The QCD String Spectrum and Conformal Field Theory

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The low energy excitation spectrum of the critical Wilson surface is discussed between the roughening transition and the continuum limit of lattice QCD. The fine structure of the spectrum is interpreted within the framework of two-dimensional conformal field theory.

1. INTRODUCTION

We believe that a deeper understanding of the string theory connection with large Wilson surfaces will require a precise knowledge of the surface excitation spectrum and the determination of the universality class of Wilson surface criticality in the continuum limit of lattice QCD. This approach will also require a consistent description of the conformal properties of the gapless Wilson surface excitation spectrum. In this short progress report we summarize our ab initio on-lattice calculations (a more extended status report was published recently[1]). In collaboration with Mike Peardon, we have also studied the spectrum of a "closed" flux loop across periodic slab geometry (Polyakov line) by choosing appropriate boundary conditions and operators for selected excitations *without* static sources[2].

2. QCD STRING FORMATION

The first attempt at a comprehensive determination of the rich energy spectrum of the gluon excitations between static sources in the fundamental representation of $SU(3)_c$ in D=4 dimensions was reported earlier[3,4] for quark-antiquark separations r ranging from 0.1 fm to 4 fm. The extrapolation of the full spectrum to the continuum limit is summarized in Fig. 1 with very different characteristic behavior on three separate physical scales. Nontrivial short distance physics dominates for $r \leq 0.3$ fermi. The transition region towards string formation is identified on the scale 0.5 fm $\leq$ 2.0 fm. String formation and the onset of string-like ordering of the excitation energies occurs in the range between 2 fm and 4 fm where we reach the current limit of our technology.

To display the fine structure with some clarity, error bars are not shown in Fig. 1. Our earlier results are compatible with extended new runs on our dedicated UP2000 Alpha cluster which was built to increase the statistics more than an order of magnitude. The notation and the origin of the quantum numbers used in the classification of the energy levels are explained in earlier publications[3,4]. The physical scale is set by the Sommer r_0 which, to a good approximation, is $r_0 = 0.5$ fm.

We also established that the main features of string formation with three separate scales is remarkably universal, independent of the gauge groups SU(2) and SU(3), and space-time dimensions D=3 and D=4: *Although the level ordering is approximately string-like in all cases at large separation, there is a surprising and rather universal fine structure in the spectrum with large displacements from the expected massless Goldstone levels.*

3. CONFORMAL THEORY

We believe that the fine structure of the Wilson surface spectrum can be understood within

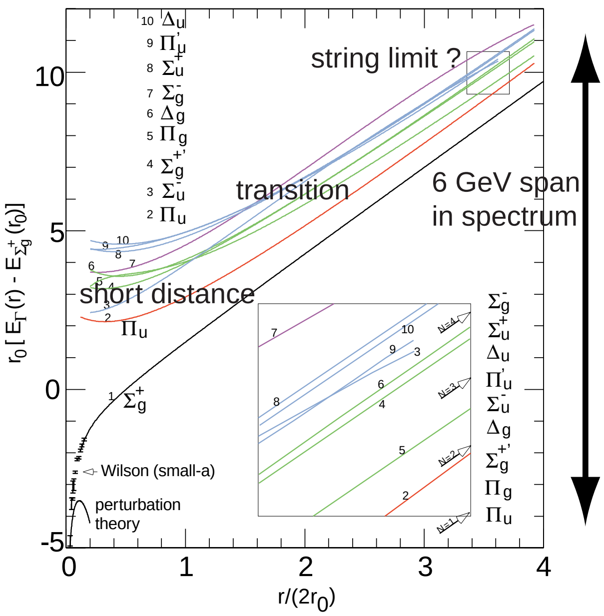

Figure 1. Continuum limit extrapolations are shown for the excitation energies where an arbitrary constant is removed by subtraction. Color coding in postscript is added to the numerical labelling of the excitations (N=0,black:1), (N=1,red:2), (N=2,green:4,5,6), (N=3,blue:3,8,9,10), and (N=4,cyan:7). The five groups represent the expected quantum numbers of a string in its ground state (N=0) and the first four excited states (N=1,2,3,4). The arrows in the inset represent the expected locations of the four lowest massless string excitations (N=1,2,3,4) which have to be compared with the energy levels of our computer simulations.

the framework of two-dimensional conformal field theory. This is illustrated first with the D=3 Z(2) gauge model. The Abelian subgroup Z(2) of SU(2) is expected to play an important role in the microscopic mechanisms of quark confinement suggesting that Wilson surface physics of the D=3 Z(2) gauge spin model should have qualitative and quantitative similarities with the theoretically more difficult QCD_3 case. In the critical region of the Z(2) model we have a rather reasonable description of continuum string formation based on the excitation spectrum of a semiclassical defect line (soliton) of the equivalent Φ^4 field theory. The surface physics of the Z(2) gauge model is closely related to the BCSOS model by universality argument and a duality transformation: *their surface spectra should show universality.*

3.1. BCSOS Surface Spectrum

The body-centered solid-on-solid (BCSOS) model is obtained from the SOS condition (accurate to a few percent around the roughening transition) on the fluctuating interface in the body-centered cubic Ising model[5]. This model can be mapped into the six-vertex formulation for which the Bethe Ansatz equations are known[6]. It follows from the Bethe Ansatz solution that the surface has a roughening phase transition at $T_R = J/(k_B \cdot 2\ln 2)$ which is of the Kosterlitz-Thouless type. For $T < T_R$ the interface is smooth with a finite mass gap in its excitation spectrum. For $T \geq T_R$ the mass gap vanishes and the surface exhibits a massless excitation spectrum.

We determined the low energy part of the full surface spectrum from direct diagonalization of the transfer matrix of the BCSOS model and from the numerical solution of the Bethe Ansatz equations. A periodic boundary condition was used, which corresponds to the spectrum of a periodic Polyakov line in the Z(2) gauge model. With a flux of period L we used exact diagonalization for $L \leq 18$, and the Bethe Ansatz equations up to L=1024. The following picture emerges from the calculation for large L values in the massless Kosterlitz-Thouless (KT) phase. The ground state energy of the flux is given by

$$E_0(L) = \sigma_\infty \cdot L - \frac{\pi}{6L} c + o(1/L) \; , \qquad (1)$$

where σ_∞ is the string tension, c designates the conformal charge, which is found to be c=1 to very high accuracy. The o(1/L) term designates the corrections to the leading 1/L behavior; they decay faster than 1/L. At the critical point of the roughening transition, the corrections can decay very slowly, like $1/(\ln L^3 \cdot L)$ for the ground state energy. Away from the critical point, the corrections decay faster than 1/L in power-like fashion. *These finite size (or equivalently, finite cut-off) effects in the fine structure of the spectrum are dominated by the Sine-Gordon operator in con-*

formal perturbation theory[7].

For each operator O_α which creates states from the vacuum (surface ground state) with quantum numbers α, there is a tower excitation spectrum,

$$E^{\alpha}_{j,j'}(L) = E_0(L) + \frac{2\pi}{L}(x_\alpha + j + j') + o(1/L) \ , \quad (2)$$

where the nonnegative integers j,j' label the conformal tower and x_α is the anomalous dimension of the operator O_α. The momentum of each excitation is given by

$$P^{\alpha}_{j,j'}(L) = \frac{2\pi}{L}(s_\alpha + j - j') \ , \quad (3)$$

where s_α is the spin of the operator O_α. We find excitations of conformal towers built on the integer scaling exponents x=1,2 which are independent of the coupling and correspond to naively expected string excitations. However, we also find scaling dimensions x_α which continuously vary with the coupling J in the rough phase. This sequence can be labelled by anomalous dimensions

$$x^{G}_{n,m} = \frac{n^2}{4\pi K} + \pi K m^2 \ , \quad (4)$$

where n,m are non-negative integers and the constant K depends in a known way on the BCSOS coupling constant J. The physical interpretation of the rather peculiar excitations of the rough gapless surface will be discussed elsewhere[7]. The surface spectrum is described by a compactified conformal Gaussian field. The nontrivial part of the spectrum corresponds to field configuration with line defects which describe the screw dislocation pairs of the fluctuating rough surface.

4. D=3 QCD STRING THEORY

If the $Q\overline{Q}$ color sources are located along one of the principal axes on the lattice in some spatial direction, the Wilson surface at strong coupling is *smooth* in technical terms. This implies the existence of a mass gap in its excitation spectrum, as seen for example in the strong coupling tests of our simulation technology. As the coupling weakens, a roughening transition is expected in the surface at some finite gauge coupling $g = g_R$ where the gap in the excitation spectrum vanishes with the characteristics of the Kosterlitz-Thouless phase transition. The correlation length in the surface diverges at g_R and it is expected to remain infinite for any value of the gauge coupling when $g \leq g_R$. At the roughening transition, the bulk behavior differs from that of the continuum theory which is located in the vicinity of $g = 0$. The low energy excitation spectrum of the Wilson surface for $g \leq g_R$ and not far from g_R, in the domain of the critical KT phase, should be essentially identical to Eqs. (1-3) of our BCSOS spectrum.

Now, is the critical Kosterlitz-Thouless picture around $g \leq g_R$ identical to what we expect for the Wilson surface in the low energy limit of continuum QCD_3 string theory at $g = 0$? We guess that according to the most likely scenario the Wilson surface remains massless throughout the $0 \leq g \leq g_R$ region but its critical behavior will cross over from the Kosterlitz-Thouless class of the conformal Gaussian behavior into the universality class of continuum QCD string theory whose precise description remains the subject of our future investigations. The low energy effective string Lagrangian will contain higher dimensional operators which will signal the deviation from the Gaussian universality class. These operators introduce a physical fine structure into the low energy spectrum. It will remain a challenge to disentangle this physical fine structure from finite cut-off effects in the surface which manifested themselves as finite size corrections in the conformal spectrum at the roughening transition.

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