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Determining the Phases α and γ from Direct CP Violation in B_u, B_d and B_s Decays to Two Vectors

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Abstract: A method for clean determination of the unitarity angles α and γ is proposed that uses only direct CP violation and does not require any time dependent measurements. The method takes advantage of helicity amplitudes for B_u, B_d and B_s decay to two vector mesons and can be used, at any B-facility, in conjunction with a large number of modes. It also allows for experimental tests of theoretical approximations involved.

Considerable progress has recently been made in experimental determination of the angle β of the unitarity triangle and improved measurements are expected in the near future [1, 2, 3]. However, even with increased precision in the extraction of β, the ability to test the CKM [4] description of CP violation in the Standard Model (SM) will be limited as the existing tests rely on theoretical calculation of hadronic matrix elements which still have considerable uncertainties [5]. Therefore clean determinations of all three angles (α, β, γ) of the unitarity triangle is important to facilitate precision tests of the SM and search for new CP-odd phase(s) due to physics beyond the SM. Currently the asymmetric B-factories are performing remarkably well and in the future hadronic B-machines may produce 1-3 orders of magnitude more B-mesons. Thus, it is very important to devise methods for clean extraction of the angles that can be used at all B-facilities. Furthermore, since time dependent measurements involving B_s mesons are extremely challenging due to

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the high frequency of $B_s - \bar{B}_s$ oscillations, it would also be helpful if methods could be devised that do not require such time dependent information.

Motivated by these considerations we propose a method with the novelty that it uses only direct CP-violation and does not involve any use of time dependent measurements. Potentially all types of B-mesons (B_u, B_d, B_s) can be used at all B-facilities. The method also allows for experimental tests of the key theoretical approximations that are involved. Specifically, we propose determination of the angles α and γ through a study of the interference of tree and penguin amplitudes in decays of B-mesons to two vector particles.

There have been several methods proposed to extract these angles. For α one can consider oscillation effects in $B^0 \to \pi^+\pi^-$ although one must account for the penguin through isospin analysis [6] by observing $B^0 \to \pi^0\pi^0$. Since the branching ratio to $\pi^0\pi^0$ is expected to be small and hard to observe, it may be preferable to consider $B^0 \to \pi^+\pi^-\pi^0$ where one can also take advantage of resonance effects in the Dalitz plot [7]; however there may be problems in precise modeling of the resonance structure. Another method for extracting α from the interference of u-penguins with t-penguins in $B^0 \to K^{(*)}K^{(*)}$ may overcome the disadvantages of the 2π and 3π final states [8] although the analogous B_s decays are required for the analysis. All three of these methods use time dependent CP violation measurements of $B^0(\bar{B}^0)$ decays driven by the mixing in that system while our method relies on direct (time independent) CP violation only.

The angle γ may be extracted at the B-factories through the interference of $b \to u\bar{c}s$ and $b \to c\bar{u}s$ [9]. In the method we discuss here, we will obtain γ through the interference of the $b \to s$ penguin and the $b \to su\bar{u}$ tree and α from the interference of the $b \to d$ penguin and the $b \to du\bar{u}$ tree.

Our method requires measurements made in two separate decay modes. One mode of the form $B \to V_1V_2$, together with $\bar{B} \to \bar{V}_1\bar{V}_2$, ($B = B_u$, B_d or B_s) which receives both tree and penguin contributions and another mode $B \to V_3V_4$ which is a pure penguin. The procedure uses helicity amplitudes of $B \to V_1V_2$, which can be inferred from decay distributions of the vectors [10, 11, 12, 13, 14]. By analyzing this data together with the helicity amplitudes for $B \to V_3V_4$ one can extract the tree-penguin weak phase difference. From a $b \to d$ penguin transition contributing to $B \to V_1V_2$, we deduce $|\frac{V_{td}}{V_{ts}} \sin\alpha|$ from which $|\sin\alpha|$ may be obtained once $|V_{td}/V_{ts}|$ is known [15]; from a $b \to s$ penguin we can obtain $|\sin\gamma|$. A list of such modes is shown in Table 1. Thus to implement this method to obtain α one needs to study one mode from the

α-mode column and one mode from the "pure penguin" column. Likewise to obtain γ one γ-mode and one pure penguin mode is required. Using more than one mode from each column will, of course, increase the analyzing power of the method.

Let us first discuss some of the tree penguin interference modes that are suitable in our method for extracting α. In charged B decays, the relevant mode is $B^{\pm} \to \rho^{\pm}\omega$. In this case the tree graph is color allowed which we find is advantageous in terms of statistical power. Since the charge of the ρ indicates the flavor of the initial state, no other tagging is required. However, this mode has two π^0, in the final state originating from decays of $\rho^{\pm}$ and ω perhaps making it difficult for hadronic machines.

From Table 1 we see see that for an analogous extraction of α via decays of B^0 we may use any one of the modes $B^0 \to \rho^0\rho^0$, $\rho^+\rho^-$, $\omega\rho^0$ or $\omega\omega$. These modes require tagging at production and C-odd interference terms are degraded somewhat by time integration. Also the tree is color suppressed; however, in the case of $\rho^0\rho^0$ it leads to a final state that does not contain any π^0 and so may be easier to implement at hadronic B machines. The mode $B^0 \to \rho^+\rho^-$ has the advantage that any electro-weak penguin (EWP) contribution is color suppressed.

Decays of B_s usable for α are $B_s \to K^{*0}\rho^0$ or $B_s \to K^{*+}\rho^-$. Both are self tagging, though in the first case only if $K^{*0} \to K^+\pi^-$. The first mode also has the advantage that the final state contains no π^0. The final state $K^{*+}\rho^-$, while suffering from at least one π^0 in the final state, has the advantage that possible contamination from the EWP is color suppressed.

Likewise, our method can also be used to extract γ through the use of $b \to s$ penguins. In B^0 the candidate modes are $B^0 \to K^{*+}\rho^-$, $B^0 \to K^{*0}\rho^0$ and $B^0 \to K^{*0}\omega$; note that $B^0 \to K^{*0}\rho^0$ has the advantage of no π^0 in the final state. For charged B's again there are two such modes: $B^+ \to K^{*+}\rho^0$ and $B^+ \to K^{*+}\omega$, either one of which could be used to obtain γ; $B^+ \to K^{*+}\rho^0$ is free of π^0.

Our method also needs input from one pure penguin mode; this should be a $b \to s$ penguin since we need the component with an intermediate u quark to be small. For pure penguin modes we can use: $B^+ \to \phi K^{*+}$, $B^0 \to \phi K^{*0}$, $B^+ \to K^{*0}\rho^+$, $B_s \to \phi\phi$ or $B_s \to K^{*0}K^{*0}$. Likewise the decays $B^0 \to \rho^0 K^{*0}$ and $B^0 \to \omega K^{*0}$ can also be used since the tree contribution is color and CKM suppressed. In Table 1 as well as listing all the modes mentioned, we indicate which have color suppressed EWP (underlined); which have color allowed tree contributions (parentheses) and which have π^0-free final states [square brackets].

Table 1: The relevant $B \to VV$ modes originating from each kind of B meson are shown. In the "α-mode" column are tree-penguin interference modes which are sensitive to α; likewise the "γ-modes" are sensitive to γ. The "pure penguin" modes proceed through $b \to s$ penguin processes only. The underlined modes have a color suppressed EWP contribution; the modes enclosed in parentheses have color allowed tree contributions while the modes enclosed in square brackets have π^0 free final states.

B-meson	α-mode	γ-mode	Pure Penguin
B^+	$(\rho^+\omega)$	$[(K^{*+}\rho^0)]$, $(K^{*+}\omega)$	$[\phi K^{*+}]$, $\underline{K^{*0}\rho^+}$
B^0	$(\underline{\rho^+\rho^-})$, $[\rho^0\rho^0]$, $\omega\omega$, $\rho^0\omega$	$(\underline{K^{*+}\rho^-})$	$[K^{*0}\phi]$
B_s	$[K^{*0}\rho^0]$, $(\underline{K^{*+}\rho^-})$, $K^{*0}\omega$	see note [16]	$[\phi\phi]$, $[\underline{K^{*0}K^{*0}}]$

In our analysis we will use the approximations that: (1) SU(3) is a valid symmetry for penguin processes, (2) the effects of the EWP are small, (3) the $q\bar{q}$ pair which arises in a strong penguin does not form a single vector meson of the final state. Recall that the EWP are assumed to be small for other proposed methods [6, 7, 8] as well for extracting α. Note also that for some of the modes in Table 1 the EWP is color suppressed. Later we will discuss ways to test each of these three approximations.

To illustrate our method, we now focus on $B^\pm \to \rho^\pm\omega$ (i.e. $V_1V_2 = \rho^+\omega$) where the discussion easily generalizes to the other modes mentioned above. The SM amplitude $\mathcal{A}$ for this process can be written:

$$\mathcal{A} = Tv_u + P_u v_u + P_c v_c + P_t v_t \equiv \hat{T}v_u + \hat{P}v_t \tag{1}$$

where T is the tree contribution to the amplitude, P_i is the penguin contribution due to the diagram with an internal quark of type u_i and $v_i = V_{ib}^* V_{id}$. Unitarity of the CKM matrix allows us to express the amplitudes in terms of

$\hat{T} = T + P_u - P_c$ which we will refer to as the corrected tree and $\hat{P} = P_t - P_c$, the corrected penguin.

For this mode it is useful to follow a (non-standard) convention for the weak phase where the corrected tree is zero. The amplitudes for $B^+ \to \rho^+\omega$ may then be written as:

$$\mathbf{A} = \mathbf{a} + \mathbf{b}e^{i\alpha} \quad \text{where} \quad \mathbf{a} = \hat{T}|v_u| \quad \text{and} \quad \mathbf{b} = \hat{P}|v_t| \tag{2}$$

and each of the bold face terms represent a set of helicity amplitudes. The amplitude for the charge conjugate process in this convention is $\bar{\mathbf{A}} = \mathbf{a} + \Pi\mathbf{b}e^{-i\alpha}$ where Π indicates parity.

Even knowing the full amplitudes in these decays we cannot hope to determine α. This is evident writing the amplitude:

$$\mathbf{A} = \mathbf{c} + i\mathbf{b}\sin\alpha, \quad \bar{\mathbf{A}} = \mathbf{c} - i\Pi\mathbf{b}\sin\alpha \quad \Rightarrow \quad \mathbf{b}\sin\alpha = (\mathbf{A} - \Pi\bar{\mathbf{A}})/(2i) \tag{3}$$

where $\mathbf{c} = \mathbf{a} + \mathbf{b}\cos\alpha$. While we may extract $\mathbf{c}$ and $\mathbf{b}\sin\alpha$, we clearly can not separately obtain $\sin\alpha$ without more information [17]. To solve for α then, we need two more pieces of information: (1) A means to fix the relative weak phase between $\mathbf{A}$ and $\bar{\mathbf{A}}$. (2) A separate normalization of the penguin contribution $\mathbf{b}$.

The measurement of pure penguin amplitudes will provide these remaining two inputs. The normalization of such a mode will clearly allow the determination of $\sin^2\alpha$ while the ratios and phase differences between the components of $\mathbf{b}$ can be matched with those determined from eqn. (3) providing the condition needed to fix the relative phase of $\mathbf{A}$ and $\overline{\mathbf{A}}$.

Let us now consider how the necessary information about the components of the amplitude may be extracted from the experimental data. For both the ω and ρ the polarization vector can be completely determined in their rest frame by their decays. For $\rho^\pm \to \pi^\pm\pi^0$ we can define the polarization vector as $\vec{E}_\rho = \vec{P}_{\pi^0}/|\vec{P}_{\pi^0}|$ while for the ω we define $\vec{E}_\omega = \vec{P}_{\pi^+} \times \vec{P}_{\pi^-}/|\vec{P}_{\pi^+} \times \vec{P}_{\pi^-}|$. Let us introduce θ_1 to be the polar angle between $\vec{E}_\rho$ and the boost axis in the ρ frame and likewise θ_2 the polar angle between $\vec{E}_\omega$ and its boost axis. In addition we introduce the azimuthal angle ψ between $\vec{E}_\rho$ and $\vec{E}_\omega$ with respect to the ρ boost axis. It is convenient to describe the final state in a helicity bases which contains the following states: $|R\rangle = |+1\rangle_\rho|+1\rangle_\omega$, $|L\rangle =$

$|-1\rangle_\rho|-1\rangle_\omega$ and $|0\rangle = |0\rangle_\rho|0\rangle_\omega$. In the following we will rewrite the transverse modes as parity eigenstates: $|S\rangle = (|R\rangle + |L\rangle)/\sqrt{2}$ and $|P\rangle = (|R\rangle - |L\rangle)/\sqrt{2}$ where the action of parity is $\Pi : |0\rangle, |S\rangle \to +|0\rangle, |S\rangle$ and $\Pi : |P\rangle \to -|P\rangle$.

The amplitudes for production of the $|0\rangle$, $|S\rangle$ and $|P\rangle$ states respectively may be expressed in the three component notation: $\mathbf{A} = [A_0,\ A_S,\ A_P]$. The angular distribution (see e.g. [12]) for the two vector decay is $d\Gamma/d\Phi = \sum_{i=1}^{6} X_i f_i(\Phi)$ where Φ represents the phase space, $d\Phi = dz_1 dz_2 d\psi/(8\pi)$ and the basic distributions are given by:

$$\begin{aligned}
&f_1 = 9z_1^2 z_2^2; \quad f_2 = (9/2)u_1^2 u_2^2 \cos^2\psi; \quad f_3 = (9/2)u_1^2 u_2^2 \sin^2\psi; \\
&f_4 = 9\sqrt{2} z_1 z_2 u_1 u_2 \cos\psi; \quad f_5 = 9\sqrt{2} z_1 z_2 u_1 u_2 \sin\psi; \\
&f_6 = 9u_1^2 u_2^2 \sin\psi\cos\psi
\end{aligned} \tag{4}$$

with $z_i = \cos\theta_i$ and $u_i = \sin\theta_i$ $(i = 1, 2)$. In terms of the helicity amplitudes, the coefficients are thus:

$$\begin{array}{lll}
X_1 = |A_0|^2; & X_2 = |A_S|^2; & X_3 = |A_P|^2; \\
X_4 = Re(A_0 A_S^*); & X_5 = Im(A_0 A_P^*); & X_6 = Im(A_s A_P^*);
\end{array} \tag{5}$$

we have normalized the coefficients so that $Br = X_1 + X_2 + X_3$.

A convenient method to determine X_i from the angular distribution is to introduce a set of operators [13] g_i such that $< g_i >= X_i/(X_1 + X_2 + X_3)$. We will use the following set of operators with this property:

$$\begin{aligned}
&g_1 = (40f_1 - 5f_2 - 5f_3)/126, \quad g_2 = (265f_2 - 20f_1 - 85f_3)/504, \\
&g_3 = (265f_3 - 20f_1 - 85f_2)/504, \quad g_{4,5} = 25f_{4,5}/36, \quad g_6 = 25f_6/72.
\end{aligned} \tag{6}$$

Once an experimental determination of the observables $\{X_i\}$ has been made, it is possible to determine the amplitudes up to the following ambiguities: (1) An overall phase, (2) the transformation $\mathbf{A} \to \Pi\mathbf{A}^*$. Thus we have six observables determining three complex (amplitudes), or equivalently six real parameters. The overall phase ambiguity means that the remaining five amplitude parameters are over determined by the six observables. An unobservable phase may be applied to both $\mathbf{A}$ and $\bar{\mathbf{A}}$ leaving one only the relative phase between the B^+ and B^- decays undetermined. To be specific, we will take this weak phase to be $\lambda = \arg(A_0(B^-)A_0^*(B^+))$. Once λ is determined, the experimental observables in B^+ and B^- decay will allow us to obtain $\mathbf{b}\sin\alpha$ from eqn. (3). To obtain $\sin\alpha$ we therefore need a means to fix λ as well as information about $\mathbf{b}$.

Consider now what may be learned from $B^0 \to \phi K^*$. In the SM we expect negligible CP violation in this mode (e.g. in [11] the CP violation is estimated to be $O(1\%)$); thus $X_{1-4}(\phi K^{*0}) = X_{1-4}(\phi \bar{K}^{*0})$ and $X_{5,6}(\phi K^{*0}) = -X_{5,6}(\phi \bar{K}^{*0})$. Within the SM and using our assumption that the gluon does not fragment to a single vector meson, the amplitude $\mathbf{A}(\phi K^{*0}) = \sqrt{2}\mathbf{b}(V_{ts}/V_{td})$ hence the observables will be related to $\mathbf{b}$ by:

$$
\begin{aligned}
&|b_0|^2 = qX_1(\mathcal{F}), \quad |b_S|^2 = qX_2(\mathcal{F}), \quad |b_P|^2 = qX_3(\mathcal{F}), \quad Re(b_0 b_S^*) = qX_4(\mathcal{F}), \\
&Im(b_0 b_P^*) = qX_5(\mathcal{F}), \quad Im(b_S b_P^*) = qX_6(\mathcal{F}), \qquad (7)
\end{aligned}
$$

where $\mathcal{F} = \phi K^*$ and $q = |V_{td}/V_{ts}|^2/2$. We are now in a position to determine λ and $\sin^2\alpha$ since eqn. (7) gives six equations in only two unknowns (i.e. λ and α) when combined with eqn. (3).

We now estimate the precision with which one may extract $\sin^2\alpha$ using this method. The statistical errors will, of course, depend on the number of B mesons which are available. To quantify this for a given decay mode $B \to X$, let us define $\hat{N}_B = (number\ of\ B)(acceptance\ for\ X)$ for each type of B-meson. For concreteness, we consider $\hat{N}_B = 10^8$ and 5×10^8 in our calculations.

The pure penguin modes $B^+ \to \phi K^{*+}$ and $B^0 \to \phi K^{*0}$ have in fact recently been observed at BaBar [18, 19] with $Br(B^+ \to \phi K^{*+}) = (9.7^{+4.2}_{-3.4} \pm 1.7) \times 10^{-6}$ and $Br(B^0 \to \phi K^{*0}) = (8.6^{+2.8}_{-2.4} \pm 1.1) \times 10^{-6}$. In our sample calculation we will take this branching ratio to be 10^{-5}. Estimating the corrected $b \to d$ penguin contributions, we get $Br_{penguin}(B^- \to \rho^-\omega) \approx 2|V_{td}/V_{ts}|^2 Br(B \to \phi K^*) \approx 8\times 10^{-7}$ where [5] $|V_{td}/V_{ts}| \approx 0.2$. The tree graph for this final state has a similar topology to the observed [20, 21] $B^+ \to \pi^+\pi^0$ decay modes with branching ratios of $\sim 5 \pm 2 \times 10^{-6}$. The branching ratio to a two vector final state should be somewhat larger because of the additional helicity states. Here we will assume that this will increase the branching ratio by roughly a factor of 3 as is the case for $Br(B^+ \to \rho^+ D^{*0})/Br(B^+ \to \pi^+ D^0)$ so that $Br_{tree}(B^+ \to \rho^+\omega) \approx 15 \times 10^{-6}$ [22].

The precision for extracting $\sin^2\alpha$ will depend on the distribution of the amplitudes between the various helicity states. As an example, which we will call Case 1, we will take the toy model for the amplitudes given by:

$$\mathbf{a} \propto [1, 1, -1] \qquad \mathbf{b} \propto [0.23, 0.23, 0.23] \tag{8}$$

which has the correct ratio of total rates. In this toy model the amplitudes have no relative phase from final state interactions. Case 2 will be the corresponding results averaged over helicity distributions and phases subject only to the condition that $|\mathbf{a}|$ and $|\mathbf{b}|$ are fixed.

To simulate the reconstruction of $\sin^2\alpha$ from experimental data we will calculate the observables X_i for each mode and estimate the correlated errors assuming that these are measured using the operators g_i. The tree-penguin mode give us A and $\bar{A}$ up to a relative phase. We can thus use eqn. (3) to get $\mathbf{b}\sin\alpha$ so that eqn. (7) further constrains the fit and a minimum value of χ^2 can be obtained for each value of $\sin^2\alpha$. The dependence of χ^2 on $\sin^2\alpha$ thus allows us to obtain the statistical error $\Delta\sin^2\alpha$ given in Table 2 where we take $\alpha = 30^\circ$ and 60°. In the last line of this Table, we consider a similar exercise for a mode sensitive to γ.

Table 2 gives results for the tree penguin modes: $B^+ \to \rho^+\omega$ where the tree is color allowed, $B^0 \to \rho^0\omega$ where the tree is color suppressed and a combined fit with both modes taken together. We also consider the B_s mode, $B_s \to K^{*+}\rho^-$. In each case we take $\hat{N}_B = 10^8$ and 5×10^8 and give results for Cases 1 and 2.

Let us now consider how the validity of the approximations used in this method may be experimentally verified. The validity of SU(3) for penguins may be tested by comparing the various pure penguin modes considered. If this symmetry is exact, the magnitude and relative phases of all the $b \to s$ pure penguin modes should be the same.

The assumption that EWPs do not play a large role is required in most isospin analysis [6, 7, 8] since the EWPs have the isospin properties of the tree but the weak phase of the penguin so it cannot be cleanly separated from the tree without additional information. Its presence may be specifically checked in the decay $B^\pm \to \rho^\pm\rho^0$. The final state must have total isospin $I = 2$ so in the SM only the tree and EWP can contribute. A significant EWP contribution which has a relative weak phase of α with respect to the tree would thus lead to CP violation. There are six possible CP violating observables

Table 2: The error in $\sin^2\alpha$ or $\sin^2\gamma$ assuming the central value of α, $\gamma = 30°$ or $60°$. The results under Case 1 use the helicity distribution given in eqn. (8) while Case 2 is a Monte Carlo average both over helicity distribution and over phases keeping the branching ratios fixed. In each entry the first number is for $\hat{N}_B = 10^8$ while the second is for $\hat{N}_B = 5\times 10^8$.

Tree-Penguin	CKM quantity	Case 1		Case 2	
		30°	60°	30°	60°
$B^+ \to \rho^+\omega$	$\sin^2\alpha$	.065; .029	.112; .050	.063;.033	.115;.062
$B^0 \to \rho^0\omega$	$\sin^2\alpha$	.220; .118	.221; .107	.095; .051	.114;.060
$B^+ \to \rho^+\omega$ & $B^0 \to \rho^0\omega$	$\sin^2\alpha$	.056; .025	.101; .045	.055;.023	.081;.040
$B_s \to K^{*+}\rho^-$	$\sin^2\alpha$	.088; .039	.151; .068	.098;.039	.146;.072
$B^+ \to K^{*+}\rho^-$	$\sin^2\gamma$	.054; .032	.069; .049	.099; .054	.095; .057

that can be obtained from the distributions: $X_{1-4}(\rho^+\rho^0) - X_{1-4}(\rho^-\rho^0)$ and $X_{5,6}(\rho^+\rho^0) + X_{5,6}(\rho^-\rho^0)$. Note that the first four are even under naive time reversal [23], T_N, thus require a rescattering phase while the last two are T_N odd and do not. The analogous test for the EWP can also be performed in the decay $B^\pm \to \pi^\pm\pi^0$ where an EWP could generate a partial rate asymmetry. The possible advantage of using the $\rho^\pm\rho^0$ final state is that there are six CP odd signals rather than one in $\pi^\pm\pi^0$.

In the $K\pi$, $K^*\pi$, $K\rho$ and $K^*\rho$ systems one can also check for EWP effects by looking for CP violation with isospin properties inconsistent with tree penguin interference [14, 24]. In the $K\pi$ system the absence of EWP implies:

$$2\Delta(B^- \to K^-\pi^0) - \Delta(B^- \to \bar{K}^0\pi^-) - \Delta(\bar{B} \to \bar{K}^-\pi^+) + 2\Delta(\bar{B} \to \bar{K}^0\rho^0) = 0 \quad (9)$$

where $\Delta(X)$ is the partial rate asymmetry for the reaction X. Eqn. (9) also applies to $K\rho$ and $K^*\pi$ while for $K^*\rho$ it applies separately to X_1, X_2 and X_3.

It is also possible to reduce the potential impact of the EWP by choosing modes in which their contribution will be color suppressed. In particular, the tree penguin interference processes $B_s \to K^{*+}\rho^-$ and $B_0 \to \rho^+\rho^-$ will color suppress the EWP as will the pure penguin process $B_s \to K^{*0}K^{*0}$.

The assumption that the gluon does not fragment into a single vector meson is true at lowest order in perturbation theory by color conservation. We can estimate how well it holds when QCD corrections are introduced, through renormalization group improved perturbation theory. If we define $\mathbf{b}'$ to be the penguin contribution where the quarks from the gluon form a single meson, the ratios of the contributions should be:

$$|\mathbf{b}'|/|\mathbf{b}| \approx (3C_3 + C_4 + 3C_5 + C_6)/(C_3 + C_4 + C_5 + C_6) \tag{10}$$

where we have used the notation of [25]. With the numerical results from that paper we find that this ratio is about 0.04 for various values of Λ_{QCD} both in naive dimensional regularization and in the 't-Hooft—Veltman schemes at next to leading order.

To summarize, we have shown that unitarity angles α and γ can be extracted by a method that is rather unique in that it only uses direct CP violation and does not need any time dependent measurements. We gave several examples of modes which can be used and also discussed how experimentally the underlying theoretical approximations can be tested. Since these angles are fundamental parameters of the SM, and their precise determinations could lead to its crucial tests, the importance of measuring them in several different ways can hardly be over emphasized.

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[15] Using the fits in [5] $|V_{td}/V_{ts}| = 0.183 \pm 0.015$. Future measurement of the rate of B_s oscillation at the Tevatron or LHC should determine this ratio with much greater accuracy.

[16] In the case of B_s, there are no analogous self tagging modes which may be used; for instance, the decay mode $B_s \to K^{*+}K^{*-}$ is not useful since it cannot be flavor tagged.

[17] Note that because the tree always has a component with the same isospin properties as the penguin, the consideration of isospin related processes alone will therefore not help to separate $\mathbf{b}\cos\alpha$ from $\mathbf{a}$.

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