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Topological Solitons from DeConstructed Extra Dimensions

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Abstract

A topological monopole-like field configuration exists for Yang-Mills gauge fields in a $4 + 1$ dimensions. When the extra dimension is compactified to $3+1$ dimensions with periodic lattice boundary conditions, these objects reappear in the low energy effective theory as a novel solution, a gauged-bosonic Skyrmion. When the low energy theory spontaneously breaks, the Nambu-Goldstone mode develops a VEV, and the gauged-bosonic Skyrmion morphs into a ‘t Hooft–Polyakov monopole.

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1 Introduction

This is a tale of three well-known topological solitons: the instanton, the Skyrmion, and the ‘t Hooft-Polyakov monopole. All three of these objects arise from a common source when Yang-Mills fields propagate in a $4+1$ bulk compactified by periodic boundary conditions to $3+1$ dimensions. A consistent description of this dimensional descent is most readily obtained through deconstruction, or latticization, of the extra compactified dimension. The structures of the conserved Chern-Simons currents neatly match, as they must, between the effective descriptions.

We start in $4+1$ dimensions with an $SU(2)$ Yang-Mills theory and note that there are “instantonic monopoles” (IM). These are static, topologically stable solutions of the pure Yang-Mills gauge theory and represent nontrivial homotopy of $\Pi_3(SU(2))$, the winding of the field configuration on the surface S_3 at infinity in four spatial dimensions. These objects were considered about a year ago by Ramond and the present author [1, 2], and they are evidently the anticipated pure-Yang-Mills solitons that can exist only in $4+1$ by Deser [3]. These are essentially instantons [4, 5] “lifted” to become the spatial configurations of a static object. For the Instantonic Monopole we can choose in $4+1$ the (noncompactified) vector potentials (where $A, B, ..$ run from 0 to 4, x^4 is our 5th dimension; time is x^0).:

$$A_4^a \frac{\tau^a}{2} = -\frac{1}{g}\frac{\vec{x}\cdot\tau}{\lambda^2+r^2} \qquad\qquad A_i^a \frac{\tau^a}{2} = \frac{1}{g}\frac{(x_4\tau_i + \vec{x}_j\epsilon^{ijk}\tau_k)}{\lambda^2+r^2} \tag{1.1}$$

This field configuration has an associated conserved topological current [1]:

$$Q_A = \frac{\overline{g}^2}{16\pi^2}\epsilon_{ABCDE}\,\mathrm{Tr}(F^{BC}F^{DE}) \tag{1.2}$$

The resulting field strength is self–dual as static configuration, i.e., $F_{AB} = \tilde{F}_{0AB}$. It has a mass given by $8\pi^2/\overline{g}^2$ where $\overline{g}$ is a $4+1$ coupling constant with dimension $(\mathrm{mass})^{-1/2}$. This mass is essentially M_{KK}/α where M_{KK} is the lowest KK-mode mass when the theory is compactified.

If we compactify the 5th dimension and “deconstruct,” or latticize the compactified dimension, we obtain an equivalent low energy effective theory in $3+1$ dimensions [6, 7, 8]. With periodic boundary conditions in our compactification, the A_4^a vector potential becomes a Nambu-Goldstone zero mode, and the product of Wilson links in the x^4 dimension becomes a low energy chiral field U, the exponentiated Nambu-Goldstone zero mode. Keeping only a single lattice brane as an approximation, the effective low energy $3+1$ theory is then the gauged chiral Lagrangian:

$$L = \frac{1}{2}v^2 \,\mathrm{Tr}[D_\mu, U^\dagger][D^\mu, U] - \frac{1}{2}\,\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} \tag{1.3}$$

where $F_{\mu\nu} \equiv F^a_{\mu\nu}\tau^a/2$, and with $U = \exp(i\phi/v)$, $\phi = \phi^a\tau^a/2$, where ϕ is essentially the Wilson line $\sim ig\int dx^4 A^4$ in $4+1$.

What then is the fate of the instantonic monopole in this low energy theory viewed as a dimensional deconstruction? In an attempt to clarify what the instantonic monopole is in a compactified theory originally motived the present author to consider latticizing the extra dimension [6, 7]. As we will see, there is a remarkable correspondence.

The first problem is to ask whether a compactified IM solution exists? This question was approached in ref.[1], but see [2]. One employs the method of images and exact multi-instanton solutions to construct a solution satisfying the periodic boundary conditions. This is well known in the case of finite temperature applications of instantons, and has been studied in detail, [9]. For compactification with periodic boundary conditions, the low energy pseudoscalar A^a_4 remains as a zero mode, while with orbifold boundary conditions this mode is absent. Correspondingly, while it is straightforward to compactify the IM with periodic boundary conditions, it is not with antiperiodic. This is because of topology; the topology is determined by the winding of field $U = \exp(ig\int dx^4 A_4)$ throughout the manifold, and this will form the basis of the correspondence with a low energy effective Lagrangian description below.

One important consequence of compactification of the IM is the following [9]. In an infinite bulk the IM is conformally invariant. The solution has a scale parameter λ but the action is independent of λ. The action density is concentrated in an arbitrarily large region $r \lesssim \lambda$, ergo arbitrarily large instantons exist. When a dimension is compactified with a length scale δ, however, the field strength configuration changes, and the action density has appreciable values only over $r \lesssim \delta$. Hence, compactification effectively cuts-off the large instantonic monopoles and gives them a size of order the compactification scale.

For the effective description of the $4+1$ IM in the $3+1$ effective Lagrangian we note that the theory of eq.(1.3), which is just a conventional gauged chiral Lagrangian, does indeed contain a novel soliton, a "bosonic gauged Skyrmion." This, we will argue, is the $3+1$ correspondence of the instantonic monopole of $4+1$. This object is an "inverted Skyrmion" built out of the $\exp(i\phi/v)$ Wilson link chiral field. At infinity $\phi/v \to \pi\hat{x}\cdot\vec{\tau}$ is a hedgehog, while at the origin $\phi/v \to 0$. This is inverted from the usual Skyrmion, but still trivially represents the nontrivial $\Pi_3(SU(2))$ mapping into the $3+1$ spatial volume (which, of course, corresponds to the spatial S_3 surface of $4+1$). There are, however, other key differences between the Bosonic Gauged Skyrmion BGS and the usual Skyrmion.

The usual Skyrmion has a nontrivial Wess-Zumino (WZ) term which gives it unusual spin and statistics. Choosing the quantized WZ term coefficient to match to $N_c = 3$ QCD, the WZ term makes the Skymion into a spin$-\frac{1}{2}$ baryon. In the present case the gauging by $SU(2)$ forbids the WZ term, and the gauged Skyrmion is a bosonic object of spin-0.

The usual Skyrmion carries a nontrivial topological charge determined from a Chern-Simons current. This current is nontrivially modified in the present case, and is seen to involve a new term which is seen to match the current of eq.(1.2) under dimensional descent.

2 Gauge Invariant Chern-Simons Current

The usual Skyrmion is associated with the conserved, normalized Chern-Simons current, and carries a unit charge:

$$Q^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left(U^\dagger(\partial_\nu U)U^\dagger(\partial_\rho U)U^\dagger(\partial_\sigma U)\right) \tag{2.4}$$

The index for the usual Skyrmion anzatz, $U = \cos(f(r)) + \vec{\hat{x}}\cdot\tau\sin(f(r))$, is then

$$\int d^3x\frac{1}{24\pi^2}\epsilon^{ijk}\,\mathrm{Tr}\left(U^\dagger\partial_i UU^\dagger\partial_j UU^\dagger\partial_k U\right) = \frac{1}{2\pi}[2(f(\infty)-f(0))+\sin(2f(\infty))-\sin(2f(0))] \tag{2.5}$$

$f(r) = \phi(r)/v$ is a kink-like configuration that runs from $f(0) = 0$ to $f(\infty) = \pi$, and thus has unit charge. Note that $f \to \pm f + N\pi$ is a discrete symmetry, so the usual QCD Skyrmion with $f(0) = \pi$ to $f(\infty) = 0$ is equivalent. Including gauge fields and the S_2 Skyrme term defined below breaks this discrete symmetry to $f \to f + N\pi$.

When we go over to the gauged case, we might guess that the gauge invariant generalization of the Chern-Simons current is:

$$Q_1^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left(U^\dagger[D_\nu,U]U^\dagger[D_\rho,U]U^\dagger[D_\sigma,U]\right) \tag{2.6}$$

Note that $Q_1^\mu = V + A$ is constructed from purely right-handed (or left-handed) chiral currents. One cannot build a conserved Chern-Simons current out of the product of mixed vector $V = (1/2)(U^\dagger[D_\rho,U] + U[D_\rho,U^\dagger])$ and axial vector $A = (1/2)(U^\dagger[D_\rho,U] - U[D_\rho,U^\dagger])$ currents, even in the ungauged case. Q_1^μ transforms, however, as a vector under parity, since $\phi \to -\phi$ hence $U \leftrightarrow U^\dagger$, $\epsilon^{\mu\nu\rho\sigma} \leftrightarrow -\epsilon^{\mu\nu\rho\sigma}$ and $D_\mu \to -D_\mu$, so $Q^\mu \to -Q^\mu$. Note also,

$$Q_1^\mu = -\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left(U[D_\nu, U^\dagger]U[D_\rho, U^\dagger]U[D_\sigma, U^\dagger]\right) \tag{2.7}$$

using $U^\dagger[D_\nu, U] = -[D_\nu, U^\dagger]U$ and cyclicity of the trace, hence Q_1 is equivalent to a current built out of pure left-handed chiral currents, i.e., the Chern-Simons current is unique.

However, Q_1 is not conserved, as seen by explicit calculation:

$$\partial_\mu Q_1^\mu \;=\; \frac{ig}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left(F_{\mu\nu}[D_\rho, U][D_\sigma, U^\dagger] - F_{\mu\nu}[D_\rho, U^\dagger][D_\sigma, U]\right) \tag{2.8}$$

On the other hand, we can introduce two new currents:

$$Q_2^\mu = \frac{ig}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left(F_{\nu\rho}U^\dagger[D_\sigma, U] - F_{\nu\rho}U[D_\sigma, U^\dagger]\right) \tag{2.9}$$

$$Q_3^\mu = \frac{ig}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left(F_{\nu\rho}U^\dagger[D_\sigma, U] + F_{\nu\rho}U[D_\sigma, U^\dagger]\right) \tag{2.10}$$

These latter currents are expected to play a role in the gauged Skyrmion because in $4+1$ dimensions we had the conserved current of the instantonic-monopole:

$$\epsilon_{ABCDE}\,\mathrm{Tr}(F^{BC}F^{DE}) \sim \epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}(F_{\mu\nu}U^\dagger[D_\sigma, U]) \tag{2.11}$$

and $U^\dagger[D_\sigma, U] \sim F_{\sigma 4}$ is the dimensional descent correspondence. We see that Q_2 has normal vectorial parity, and it can thus form a vector combination with Q_1. Q_3 is an axial vector under parity. Computing the divergence of Q_2 we obtain the opposite of the *rhs* eq.(2.8), and we thus arrive at the conclusion that there is a new conserved current:

$$\widetilde{Q}^\mu = Q_1^\mu + Q_2^\mu \qquad\qquad \partial_\mu\left(\widetilde{Q}^\mu\right) = 0 \tag{2.12}$$

$\widetilde{Q}^\mu$ is the $3+1$ current corresponding to the $4+1$ eq.(1.2) under dimensional descent. For completeness, notice that the axial current is not conserved:

$$\begin{aligned}\partial_\mu Q_3^\mu \;=\; &-\frac{ig}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left(F_{\mu\nu}[D_\rho, U][D_\sigma, U^\dagger] + F_{\mu\nu}[D_\rho, U^\dagger][D_\sigma, U]\right)\\ &+\frac{ig}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left(F_{\mu\nu}U^\dagger F_{\rho\sigma}U - F_{\mu\nu}F_{\rho\sigma}\right)\end{aligned} \tag{2.13}$$

The latter term somewhat resembles an anomaly. If, for the usual Skyrmion, we gauged only the left-handed or right-handed pieces of U, e.g., as in the electroweak theory, then we would obtain the normal current algebra anomalies through these manipulations, but our baryon number current must be modified as in eq.(2.12).

The usual Skyrmion in a pure chiral lagrangian is unstable to core collapse. It's core is stabilized by adding the "Skyrme term" which is a short-distance correction to the action:

$$S_0 = \frac{1}{32}\,\mathrm{Tr}\,([U^\dagger \partial_\mu U, U^\dagger \partial_\nu U])^2 \tag{2.14}$$

In the present case there will indeed be gauge invariant generalizations of the Skyrme-term induced by higher order effects of KK-modes and loops,

$$S_1 = \frac{1}{32}\,\mathrm{Tr}\,([U^\dagger [D_\mu, U], U^\dagger [D_\nu, U]])^2 = \frac{1}{32}\,\mathrm{Tr}\,([[D_\mu, U^\dagger], [D_\nu, U]])^2 \tag{2.15}$$

The gauge invariant Skyrme term S_1 stabilizes the solution on distance scales $\delta \sim 1/v$. There are also candidate new $d = 4$ Skyrme terms in the present case,

$$\mathcal{O}_1 = i\,\mathrm{Tr}\,F_{\mu\nu}(D^\mu U^\dagger)(D^\nu U) \qquad \mathcal{O}_2 = i\,\mathrm{Tr}\,F_{\mu\nu}(D^\mu U)(D^\nu U^\dagger) \tag{2.16}$$

Under parity we have $U \leftrightarrow U^\dagger$, and hence $P : \mathcal{O}_1 \leftrightarrow \mathcal{O}_2$ and under charge conjugation $C : \mathcal{O}_i \to \mathcal{O}_i$ (note that $C : F_{\mu\nu} \to -F_{\mu\nu}$, but we must treat $C : ig \to -ig$ for consistency with $C : D_\mu \to D_\mu$). Hence, an odd combination under CP, but odd under P and even in C is:

$$S_2 = -\frac{ig}{8}\,\mathrm{Tr}\,F^{\mu\nu}[U^\dagger[D_\mu, U], U^\dagger[D_\nu, U]] = -\frac{ig}{8}\,\mathrm{Tr}\,F^{\mu\nu}([D_\mu U^\dagger][D_\nu, U]] - [D_\mu U][D_\nu, U^\dagger]) \tag{2.17}$$

while:

$$S_3 = -\frac{ig}{8}\,\mathrm{Tr}\,F^{\mu\nu}([D_\mu U^\dagger][D_\nu, U] + [D_\mu U][D_\nu, U^\dagger]) \tag{2.18}$$

is even under CP, even under P and even under C. These terms are always destabilizing, and determine the sign of the Skyrmionic field configuration.

The stable Skyrmion solution necessarily involves the nontrivial near-zone gauge field configuration in the core, which we will see below is identical to the short-distance core of a BPS monopole, as well as the conventional gauge invariant Skyrme term.

3 Energetics

The existence of the conserved current $\widetilde{Q}^\mu$ guarantees that there are nontrivial Skyrmionic configurations including the gauge fields. The core profile of the Skyrmion must act as a source to Yang-Mills fields. Moreover, the GBS is the deconstructed dimensional analogue of the original instantonic monopole. The chiral field ϕ/v is identified with the Wilson line $\int dx^4 A_4$ and we thus choose the anzatz:

$$\int dx^4 A_4 \sim \phi/v = f(r)\hat{x}\cdot\vec{\tau} \tag{3.19}$$

For the vector potential we choose:

$$A_i^a \frac{\tau^a}{2} = \frac{h(r)}{g} \vec{x}_j \epsilon^{ijk} \tau_k \tag{3.20}$$

The energy ($(-1)\times$action for static configurations) then takes the form:

$$\begin{aligned} E &= \frac{4\pi}{g^2} \int dr \left[r^2 \left(h'(r) + \frac{h}{r} \right)^2 + \frac{1}{2} r^2 \left(h^2(r) - \frac{h^2}{r} \right) \right] \\ &\quad + \frac{1}{2} v^2 \int dr \left[r^2 (f'(r))^2 + 2(H(r))^2 \sin^2(f(r)) \right] \end{aligned} \tag{3.21}$$

where it is convenient to introduce the combination:

$$H(r) = 1 - rh(r) \tag{3.22}$$

If we substitute any particular anzatz into eq.(3.21) we obtain:

$$E = \frac{4\pi}{\lambda g^2} c_0 + \frac{1}{2} v^2 \lambda c_1 \tag{3.23}$$

where c_0 and c_1 are determined from the Yang-Mills and Skyrmionic energies respectively. The energy of the particular anzatz is then relaxed to the minmum of eq.(3.23) with the choice:

$$\lambda^2 = \frac{8\pi c_0}{g^2 v^2 c_1} \quad (a); \qquad E = \frac{2v}{g} \sqrt{2\pi c_0 c_1} \quad (b) \tag{3.24}$$

The energy is equipartitioned between the two terms of eq.(3.23), which accounts for the factor of 2 in eq.(3.24.b).

To verify that a nontrivial Yang-Mills field is part of a stable solution we check that it is required for binding. We can compare to the energy of the same Skyrme profile in the case that the Yang-Mills field is switched off:

$$E_{off} = \frac{1}{2} v^2 \int dr \left[r^2 (f'(r))^2 + 2 \sin^2(f(r)) \right] \equiv \frac{1}{2} v^2 c_2 \tag{3.25}$$

Thus, we must have:

$$\frac{E}{E_{off}} < 1, \qquad \text{or,} \qquad \frac{c_1}{c_2} < \frac{1}{2}. \tag{3.26}$$

Various choices of anzatze for the GBS have been explored numerically. One is inspired from the instantonic monopole. Matching $f(r)$ to $\int dx^4 A_4$ and $h(r)$ to the $x^4 = 0$ behavior of A_i we obtain:

$$f(r) = \frac{\pi r}{\lambda^2 + r^2}, \qquad h(r) = \frac{2r}{\sqrt{\lambda^2 + r^2}}, \tag{3.27}$$

In fact, we find that this anzatz *is not bound*, and numerically $E/E_{off} = 1.4$, not close to a binding a solution. The reason is as follows; we can easily see that for small r, and $f(r)$, our action is equivalent to that of a BPS monopole (e.g., see the analysis of [10]). The core structure of the previous anzatz is far from that of a BPS monopole. After some numerical experimentation we are led to the following:

$$\tilde{f}(r) = \frac{\pi\sqrt{r}}{\sqrt{\epsilon\lambda + r}}, \qquad \tilde{h}(r) = \frac{2}{\lambda + r}, \tag{3.28}$$

Let us initially choose $\epsilon = 1$. Then we find:

$$\begin{aligned} \tilde{c}_1 &= \int_0^\infty dx \frac{\pi^2 x}{4(1+x)^3} + 2\sin^2\left(\frac{\pi\sqrt{x}}{\sqrt{1+x}}\right)\left(\frac{1-x}{1+x}\right)^2 \\ \tilde{c}_2 &= \int_0^\infty dx \frac{\pi^2 x}{4(1+x)^3} + 2\sin^2\left(\frac{\pi\sqrt{x}}{\sqrt{1+x}}\right) \end{aligned} \tag{3.29}$$

and this leads to a net binding:

$$\frac{E}{E_{off}} = \frac{2\tilde{c}_1}{\tilde{c}_2} = 0.883 < 1 \tag{3.30}$$

While the form of eq.(3.23) suggests stability of the core of a solution supported by the Yang-Mills field, generally we find that the Skyrme profile can be deformed to collapse and reduce the energy in the absence of the Skyme S_1. We can, for example, deform the above solution by choosing $\epsilon \neq 1$. We find that the energy is reduced, and the Skyrme core is unstable.

The Skyrme terms can be added and take the form in an anzatz:

$$S_1 = 2\pi \int dr \left[\sin^2(f)(1-rh)^2\left(2(f')^2 + \frac{1}{r^2}\sin^2(f)(1-rh)^2\right)\right] \tag{3.31}$$

and, for example, should we choose to include it:

$$S_2 = 2\pi \int dr \left[-f'\sin(2f)H(r)(rh'+h) + \sin^2(f)H^2(r)\left(h^2 - \frac{2h}{r}\right)\right] \tag{3.32}$$

S_1 must enter the energy with a positive coefficient and always dominates at extreme core collapse. Note that positivity of the energy is gauranteed if one defines $\tilde{F}_{ij} = F_{ij} + \tilde{c}[[D_i, U^\dagger][D_j, U]]$ and defines the energy density by $\mathrm{Tr}\,\tilde{F}_{ij}\tilde{F}^{ij}/2$. This fixes the Skyrme term coefficients.

4 Spontaneouly Broken $SU(2)$

We can imagine adding terms to the Lagrangian consistent with the $SU(2)$ symmetry of the form $\sum_p c_p(\mathrm{Tr}(U))^p + h.c.$. Indeed, such terms must arise at the quantum level, as in a computation of the Coleman-Weinberg potential ref.[8]. We presently add them by hand. With such terms we can then destabilize the vacuum; ϕ becomes a Higgs-field which breaks $SU(2) \to U(1)$, (as in the recent model of ref.[8], though we do not presently want an $I = \frac{1}{2}$ Higgs). This means that $\langle\phi\rangle = (1-\epsilon)v$ where $\epsilon \neq 0$ is not gauge equivalent to the unbroken vacuum,

When ϕ is an isovector field we have all of the conditions required for a nontrivial $\Pi_2(SU(2)/U(1))$. In this case the the Gauged-Bosonic Skyrmion grows into a 't Hooft-Polyakov monopole. The monopole charge is measured by a Chern-Simons charge in one less dimension, integrated over the surface at infinity. This contains the dual of F_{ij}, e.g., $\mathrm{Tr}\,\chi\epsilon^{ijk}F_{ij}$ which is integrated over the surface $d\Sigma^k$ at infinity. We have:

$$\int r^2 \sin\theta d\theta d\phi \; \tilde{F}_r\Big|_{r^2\to\infty} = 4\pi r^2 \frac{H^2-1}{2gr^2} = -\frac{2\pi}{g} \tag{4.33}$$

where $H(r)$ is defined in eq.(3.22), and we see that asymptotically $H(\infty)$ (as well as $H(0)$) tends to zero [10]. The Skyrme terms now play no role in the core stability since the nontrivial potential is determining the field value at infinity and it costs energy to shrink the core.

Remarkably, however, we see that our monopole is nontrivially charged under the original $3+1$ Chern-Simons charge $\tilde{Q}$ as well. Including the gauge degrees of freedom in the Chern-Simons current in $3+1$ we find that the Chern-Simons charge density is an exact diffential and the result:

$$\int d^3x \frac{1}{24\pi^2}\tilde{Q}_0 = \frac{1}{2\pi}[2(f(\infty)-f(0)) + \sin(2f(\infty))H(\infty) - \sin(2f(0))H(0)] \tag{4.34}$$

Note that no manipulations involving the Chern-Simons current rely upon the use of equations of motion. The monopole anzatz for $f(r)$ is similar to the Gauge Bosonic Skyrmion, with $f(0) = 0$ but now the asymptotic value $f(\infty) = (1-\epsilon)\pi$ is not π. The Chern-Simons charge is now $(1-\epsilon)$, and is an arbitrary fractional quantity. The reason is that by forcing $f(\infty)$ to a value less than π we do not completely, but only partially, map $SU(2)$ into the 3-volume. Essentially, some fraction of the Skymion's charge has flowed out to infinity as the field relaxes into it's nontrivial VEV.

5 Conclusions

We have explored the dimensional descent of a pure Yang-Mills gauge theory in $4+1$ dimensions, via deconstruction, into a $3+1$ effective low energy description. We have seen that topologically nontrivial objects, Instantonic Monopoles, exist in $4+1$ with nontrivial conserved charges. Under deconstruction these objects morph into Gauged Bosonic Skyrmions, carrying a conserved gauge Chern-Simons charge. The scale size and masses are determined by the compactification scale. With spontaneous symmetry breaking, the GBS's further morph into 't Hooft–Polyakov monopoles. The latter objects carry the usual magnetic charge, i.e., the magnetic flux crossing the surface at infinity, as well as a fractional Chern-Simons charge in $3+1$

All of this occurs with pure-gauge $SU(2)$ Yang-Mills theory (it is imbeddible into $SU(N)$) with no explicit Higgs fields, or explicit chiral fields! It is a consequence of dimensional compactification, and deconstruction, which requires the latticization of the extra dimensions to maintain the explicit manifest gauge invariance. It is an explicit demonstration of the descent cohomology of the classical topological solutions themselves. Moreover, such objects appear to be a necessary consequence of Yang-Mills gauge theories propagating in the bulk with periodic boundary conditions.

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