belle Collaboration) and C. Voena (BaBar Collaboration). Talks at ICHEP02, Amsterdam, July 2002. [http://www.ichep02.nl/].
2. K. Anikeev *et al.*, *B Physics at the Tevatron: Run II and Beyond*, arXiv:hep-ph/0201071, http://www-theory.lbl.gov/Brun2/report/.
3. N. Yamada, these proceedings.
4. B. Grinstein *et al.* Nucl. Phys. B 380 (1992) 369; S.R. Sharpe, Y. Zhang, Phys. Rev. D 53 (1996) 5125; J. L. Goity, Phys. Rev. D 46 (1992) 3929.
5. JLQCD Collaboration, Nucl. Phys. Proc. Suppl. 106 (2002) 397-399 .
6. S. Ryan, Nucl. Phys. Proc. Suppl. 106 (2002) 86-97.
7. A.S. Kronfeld and S.M. Ryan. Phys. Lett. B 543 (2002) 59. arXiv:hep-ph/0206058.
8. A. Anastassov *et al.* (CLEO Collaboration), Phys. Rev. D 65 (2002) 032003.
9. Panel discussion on Chiral Extrapolation of Lattice Observables, these proceedings.
10. A.X. El-Khadra *et al.*, Phys. Rev. D 64 (2001) 014502; J. Shigemitsu *et al.*, arXiv:hep-lat/0207011.