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Lifetimes of heavy hadrons beyond leading logarithms*

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Abstract

The lifetime splitting between the B^+ and B_d^0 mesons has recently been calculated in the next-to-leading order of QCD. These corrections are necessary for a reliable theoretical prediction, in particular for the meaningful use of hadronic matrix elements computed with lattice QCD. Using results from quenched lattice QCD we find $\tau(B^+)/\tau(B_d^0) = 1.053 \pm 0.016 \pm 0.017$, where the uncertainties from unquenching and $1/m_b$ corrections are not included. The lifetime difference of heavy baryons Ξ_b^0 and Ξ_b^- is also discussed.

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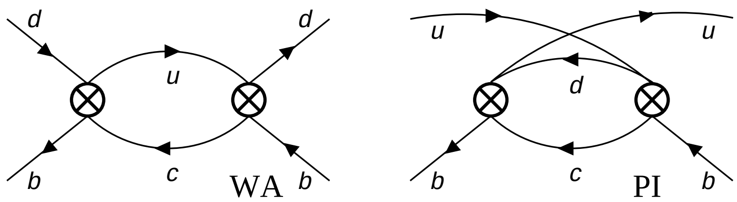

Figure 1: *Weak annihilation* (WA) and *Pauli interference* (PI) diagrams in the leading order of QCD. They contribute to $\Gamma(B_d^0)$ and $\Gamma(B^+)$, respectively. The crosses represent $|\Delta B| = 1$ operators, which are generated by the exchange of W bosons. CKM-suppressed contributions are not shown.

1 Introduction

In my talk I present work done in collaboration with Martin Beneke, Gerhard Buchalla, Christoph Greub and Alexander Lenz [1].

Twenty years ago the hosts of this conference showed that inclusive decay rates of hadrons containg a heavy quark can be computed from first principles of QCD. The *Heavy Quark Expansion* (HQE) technique [2] exploits the heaviness of the bottom (or charm) quark compared to the fundamental QCD scale Λ_{QCD}. In order to study the lifetime of some b-flavored hadron H containing a single heavy quark one needs to compute its total decay rate $\Gamma(H_b)$. Now the HQE is an operator product expansion (OPE) expressing $\Gamma(H_b)$ in terms of matrix elements of local $\Delta B = 0$ (B denotes the bottom number) operators, leading to an expansion of $\Gamma(H_b)$ in terms of Λ_{QCD}/m_b. In the leading order of Λ_{QCD}/m_b the decay rate of H_b equals the decay rate of a free b-quark, unaffected by the light degrees of freedom of H_b. Consequently, the lifetimes of all b-flavored hadrons are the same at this order. The dominant source of lifetime differences are weak interaction effects between the b-quark and the light valence quark. They are depicted in Fig. 1 for the case of the B^+–B_d^0 lifetime difference. The relative size of these weak non-spectator effects to the leading free-quark decay is of order $16\pi^2(\Lambda_{QCD}/m_b)^3 = \mathcal{O}(5\text{–}10\%)$. The measurement of lifetime differences among different b-flavored hadrons therefore tests the HQE formalism at the third order in the expansion parameter.

The optical theorem relates the total decay rate $\Gamma(H_b)$ to the self-energy of H_b:

$$\Gamma(H_b) \quad = \quad \frac{1}{2M_{H_b}} \langle H_b | \mathcal{T} | H_b \rangle. \tag{1}$$

Here we have introduced the transition operator:

$$\mathcal{T} \quad = \quad \mathrm{Im}\, i \int d^4x \, T[H(x)\, H(0)] \tag{2}$$

with the effective $|\Delta B| = 1$ Hamiltonian H describing the W-mediated decay of the b quark. The HQE amounts to an OPE applied to $\mathcal{T}$ which effectively integrates out the hard loop momenta (corresponding to the momenta of the final state quarks). We decompose the result as

$$\begin{aligned}\mathcal{T} &= [\mathcal{T}_0 + \mathcal{T}_2 + \mathcal{T}_3]\left[1 + \mathcal{O}(1/m_b^4)\right] \\ \mathcal{T}_3 &= \mathcal{T}^u + \mathcal{T}^d + \mathcal{T}_{sing}. \end{aligned} \tag{3}$$

Here $\mathcal{T}_n$ denotes the portion of $\mathcal{T}$ which is suppressed by a factor of $1/m_b^n$ with respect to $\mathcal{T}_0$ describing the free quark decay. The contributions to $\mathcal{T}_3$ from the weak interaction with the valence quark read

$$\begin{aligned}\mathcal{T}^u &= \frac{G_F^2 m_b^2 |V_{cb}|^2}{6\pi}\Big[\ |V_{ud}|^2\left(F^u Q^d + F_S^u Q_S^d + G^u T^d + G_S^u T_S^d\right) \\ &\qquad + |V_{cd}|^2\left(F^c Q^d + F_S^c Q_S^d + G^c T^d + G_S^c T_S^d\right)\Big] \\ &\qquad + (d \to s) \\ \mathcal{T}^d &= \frac{G_F^2 m_b^2 |V_{cb}|^2}{6\pi}\left[F^d Q^u + F_S^d Q_S^u + G^d T^u + G_S^d T_S^u\right]. \end{aligned} \tag{4}$$

Here G_F is the Fermi constant, m_b is the bottom mass and the V_{ij}'s are elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The superscript q of the coefficients F^q, F_S^q, G^q, G_S^q refers to the cq intermediate state. The leading contributions to $\mathcal{T}^u$ and $\mathcal{T}^d$ are obtained from the left and right diagram in Fig. 1, respectively. They involve the local dimension-6, $\Delta B = 0$ operators

$$\begin{aligned} Q^q &= \overline{b}\gamma_\mu(1-\gamma_5)q\,\overline{q}\gamma^\mu(1-\gamma_5)b, \\ Q_S^q &= \overline{b}(1-\gamma_5)q\,\overline{q}(1+\gamma_5)b, \\ T^q &= \overline{b}\gamma_\mu(1-\gamma_5)T^a q\,\overline{q}\gamma^\mu(1-\gamma_5)T^a b, \\ T_S^q &= \overline{b}(1-\gamma_5)T^a q\,\overline{q}(1+\gamma_5)T^a b, \end{aligned} \tag{5}$$

where T^a is the generator of color SU(3). The Wilson coefficients $F^u \ldots G_S^d$ contain the physics from scales above m_b and are computed in perturbation theory. The remainder $\mathcal{T}_{sing}$ in (3) involves additional dimension-6 operators, which are $SU(3)_F$ singlets and do not contribute to the lifetime splitting within the (B^+, B_d^0) and (Ξ_b^0, Ξ_b^-) isodoublets. In order to predict the widths $\Gamma(B_d^0)$ and $\Gamma(B^+)$ one needs to compute the hadronic matrix elements of the operators in (5). After using the isospin relation $\langle B_d^0|Q^{d,u}|B_d^0\rangle = \langle B^+|Q^{u,d}|B^+\rangle$ the matrix elements will enter $\Gamma(B_d^0) - \Gamma(B^+)$ in isospin-breaking combinations, which are conventionally parametrized as [3, 4]

$$\begin{aligned} \langle B^+|(Q^u - Q^d)|B^+\rangle &= f_B^2 M_B^2 B_1, \quad \langle B^+|(Q_S^u - Q_S^d)|B^+\rangle = f_B^2 M_B^2 B_2, \\ \langle B^+|(T^u - T^d)|B^+\rangle &= f_B^2 M_B^2 \epsilon_1, \quad \langle B^+|(T_S^u - T_S^d)|B^+\rangle = f_B^2 M_B^2 \epsilon_2. \end{aligned} \tag{6}$$

Here f_B and M_B are decay constant and mass of the B meson, respectively. In the *vacuum saturation approximation* (VSA) one has $B_1 = 1$, $B_2 = 1 + \mathcal{O}(\alpha_s(m_b), \Lambda_{QCD}/m_b)$ and $\epsilon_{1,2} = 0$. Corrections to the VSA results are order $1/N_c$, where $N_c = 3$ is the number of colors.

We now find from (1) and (4):

$$\Gamma(B_d^0) - \Gamma(B^+) = \frac{G_F^2 m_b^2 |V_{cb}|^2}{12\pi} f_B^2 M_B \left(|V_{ud}|^2 \vec{F}^u + |V_{cd}|^2 \vec{F}^c - \vec{F}^d \right) \cdot \vec{B}. \quad (7)$$

Here we have introduced the shorthand notation

$$\vec{F}^q(z) = \begin{pmatrix} F^q(z) \\ F_S^q(z) \\ G^q(z) \\ G_S^q(z) \end{pmatrix}, \qquad \vec{B} = \begin{pmatrix} B_1 \\ B_2 \\ \epsilon_1 \\ \epsilon_2 \end{pmatrix} \qquad \text{for } q = d, u, c. \quad (8)$$

Since the hard loops involve the charm quark, the coefficient $\vec{F}^q$ depends on the ratio $z = m_c^2/m_b^2$. The minimal way to include QCD effects is the leading logarithmic approximation, which includes corrections of order $\alpha_s^n \ln^n(m_b/M_W)$, $n = 0, 1, \ldots$ in $\vec{F}^q$ in (7). The corresponding leading order (LO) calculation of the width difference in (7) involves the diagrams in Fig. 1 [2, 3]. Yet LO results are too crude for a precise calculation of lifetime differences. The heavy-quark masses in (7) cannot be defined in a proper way and one faces a large dependence on unphysical renormalization scales. Furthermore, results for $B_{1,2}$ and $\epsilon_{1,2}$ from lattice gauge theory cannot be matched to the continuum theory in a meaningful way at LO. Finally, as pointed out in [3], at LO the coefficients F, F_S in (7) are anomalously small. They multiply the large matrix elements parametrized by $B_{1,2}$, while the larger coefficients G, G_S come with the small hadronic parameters $\epsilon_{1,2}$, rendering the LO prediction highly unstable. To cure these problems one must include the next-to-leading-order (NLO) QCD corrections of order $\alpha_s^{n+1} \ln^n(m_b/M_W)$.

The first calculation of a lifetime difference beyond the LO was performed for the B_s^0–B_d^0 lifetime difference [5], where $\mathcal{O}(\alpha_s)$ corrections were calculated in the SU(3)$_\mathrm{F}$ limit neglecting certain terms of order z. In this limit only a few penguin effects play a role. A complete NLO computation has been carried out for the lifetime difference between the two mass eigenstates of the B_s^0 meson in [6]. In particular the correct treatment of infrared effects, which appear at intermediate steps of the calculation, has been worked out in [6]. The recent computation in [1] is conceptually similar to the one in [6], except that the considered transition is $\Delta B = 0$ rather than $\Delta B = 2$ and the quark masses in the final state are different. The NLO calculation of $\Gamma(B_d^0) - \Gamma(B^+)$ involves the diagrams of Fig. 2. In [4] the NLO corrections to $\Gamma(B_d^0) - \Gamma(B^+)$ have been calculated for the limiting case $z = 0$. The corrections to this limit are of order $z \ln z$ or roughly 20%. The first NLO calculation with the complete z dependence was presented in [1] and subsequently confirmed in [7].

2 Lifetime differences at next-to-leading order

The analytic expressions for the Wilson coefficients $F_{ij}^{u,(1)} - F_{ij}^{d,(1)} \ldots G_{S,ij}^{u,(1)} - G_{S,ij}^{d,(1)}$ are cumbersome functions of z involving dilogarithms. They depend on the renormalization scheme chosen for the $\Delta B = 0$ operators in (5) and also on the renormalization scale $\mu_0 = \mathcal{O}(m_b)$ at which these operators are defined. These dependences properly cancel between $\vec{F}^q$ and $\vec{B}$ in physical observables like (7). When our results for

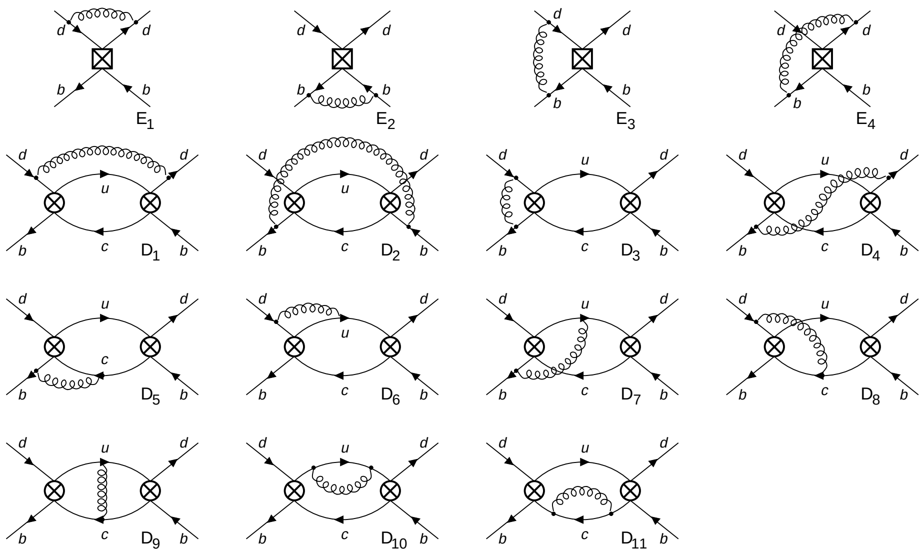

Figure 2: WA contributions in the next-to-leading order of QCD. The PI diagrams are obtained by interchanging u and d and reversing the fermion flow of the u and d lines. The first line shows the radiative corrections to $\Delta B = 0$ operators, which are necessary for the proper infrared factorization. Not displayed are the diagrams E'_3, E'_4 and D'_{3-8} which are obtained from the corresponding unprimed diagrams by left-right reflection and the reverse of the fermion flow.

$F^{u,(1)}_{ij} - F^{d,(1)}_{ij} \ldots G^{u,(1)}_{S,ij} - G^{d,(1)}_{S,ij}$ are combined with some non-perturbative computation of $B_1, \ldots \epsilon_2$, one has to make sure that the numerical values of these hadronic parameters correspond to the same renormalization scheme. Our scheme is defined by the use of dimensional regularization with $\overline{\mathrm{MS}}$ [8] subtraction, an anticommuting γ_5 and a choice of evanescent operators preserving Fierz invariance at the loop level [9]. Choosing further $\mu_0 = m_b$ the desired lifetime ratio can be compactly written as

$$\begin{aligned}\frac{\tau(B^+)}{\tau(B_d^0)} - 1 &= \tau(B^+)\,\left[\Gamma(B_d^0) - \Gamma(B^+)\right] \\ &= 0.0325 \left(\frac{|V_{cb}|}{0.04}\right)^2 \left(\frac{m_b}{4.8\,\mathrm{GeV}}\right)^2 \left(\frac{f_B}{200\,\mathrm{MeV}}\right)^2 \times \\ &\quad \Big[(1.0 \pm 0.2)\,B_1 + (0.1 \pm 0.1)\,B_2 - (18.4 \pm 0.9)\,\epsilon_1 + (4.0 \pm 0.2)\,\epsilon_2\Big]. \quad (9)\end{aligned}$$

Here $\tau(B^+) = 1.653\,\mathrm{ps}$ has been used in the overall factor.

The hadronic parameters have been computed in [10] with quenched lattice QCD using the same renormalization scheme as in the present paper. They read

$$(B_1, B_2, \epsilon_1, \epsilon_2) = (1.10 \pm 0.20,\, 0.79 \pm 0.10,\, -0.02 \pm 0.02,\, 0.03 \pm 0.01). \quad (10)$$

Inserting $|V_{cb}| = 0.040 \pm 0.0016$ from a CLEO analysis of inclusive semileptonic B

decays [13], the world average $f_B = (200 \pm 30)$ MeV from lattice calculations [14] and $m_b = 4.8 \pm 0.1$ GeV for the one-loop bottom pole mass into (9), our NLO prediction reads

$$\frac{\tau(B^+)}{\tau(B_d^0)} = 1.053 \pm 0.016 \pm 0.017 \tag{11}$$

compared to

$$\left[\frac{\tau(B^+)}{\tau(B_d^0)}\right]_{\rm LO} = 1.041 \pm 0.040 \pm 0.013. \tag{12}$$

Here the first error is due to the errors on the coefficients and the hadronic parameters (10), and the second error is the overall normalization uncertainty due to m_b, $|V_{cb}|$ and f_B in (9). The Wilson coefficients also depend on the renormalization scale μ_1 at which the $\Delta B = 1$ operators entering the diagrams in Figs. 1 and 2 are defined. This dependence stems from the truncation of the perturbation series and diminishes order-by-order in α_s. The dependence on μ_1 is the dominant uncertainty of the LO prediction of the lifetime ratio. In Fig. 3 the μ_1-dependence of the LO and NLO predictions for $\tau(B^+)/\tau(B_d^0) - 1$ is shown. The substantial reduction of scale dependence at NLO leads to the improvement in the NLO vs. LO results in (11),(12). Note that the NLO calculation has firmly established that $\tau(B^+) > \tau(B_d^0)$, a conclusion which could not be drawn from the old LO result. The result in (11) is compatible with recent measurement from the B factories [11, 12]:

$$\frac{\tau(B^+)}{\tau(B_d^0)} = \begin{cases} 1.082 \pm 0.026 \pm 0.012 & (\text{BABAR}) \\ 1.091 \pm 0.023 \pm 0.014 & (\text{BELLE}) \end{cases}$$

The calculated Wilson coefficients can also be used to predict the lifetime splitting within the iso-doublet ($\Xi_b^0 \sim bus, \Xi_b^- \sim bds$) with NLO precision. The corresponding LO diagrams are shown in Fig. 4. Note that the role of $\mathcal{T}^u$ and $\mathcal{T}^d$ is interchanged compared to the meson case with $\mathcal{T}^u$ describing the Pauli interference effect. The lifetime difference between $\Lambda_b \sim bud$ and Ξ_b^0 is expected to be small, as in the case of B_s^0 and B_d^0, because it mainly stems from the small U-spin breaking effects in the matrix elements appearing at order $1/m_b^2$.

For Ξ_b's the weak decay of the valence s-quark could be relevant: the decays $\Xi_b^- \to \Lambda_b \pi^-$, $\Xi_b^- \to \Lambda_b e^- \overline{\nu}_e$ and $\Xi_b^0 \to \Lambda_b \pi^0$ are triggered by $s \to u$ transitions and could affect the total rates at the $\mathcal{O}(1\%)$ level [15]. Once the lifetime measurements reach this accuracy, one should correct for this effect. To this end we define

$$\overline{\Gamma}(\Xi_b) \equiv \Gamma(\Xi_b) - \Gamma(\Xi_b \to \Lambda_b X) = \frac{1 - B(\Xi_b \to \Lambda_b X)}{\tau(\Xi_b)} \equiv \frac{1}{\overline{\tau}(\Xi_b)} \quad \text{for } \Xi_b = \Xi_b^0, \Xi_b^-, \tag{13}$$

where $B(\Xi_b \to \Lambda_b X)$ is the branching ratio of the above-mentioned decay modes. Thus $\overline{\Gamma}(\Xi_b)$ is the contribution from $b \to c$ transitions to the total decay rate. In contrast to the B meson system, the matrix elements of the four operators in (5) are not independent at the considered order in Λ_{QCD}/m_b. Since the light degrees of freedom

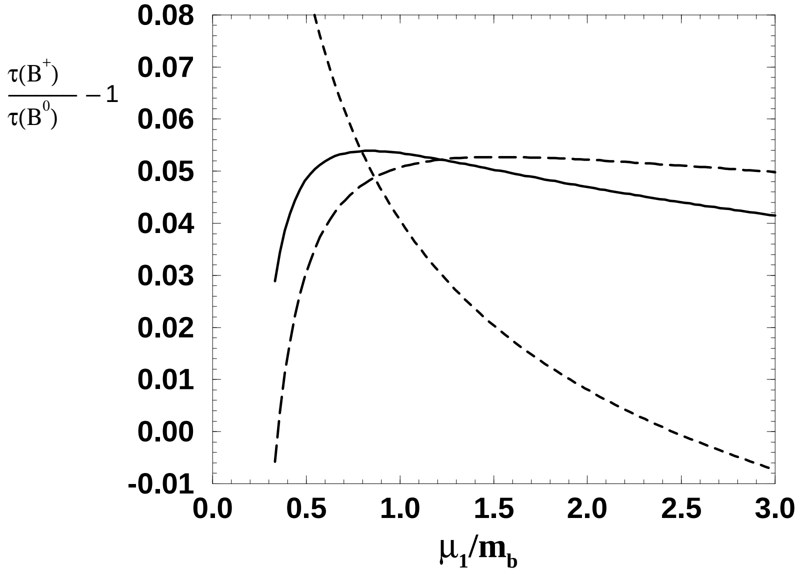

Figure 3: Dependence of $\tau(B^+)/\tau(B_d^0) - 1$ on μ_1/m_b for the central values of the input parameters and $\mu_0 = m_b$. The solid (short-dashed) line shows the NLO (LO) result. The long-dashed line shows the NLO result in the approximation of [4], i.e. z is set to zero in the NLO corrections.

are in a spin-0 state, the matrix elements $\langle \Xi_b | 2Q_S^q + Q^q | \Xi_b \rangle$ and $\langle \Xi_b | 2T_S^q + T^q | \Xi_b \rangle$ are power-suppressed compared to those in (14) (see e.g. [2,3]). This, however, is not true in all renormalization schemes, in the $\overline{\mathrm{MS}}$ scheme used by us $2Q_S^q + Q^q$ and $2T_S^q + T^q$ receive short-distance corrections, because hard gluons can resolve the heavy b-quark mass. A priori one can choose the renormalization of e.g. Q_S^q independently from Q^q, so that $\langle \Xi_b | 2Q_S^q + Q^q | \Xi_b \rangle = \mathcal{O}(\Lambda_{QCD}/m_b)$ can only hold in certain renormalization schemes. This is also the case, if the operators are defined in heavy quark effective theory (HQET) rather than in full QCD. After properly taking into account these short-distance corrections, one can express the desired lifetime ratio solely in terms of two hadronic parameters defined as

$$\begin{aligned} \langle \Xi_b^0 | (Q^u - Q^d)(\mu_0) | \Xi_b^0 \rangle &= f_B^2 M_B M_{\Xi_b} L_1^{\Xi_b}(\mu_0), \\ \langle \Xi_b^0 | (T^u - T^d)(\mu_0) | \Xi_b^0 \rangle &= f_B^2 M_B M_{\Xi_b} L_2^{\Xi_b}(\mu_0). \end{aligned} \tag{14}$$

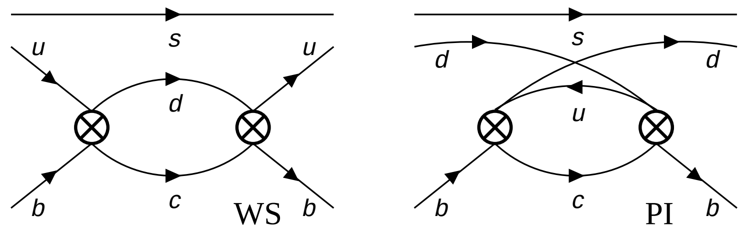

Figure 4: *Weak scattering* (WS) and PI diagrams for Ξ_b baryons in the leading order of QCD. They contribute to $\Gamma(\Xi_b^0)$ and $\Gamma(\Xi_b^-)$, respectively. CKM-suppressed contributions are not shown.

Then one finds

$$\begin{aligned}\frac{\overline{\tau}(\Xi_b^0)}{\overline{\tau}(\Xi_b^-)} - 1 &= \overline{\tau}(\Xi_b^0)\,\left[\Gamma(\Xi_b^-) - \Gamma(\Xi_b^0)\right] \\ &= 0.59\left(\frac{|V_{cb}|}{0.04}\right)^2 \left(\frac{m_b}{4.8\,\text{GeV}}\right)^2 \left(\frac{f_B}{200\,\text{MeV}}\right)^2 \frac{\overline{\tau}(\Xi_b^0)}{1.5\,\text{ps}} \times \\ &\qquad \left[(0.04 \pm 0.01)\,L_1 \;-\; (1.00 \pm 0.04)\,L_2\right], \qquad (15)\end{aligned}$$

with $L_i = L_i^{\Xi_b}(\mu_0 = m_b)$. For the baryon case there is no reason to expect the color-octet matrix element to be much smaller than the color-singlet ones, so that the term with L_2 will dominate the result. The hadronic parameters $L_{1,2}$ have been analyzed in an exploratory study of lattice HQET [16] for Λ_b baryons. Up to SU(3)$_\text{F}$ corrections, which are irrelevant in view of the other uncertainties, $L_i^{\Xi_b}$ and $L_i^{\Lambda_b}$ are equal.

3 Conclusions

Twenty years ago the ITEP group has developed the Heavy Quark Expansion, which allows to study inclusive decay rates of heavy hadrons in a model-free, QCD-based framework [2]. The HQE expresses these decay rates as a series in both Λ_{QCD}/m_b and $\alpha_s(m_b)$. With the advent of precision measurements of lifetimes of b-flavored hadrons at the B factories and the Tevatron correspondingly precise theory predictions are desirable. This requires the calculation of higher-order terms in the HQE. The inclusion of the α_s corrections presented in this talk is in particular mandatory for any meaningful use of hadronic matrix elements computed in lattice gauge theory. The calculated QCD corrections to the WA and PI diagrams in Figs. 1,2 allow to study the lifetime splitting within the (B^+, B_d^0) and (Ξ_b^0, Ξ_b^-) iso-doublets with NLO accuracy. It is gratifying that these corrections have been independently calulated by two groups finding agreement in their analytic expressions for the Wilson coefficients [1, 7].

Current lattice calculations, which are still in a relatively early stage in this case, yield, when combined with our calculations, $\tau(B^+)/\tau(B_d^0) = 1.053 \pm 0.016 \pm 0.017$ [see (11)]. The effects of unquenching and $1/m_b$ corrections are not included in the error estimate, but the unquenching effects can well be sizable. A substantial improvement of the NLO calculation is the large reduction of perturbative uncertainty reflected in the scale dependence stemming from the $\Delta B = 1$ operators. This scale dependence had been found to be very large at leading order, preventing even an unambiguous prediction of the sign of $\tau(B^+)/\tau(B_d^0) - 1$ up to now [3].

At present the experimentally measured Λ_b lifetime falls short of $\tau(B_d^0)$ by roughly 20% [17], which has raised concerns about the applicability of the HQE to baryons. Unfortunately this interesting topic cannot yet be addressed at the NLO level for two reasons: First, $\tau(\Lambda_b)/\tau(B_d^0)$ receives contributions from the yet uncalculated SU(3)$_{\mathrm{F}}$-singlet portion $\mathcal{T}_{sing}$ of the transition operator in (3). Second, the hadronic matrix elements entering $\tau(\Lambda_b)/\tau(B_d^0)$ involve penguin (also called 'eye') contractions of the operators in (5), which are difficult to compute. These penguin contractions are contributions to the matrix elements in which the light q and $\overline{q}$ quark fields of the operator are contracted with each other, not with the hadron's valence quarks.

Acknowledgments

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