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The Standard Model in the Latticized Bulk

Hsin-Chia Cheng[1]

Christopher T. Hill[2]

Stefan Pokorski[3]

Jing Wang[2]

[1] *Enrico Fermi Institute, The University of Chicago*
Chicago, Illinois, 60637 USA

[2] *Fermi National Accelerator Laboratory*
P.O. Box 500, Batavia, Illinois 60510, USA *

[3] *Institute for Theoretical Physics*
Hoza 69, 00-681, Warsaw, Poland

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Abstract

We construct the manifestly gauge invariant effective Lagrangian for the Standard Model in 4+1 dimensions, following the transverse lattice technique. Extra dimensions do not ameliorate the naturalness problem as Higgs mass terms are always required to be unnaturally small.

*e-mail: hill@fnal.gov, jingw@fnal.gov, Stefan.Pokorski@fuw.edu.pl

1 Introduction

Recently we introduced the low energy effective Lagrangian of an extra-dimensional Yang-Mills gauge theory in which gauge fields, fermions, and scalars propagate in the bulk [?], [?]. The idea is to ask how an experimentalist would describe the first few KK-modes seen in her detector in an effective Lagrangian in $3+1$ dimensions? We exploited universality and find that the solution to this problem is the transverse Wilson lattice of Bardeen and Pearson [?]. Thus we arrive at a local gauge invariant $3+1$ effective Lagrangian description of the KK-modes. For example, Pure QCD in the $4+1$ bulk may be an "aliphatic" model $\Pi_{i=1}^{N+1} SU(3)_i$ with N chiral $(\overline{3}_i, 3_{+1})$ Φ_i fields connecting the groups sequentially. This can be viewed as a Wilson action for a transverse lattice in x^5, and is shown explicitly to match a compactified continuum $4+1$ Lagrangian truncated in p^5 momentum space. With periodic boundary conditions the spectrum is the same, but the KK modes are doubled and the overall scale of the KK masses increases by $\sqrt{2}$, as seen below.

In the present letter we turn our attention to the full Standard Model. Our goal presently is not ambitious; rather than contructing a new dynamics for EWSB, we wish to use the usual Higgs mechanism to describe the EWSB, and to understand the immediate ramifications of extra-dimensions from the point of view of the latticized effective Lagrangian. Unfortunately, it appears that extra-dimensions cannot solve the naturalness problem of the electroweak hierarchy with a fundamental Higgs, and, indeed, in certain instances makes it much worse.

2 Standard Model Effective Lagrangian

2.1 Incorporation of QCD

We wish to describe the low energy effective Lagrangian of the Standard Model in $4+1$ dimensions. Let us review the compactification of the pure gauge-field degrees of freedom of QCD. The spectrum of KK modes is sensitive to the structure of the effective Lagrangian in $3+1$, which incorporates the global boundary conditions of the underlying $4+1$ theory. First we examine the simplest case, the *aliphatic model* corresponding to a linear system with free boundary conditions.[1] Then we examine the *periodic model* in which we attach the zeroth and Nth fields together. These are distinct global systems

[1]The name follows the chemical nomenclature for hydrocarbons; aliphatic means "in a line"

with characteristically distinct spectra. Which one occurs depends upon the detailed compactification scheme of nature.

Consider the pure gauge Lagrangian in $3+1$ dimensions:

$$\mathcal{L}_{QCD} = -\frac{1}{4}\sum_{i=0}^{N} G^a_{i\mu\nu}G^{i\mu\nu a} + \sum_{i=1}^{N} D_\mu \Phi_i^\dagger D^\mu \Phi_i \tag{2.1}$$

in which we have $N+1$ gauge groups $SU(3)_i$ and N link-Higgs fields, Φ_i forming $(\overline{3}_i, 3_{i-1})$ representations. The covariant derivative is defined as $D_\mu = \partial_\mu + i\tilde{g}_3 \sum_{i=0}^{N} A^a_{i\mu} T^a_i$. $\tilde{g}_3$ is a dimensionless gauge coupling constant that is common to all of the $SU(3)_i$ local gauge symmetries. The physical observed low energy QCD coupling will be $g_3 \propto [\sqrt{2}]\tilde{g}_3/\sqrt{N+1}$ (where the $[\sqrt{2}]$ factor occurs in the aliphatic case, but is absent in the periodic case). T^a_i are the generators of the ith $SU(3)_i$ gauge symmetry, where a is the color index. Thus, $[T^i, T^j] = 0$ for $i \neq j$; T^a_i annihilates a field that is singlet under the $SU(3)_i$; when the covariant derivative acts upon Φ_i we have a commutator of the gauge part with Φ, $T^{a\dagger}_i$ acting on the left and T^a_{i-1} acting on the right; the ith field strength is $F^{ai}_{\mu\nu} \propto \mathrm{tr}\, T^{ai}[D_\mu, D_\nu]$, etc.

A common renormalizable potential can be constructed for each of the link-Higgs fields,

$$V(\Phi_j) = \sum_{j=1}^{N}\left[-M^2\mathrm{Tr}(\Phi_j^\dagger\Phi_j) + \lambda_1\mathrm{Tr}(\Phi_j^\dagger\Phi_j)^2 + \lambda_2(\mathrm{Tr}(\Phi_j^\dagger\Phi_j))^2 + M'\left(e^{i\theta}\det(\Phi_j) + h.c.\right)\right], \tag{2.2}$$

We can always arrange the parameters in the potential such that the diagonal components of each Φ_j develop a vacuum expectation value v, and the Higgs and $U(1)$ PNGB are heavy. Hence, we can arrange that each Φ_i becomes effectively a nonlinear-σ model field:

$$\Phi_i \to v\exp(i\phi^a_i\lambda^a/2v) \tag{2.3}$$

Thus, the Φ_i kinetic terms lead to a mass-squared matrix for the gauge fields:

$$\sum_{i=1}^{N}\frac{1}{2}\tilde{g}_3^2 v^2 (A^a_{(i-1)\mu} - A^a_{i\mu})^2 \tag{2.4}$$

This mass-squared matrix has the structure of a nearest neighbor coupled oscillator Hamiltonian. It can be written as an $(N+1)\times(N+1)$ matrix sandwiched between the column vector $A = (A^a_{0\mu}, A^a_{1\mu}, ..., A^a_{N\mu})$, and it's transpose, as $A^T M A$ where:

$$M = \frac{1}{2}\tilde{g}_3^2 v^2 \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \cdots & 0 \\ 0 & -1 & 2 & \cdots & 0 \\ & & & \cdots & \\ 0 & 0 & \cdots & -1 & 1 \end{pmatrix}. \tag{2.5}$$

We can diagonalize the matrix as follows. The gauge fields A^j_μ can be expressed as real linear combinations of the mass eigenstates $\tilde{A}^n_\mu$ as:

$$A^j_\mu = \sum_{n=0}^{N} a_{jn} \tilde{A}^n_\mu. \tag{2.6}$$

The a_{jn} form a normalized eigenvector ($\vec{a}_n$) associated with the nth $n \neq 0$ eigenvalue and has the following components:

$$a_{jn} = \sqrt{\frac{2}{N+1}} \cos\left(\frac{2j+1}{2}\gamma_n\right) , \qquad j = 0, 1, \ldots, N, \tag{2.7}$$

where $\gamma_n = \pi n/(N+1)$. The mass terms take the form:

$$\mathcal{L}_{mass} = \frac{1}{2}\tilde{g}_3^2 v^2 \sum_{j=1}^{N} (A_{j-1} - A_j)^2 \tag{2.8}$$

$$= \tilde{g}_3^2 v^2 \sum_{n=0}^{N} \sin\left(\frac{\gamma_n}{2}\right)^2 (\tilde{A}^n)^2. \tag{2.9}$$

hence the KK tower of masses are:

$$M_n = \sqrt{2}\tilde{g}_3 v \sin\left[\frac{\gamma_n}{2}\right] \qquad \gamma_n = \frac{n\pi}{N+1} , \qquad n = 0, 1, \ldots, N. \tag{2.10}$$

Thus we see that for small n this system has a geometrical KK tower of masses given by:

$$M_n \approx \frac{\tilde{g}_3 v \pi n}{\sqrt{2}(N+1)} \qquad n << N \tag{2.11}$$

and $n = 0$ corresponds to the zero-mode gluon. To match on to the spectrum of the KK modes, we require

$$\frac{\tilde{g}_3 v}{\sqrt{2}(N+1)} = \frac{1}{R}. \tag{2.12}$$

Hence, the aliphatic system with $SU(3)^{N+1}$ and N Φ_i provides a gauge invariant description of the first n KK modes by generating the same mass spectrum.

The zero mode theory is pure QCD with a massless gluon. The zero-mode trilinear coupling constant is $g_3 = \tilde{g}_3\sqrt{2/N+1}$ [?]. In a geometric picture, the aliphatic model corresponds to a "transverse lattice" description of a full $4+1$ gauge theory [?]. Here the $4+1$ theory is compactified between two parallel branes at $x^5 = 0$ and $x^5 = R$ and the boundary conditions on the branes are $F^a_{\mu 5} = -F^a_{5\mu} = 0$. These boundary conditions insure that no vector gauge invariant field strength is "observable" on the branes.

Of course, we can always make a periodic extension of the interval $[0, R]$. This leads to a Lagrangian in which we have $N+1$ branes, hence $N+1$ $SU(3)_i$ as before, but now, $N+1$ linking Φ_i Higgs fields,

$$\mathcal{L} = -\frac{1}{4}\sum_{i=0}^{N} F^a_{i\mu\nu}F^{i\mu\nu a} + \sum_{i=0}^{N} D_\mu\Phi_i^\dagger D^\mu\Phi_i \tag{2.13}$$

We now have the additional Φ_0 which is a $(\overline{3}, 3_N)$ representaion linking the first $SU(3)_0$ gauge group to the last $SU(3)_N$. The resulting gauge field mass-squared term becomes:

$$\sum_{i=1}^{N+1} \frac{1}{2}\tilde{g}_3^2 v^2 (A^a_{(i-1)\mu} - A^a_{i\mu})^2 \tag{2.14}$$

where we identify $A^a_{(N+1)\mu} \equiv A^a_{(0)\mu}$. Thus, the mass-squared matrix is now:

$$M = \frac{1}{2}\tilde{g}_3^2 v^2 \begin{pmatrix} 2 & -1 & 0 & \cdots & -1 \\ -1 & 2 & -1 & \cdots & 0 \\ 0 & -1 & 2 & \cdots & 0 \\ & & & \cdots & \\ -1 & 0 & \cdots & -1 & 2 \end{pmatrix}. \tag{2.15}$$

The diagonalization is now done with a complex representation (suppressing gauge and Lorentz indices; consider N even):

$$A^j_\mu = \sum_{n=-N/2}^{N/2} a_{jn}\tilde{A}^n_\mu. \tag{2.16}$$

where now:

$$a_{jn} = \frac{1}{\sqrt{N+1}} \exp\left(2\pi i n j/(N+1)\right), \qquad j = 0, 1, \ldots, N, \tag{2.17}$$

Note with this definition A_j is periodic, $A_{(N+1)} = A_0$. Reality of A_i dictates that $\tilde{A}^n = \tilde{A}^{-n*}$. One thus obtains for the mass matrix:

$$2\tilde{g}_3^2 v^2 \sum_{n=-N/2}^{N/2} \sin^2\left(\frac{\pi n}{N+1}\right) |\tilde{A}^n|^2 \tag{2.18}$$

The spectrum is now:

$$2\tilde{g}_3 v \sin\left(\frac{\pi n}{N+1}\right) \qquad n = 0, 1, 2, ..., N/2 \tag{2.19}$$

We now require:

$$\frac{\tilde{g}_3 v}{(N+1)} = \frac{1}{R}. \tag{2.20}$$

Hence, the periodic system with $SU(3)^{N+1}$ and N Φ_i provides a gauge invariant description of the first n doubled KK modes, generating the same mass spectrum as in the aliphatic case up to an overall scale factor of $\sqrt{2}$. (Note that if N were odd the spectrum would include an additional singlet level with $n = (N+1)/2$). There remains the zero-mode in the spectrum $n = 0$, which is a singlet since the reality condtion $\tilde{A}^n = \tilde{A}^{-n*}$ inplies that $\tilde{A}^0$ is real. However, every nonzero n corresponds to a degenerate doublet of levels. The zero-mode theory is again QCD with a massless gluon and a coupling constant $g_3 = \tilde{g}_3/\sqrt{N+1}$.

2.2 Incorporating $SU(2)_L \times U(1)_Y$

We now consider the pure gauge Lagrangian in $3+1$ dimensions:

$$\mathcal{L}_{ew} = -\frac{1}{4}\sum_{i=0}^{N} F^a_{i\mu\nu}F^{i\mu\nu a} - \frac{1}{4}\sum_{i=0}^{N} F_{i\mu\nu}F^{i\mu\nu} + \sum_{i=1}^{N} D_\mu \Phi'^\dagger_i D^\mu \Phi'_i + + \sum_{i=1}^{N} D_\mu \phi^\dagger_i D^\mu \phi_i \tag{2.21}$$

Here we have $N+1$ copies of the $SU(2)_L \times U(1)_Y$ electroweak Standard Model. Thus the gauge group is $\Pi_{i=0}^N SU(2)_{iL} \times U(1)_{iY}$ where $F^a_{i\mu\nu}$ ($F_{i\mu\nu}$) is the $SU(2)_{iL}$ ($U(1)_{iY}$) field strength. The N Φ'_i and ϕ_i are elementary scalars The Φ'_i carry $SU(2)$ charges $(\frac{1}{2}_i, \frac{1}{2}_{i+1})$,and the ϕ_i carry weak hypercharges (Y_i, Y_{i+1}). These fields correspond to the links of a transverse Wilson lattice in the fifth dimension, x^5. We will choose $Y_i = Y = 1$ throughout (the choice of arbitrary $Y \neq 1$ only renormalizes the masses of the $U(1)$ KK modes relative to $SU(2)$ modes and is dictated by compactification and matching). We arrange potentials for the Φ'_i and ϕ_i so they each acquire a common VEV's. Hence, we can again arrange that each field becomes effectively a nonlinear-σ model:

$$\Phi'_i \rightarrow v_2 \exp(i\phi^a_i \tau^a/2v_2) \qquad \phi_i \rightarrow v_1 \exp(i\phi_i/v_1) \tag{2.22}$$

Note that we could without loss of generality amalgamate the product $\Phi_i \phi_i$ into a single link-Higgs field $\tilde{\Phi}_i = \Phi_i \phi_i$ (it is, in fact, expedient to do so in discussing the fermion incorporation). Thus, the Φ_i and ϕ_i kinetic terms lead to a mass-squared matrix for the $SU(2)$ and $U(1)$ gauge fields:

$$\sum_{i=1}^{N} \frac{1}{2} \tilde{g}_2^2 v_2^2 (A^a_{(i-1)\mu} - A^a_{i\mu})^2 + \sum_{i=1}^{N} \frac{1}{2} \tilde{g}_1^2 v_1^2 (A_{(i-1)\mu} - A_{i\mu})^2 \tag{2.23}$$

The gauge fields A^j_μ can again be expressed as linear combinations of the mass eigenstates $\tilde{A}^n_\mu$ as:

$$A^j_\mu = \sum_{n=0}^{N} a_{jn} \tilde{A}^n_\mu. \tag{2.24}$$

with (in the aliphatic case):

$$a_{jn} = \sqrt{\frac{2}{N+1}} \cos\left(\frac{2j+1}{2}\gamma_n\right) , \qquad j = 0, 1, \ldots, N, \tag{2.25}$$

where $\gamma_n = \pi n/(N+1)$. The mass eigenvalues are:

$$M_n^{(2)} = \sqrt{2}\tilde{g}_2 v_2 \sin\left[\frac{\gamma_n}{2}\right] \qquad M_n^{(1)} = \sqrt{2}\tilde{g}_1 v_1 \sin\left[\frac{\gamma_n}{2}\right] \qquad \gamma_n = \frac{n\pi}{N+1} , \qquad n = 0, 1, \ldots, N. \tag{2.26}$$

Thus we see that for small n this system has a KK tower of masses given by:

$$M_n^{(2)} \approx \frac{\tilde{g}_2 v_2 \pi n}{\sqrt{2}(N+1)}; \qquad M_n^{(1)} \approx \frac{\tilde{g}_1 v_1 \pi n}{\sqrt{2}(N+1)} \qquad\qquad n << N \tag{2.27}$$

and $n = 0$ again corresponds to the zero-mode gauge fields.

The geometrical constraints are as follows. To match on to the spectrum of the KK modes, we require

$$\frac{\tilde{g}_2 v_2}{\sqrt{2}(N+1)} = \frac{\tilde{g}_1 v_1}{\sqrt{2}(N+1)} = \frac{1}{R}. \tag{2.28}$$

The KK modes should have common values owing to geometry. Thus we require for matching:

$$\frac{v_2}{v_1} = \frac{\tilde{g}_1}{\tilde{g}_2} = \tan\theta_W \tag{2.29}$$

This corresponds to an aliphatic system with $SU(2)_L^{N+1} \times U(1)^{N+1}$ and N Φ_i' and ϕ_i providing a gauge invariant description of the first n KK modes.

We can alternatively use the periodic extension. This again leads to a Lagrangian in which we have $N+1$ branes, hence $N+1$ Φ_i and ϕ_i, with new link-Higgs field connections $[0, N]$, as in the QCD case. The diagonalization is again done with the complex representation and the spectrum becomes:

$$M_n^{(2)} = 2\tilde{g}_2 v_2 \sin\left(\frac{\pi n}{N+1}\right) \qquad M_n^{(1)} = 2\tilde{g}_1 v_1 \sin\left(\frac{\pi n}{N+1}\right) \qquad n = 0, 1, 2, ..., N/2 \quad (2.30)$$

If N were odd the spectrum includes an additional singlet level with $n = (N+1)/2$. Note that there remains the zero-mode, in the spectrum $n = 0$. Moreover, the zero-mode is a singlet since the reality condtion $\tilde{A}^n = \tilde{A}^{-n*}$ inplies that $\tilde{A}^0$ is real. However, every nonzero n corresponds to a degenerate doublet of levels.

The zero modes of this pure gauge theory are described by the effective Lagrangian in $3+1$ dimensions:

$$\mathcal{L}_{gauge} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad (2.31)$$

where $F^a_{\mu\nu}$ ($F_{\mu\nu}$) is the $SU(2)_L$ ($U(1)_Y$) field strength. The physical $SU(2)_L$ ($U(1)_Y$) gauge coupling constant is $g_2 \equiv [\sqrt{2}]\tilde{g}_2/\sqrt{N+1}$, ($g_2 \equiv [\sqrt{2}]\tilde{g}_1/\sqrt{N+1}$) a consequence of using the expansions of eq.(**?? ??**). The fact that the physical coupling constants are suppressed by $\sim 1/\sqrt{N}$ is just the classical "power-law" running of the coupling in the $4+1$ dimensional theory.

3 Incorporating Electroweak Higgs Fields

We now introduce $N+1$ Higgs fields, H_i each transforming as $\left(\frac{1}{2}{}_i\right)$ (and singlet under $j \neq i$, and with weak hypercharges $Y_i = 1$ and $Y_j = 0$ $j \neq i$. The Lagrangian for the Higgs fields is:

$$\mathcal{L}_{Higgs} = \sum_{n=0}^{N}(D_\mu H_i)^\dagger(D^\mu H_i) - M^2|H_{i+1} - (\Phi_i'\phi_i/v_1 v_2)H_i|^2 - V(H_i) \qquad (3.32)$$

where we identify $H_{N+1} \equiv H_0$ in the above summation for the periodic case, and $H_{N+1} = 0$ in the aliphatic case. Note that the second term is a latticized covariant derivative in the x^5 direction.

First we ignore the Higgs potentials, and we gauge away the chiral field components, so $\Phi_i = v_1$ and $\phi_i = v_2$. We thus have the nearest neighbor mass terms:

$$\mathcal{L}_{Higgs} = -\sum_{n=0}^{N} M_0^2|H_{i+1} - H_i|^2 \qquad (3.33)$$

which leads to the spectroscopy:

$$M_n^2 = [2]M_0^2 \sin^2\left[\frac{\gamma_n}{2}\right] \qquad n = 0, 1, \ldots, N. \qquad (3.34)$$

where the coefficient [2] here and below applies in the periodic case. Matching onto the spectrum of the KK modes requires:

$$[2]\frac{M}{\sqrt{2}(N+1)} = \frac{1}{R}. \tag{3.35}$$

The eigenfields are given by:

$$H^j = \sum_{n=0}^{N} a_{jn}\tilde{H}^n. \tag{3.36}$$

with the a_{jn} as in eq.(**??**) for the aliphatic model, or eq.(**??**) for the periodic case..

We now incorporate a Higgs potential. We take a universal Higgs potential common to each brane i:

$$V(H_i) = -\tilde{m}^2 H_i^\dagger H_i + \frac{\tilde{\lambda}}{2}(H_i^\dagger H_i)^2 \tag{3.37}$$

The presence of the Higgs potential adds a common mass term $-\tilde{m}^2 \sum H_i^\dagger H_i$ to each of the H_i in the Lagrangian. This modifies the eigenvalues:

$$M_n^2 = 2M^2 \sin^2\left[\frac{\gamma_n}{2}\right] - \tilde{m}^2 \qquad n = 0, 1, \ldots, N. \tag{3.38}$$

We see that $-\tilde{m}^2$ is the mass for the zero mode. Hence the zero-mode Lagrangian corresponds to the Standard Model with a tachyonic Higgs of negative mass-squared $-\tilde{m}^2$.

Let us go to mass eignebases and truncate on the zero-mode. hence the zero-mode Higgs potential is:

$$V(\tilde{H}_0) = -\tilde{m}^2 \tilde{H}_0^\dagger \tilde{H}_0 + \frac{\tilde{\lambda}}{[2]2(N+1)}(\tilde{H}_0^\dagger \tilde{H}_0)^2 \tag{3.39}$$

Notice the large suppression factor of the quartic interaction term, a consequence of the normalization of the zero-mode component of the Higgs field. This may be interpreted as power-law running of the quartic coupling constant in the extra-dimensional theory. Thus, we define the low energy physical quartic coupling as $\lambda = \tilde{\lambda}/\sqrt{[2](N+1)}$. The VEV of the zero mode Higgs thus becomes $v_0^2 = \tilde{m}^2/\lambda = [2](N+1)\tilde{m}^2/\tilde{\lambda}$. Substituting the zero-mode Higgs field with VEV, the zero-mode Higgs boson kinetic term becomes:

$$\mathcal{L}_{Higgs} = \sum_{j=0}^{N}(D_\mu H_j)^\dagger(D^\mu H_j) \rightarrow \frac{[2]}{(N+1)}\sum_{j=0}^{N}|(i\tilde{g}_2 A^a_{j,\mu}\frac{\tau^a}{2} + i\tilde{g}_1 A_{j,\mu}\frac{Y}{2})\begin{pmatrix} v_0 \\ 0 \end{pmatrix}|^2 \tag{3.40}$$

where the $[2]/(N+1)$ comes from the zero-mode normalization. We can absorb it into renormalized physical couplings, g_1 and g_2:

$$\mathcal{L}_{Higgs} \rightarrow \sum_{j=0}^{N}|(ig_2 A^a_{j,\mu}\frac{\tau^a}{2} + ig_1 A_{j,\mu}\frac{Y}{2})\begin{pmatrix} v_0 \\ 0 \end{pmatrix}|^2 \tag{3.41}$$

These terms may be rewritten in term of W, Z and γ fields on each brane:

$$\mathcal{L}_{Higgs} = \sum_{j=0}^{N} M_W^2 W_{j\mu}^+ W^{j\mu -} + \frac{1}{2} M_Z^2 Z_{j\mu} Z^{j\mu} \tag{3.42}$$

The W_i and Z_i fields are combined with the Nambu-Goldstone bosons π^a. The combined fields are defined as:

$$\begin{aligned} W_{j\mu}^{\pm} &= (A_{j,\mu}^1 \pm i A_{j,\mu}^2)/\sqrt{2} \\ \gamma_{j,\mu} &= \sin\theta\ A_{j,\mu}^3 + \cos\theta\ A_{j,\mu} \\ Z_{j,\mu} &= \cos\theta\ A_{j,\mu}^3 - \sin\theta\ A_{j,\mu} = \frac{(\tilde{g}_2 A_{j,\mu}^3 - \tilde{g}_1 A_{j,\mu})}{\sqrt{\tilde{g}_1^2 + \tilde{g}_2^2}} \end{aligned} \tag{3.43}$$

where $\gamma_{j,\mu}$ is a photon field, while Z_j ($W_{j,\mu}$) is a Z-boson (W-boson) mode.

The masses M_W and M_Z are universal to all the $SU(2) \times U(1)$'s, i.e., to all branes, and they are just the masses of the W and Z measured in the low energy theory:

$$M_W = \frac{\tilde{g}_2^2 v_0^2}{2} = \frac{g_2^2 \tilde{v}_0^2}{2} \tag{3.44}$$

$$M_Z = \frac{(\tilde{g}_2^2 + \tilde{g}_1^2) v_0^2}{2} = \frac{(g_2^2 + g_1^2)\tilde{v}_0^2}{2}, \tag{3.45}$$

where g_1, g_2 and $\tilde{v}_0$ are measured at low energies.

Combining these expressions with the full KK mass formula, we find that the W, Z and γ KK towers are given by:

$$M_\gamma^{n\,2} = M_0^2 \sin^2 \gamma_n \tag{3.46}$$

$$M_W^{n\,2} = M_W^2 + M_0^2 \sin^2 \gamma_n \tag{3.47}$$

$$M_Z^{n\,2} = M_Z^2 + M_0^2 \sin^2 \gamma_n \tag{3.48}$$

Each of the KK mode levels thus has a fine-structure determined by the electroweak symmetry breaking.

4 Incorporating Fermions

In $4 + 1$ dimensions free fermions are vectorlike. Chiral fermion zero modes can be readily engineered. For example, one can use domain wall kinks in a background field which couples to the fermion like a mass term. This can trap a chiral zero-mode on the kink [?]. The magnitude of the kink field away from the domain wall can be arbitrarily large, so the vectorlike fermion masses can be made arbitrarily large, and are not directly related to the compactifaction scale. This means that we need be concerned at present *only* with the chiral zero-modes. That is, from the point of view of our $3+1$ effective Lagrangian approach, if we are only interested in the fermionic zero modes then we can simply incorporate the chiral fermions by hand.

Consider one complete generation of left-handed quarks and leptons, ℓ_L, q_L which are doublets under the specific $SU(2)_{jL}$ and carrying weak hypercharges $Y_\ell = -1$, $Y_q = 2/3$ under the $U(1)_{jY}$; the quarks carry color under $SU(3)_j$; the fermions are sterile under all other gauge groups $i \neq j$. Likewise, we have right-handed $SU(2)$ singlets, ℓ_R, q_{uR}, and q_{dR} carrying weak hypercharges under the $U(1)_{jY}$. Additional generations can be incorporated with additional fields.

The chiral fermions of a given generation can be placed at a unique brane, distinct from the others. One could go further and split members within a single generation. In a sense this latter approach would emulate the split-fermion construction of Arkani-Hamed and Schmaltz, [**?**]. It leads us into interesting issues involving anomalies, and Wess-Zumino terms in the present formulation which we prefer to address elsewhere. We will emulate more closely the split family model of Dvali and Shifman [**?**], as we will presently consider a complete anomaly free generation on any given brane.

Let us designate the branes which receive the generations by $j = (j_1, j_2, j_3)$, thus the full fermionic Lagrangian becomes:

$$\mathcal{L}_{fermion} = \sum_j \left(\overline{\ell}_{j,L} \not{D}_j \ell_{j,L} + \overline{q}_{j,L} \not{D}_j q_{j,L} + \overline{\ell}_{j,R} \not{D}_j \ell_{j,R} + \overline{q}_{j,uR} \not{D}_j q_{j,uR} + \overline{q}_{j,dR} \not{D}_j q_{j,dR} \right) \tag{4.49}$$

where $\not{D}_j = \gamma^\mu(\partial - i\tilde{g}_2 A^a_{j,\mu} \frac{\tau^a}{2} - i\tilde{g}_1 A_{j,\mu} \frac{Y_{psi}}{2})$, and the sum extends over $j = (j_1, j_2, j_3)$. The couplings to the zero–mode gauge boson of e.g., the quarks, are therefore

$$\mathcal{L}_0 = \sum_j \left(\overline{q}_{j,L} \tilde{\not{D}}\, q_{j,L} + \overline{q}_{j,uR} \tilde{\not{D}}\, q_{j,uR} + \overline{q}_{j,dR} \tilde{\not{D}}\, q_{j,dR} \right) \tag{4.50}$$

where $\tilde{\not{D}} = \gamma^\mu(\partial - ig_2 \tilde{A}^a_{0,\mu} \frac{\tau^a}{2} - ig_1 \tilde{A}_{0,\mu} \frac{Y_{psi}}{2})$, in which g_1 and g_2 are the physical gauge coupling constants.

In the above discussion of we considered a universal Higgs field in the bulk. This translated into $N+1$ Higgs fields, H_i each transforming as $\left(\frac{1}{2}_i\right)$ (and singlet under $j \neq i$, and with weak hypercharges $Y_i = 1$ and $Y_j = 0$ $j \neq i$. This led to the zero mode gauge fields feeling a Higgs VEV of order $m_H^2/\lambda \sim (N+1)m_H^2/\tilde{\lambda}$, which is the conventional Standard Model resul where λ is the physical (renormalized) low energy quartic coupling.

Hence, one requires a tiny and unnaturally small Higgs boson mass,m_H to generate the electroweak symmetry breaking scale. The power law running of the coupling $\tilde{\lambda}$ brings $\tilde{\lambda}$ at the fundamental high evergy scale (M_s) down to a low scale $\lambda = \tilde{\lambda}/(N+1)$. To match on to the measured EW theory, one requires the mass-squared in the Higgs potential $m_H^2 \lesssim v_0^2$ which me be viewed as the present electroweak radiative bound, whence $\lambda \lesssim 1$. If one saturates perturbative unitarity and assumes $\tilde{\lambda} \sim 4\pi$ at M_s, then the KK tower is bounded by $N \lesssim 4\pi$.

We would have expected that the natural scale for the Higgs mass is of order the fundamental scale of the theory, M_s. Can we modify the approach to introducing the Higgs in such a way that the light Higgs boson becomes natural? For example, can we engineer a Higgs mass of order M_s^2/N by judicious choice of the structure of the model?

One possibility is to assume that the Higgs potential is non-universal, i.e., takes different values of it's parameters for different values of j. The simplest idea is to assume that a single Higgs on the kth brane has a large negative mass-squared $\sim m_H^2$ and the Higgs gets a VEV on that brane only. This helps considerably, but does not alleviate the naturalness problem. If $\langle H_k \rangle \sim v$ then we get a gauge mass term $\tilde{g}^2(A_k)^2v^2$ where k is unsummed. However $A_k = A_0/\sqrt{N} + ...$ so again the zero-mode mass term becomes $\tilde{g}^2(A_0)^2v^2/N \sim g^2A_0^2v^2$. This requires that $v = v_0$, which implies that on the kth brane the Higgs mass is given by $v_0^2 = m_H^2/\tilde{\lambda}$. Note that now there is no large $(N+1)$ prefactor. Using perturbative unitarity for $\tilde{\lambda} \lesssim 4\pi$, we have an upper limit on $m_H \sim 1$ TeV (the Lee-Quigg Thacker bound [**?**]). Thus, this localization of the Higgs allows us to raise the scale of the Higgs boson somewhat. However, given that we typically want $N >> 1$ we require $m_H << M_s$, so again we have an unnatural situation.

These are the two extreme limits of a zero-momentum VEV and a localized (all momentum) VEV. The only remaining possibility is to consider a high momentum VEV. For example, if $H_j = (-1)^j v_0/\sqrt{N}$ we still obtain a substantial gauge field mass $\sim gv_0$ since it is $|H_j|^2$ that contributes.

Despite the fact that the fundamental Higgs field is unnatural in these schemes, it is interesting to examine a latticized version of the Dvali-Shifman model. Thus we consider a model in which there is a strongly localized Higgs VEV [**?**]. We assign the Higgs VEV v_0 only to the kth brane, then the zero mode gauge fields acquire masses of order $\tilde{g}^2v_0^2/N \sim g_2^2v_0^2$.

The Higgs VEV exponentially attenuates away from the localization point and fermions that are at various distances from the localized VEV will receive different masses. We assume the same structure as in eq.(**??** where now the Higgs potentials have an i-dependent mass term:

$$V(H_i) = M_i^2 H_i^\dagger H_i + \frac{\lambda}{2}(H_i^\dagger H_i)^2 \tag{4.51}$$

For concreteness as an explicit example we choose:

$$M_0^2 = -\kappa M^2 \qquad M_{i\neq 0}^2 = +M^2 \qquad \lambda_{i\neq 0} = 0 \tag{4.52}$$

κ is a phenomenological parameter. The full Higgs-only potential can be written:

$$-V_{Higgs} = -\tilde{M}^2 H_0^\dagger H_0 - \frac{\lambda}{2}(H_0^\dagger H_0)^2 - \sum_{i=1}^{N} \Lambda^2 H_i^\dagger H_i + (M_0^2 H_{i+1}^\dagger H_i + h.c.) \tag{4.53}$$

where we identify $H_{N+1} = 0$ ($H_{N+1} = H_0$) in the aliphatic (periodic) case and thus

$$\tilde{M}^2 = (1+\kappa)M^2 - [2]M_0^2 \qquad \Lambda^2 = M^2 + 2M_0^2 \tag{4.54}$$

(where [2] again refers to the periodic case). The equation of motion of the H_i is thus:

$$\Lambda^2 H_i = M_0^2 H_{i+1} + M_0^2 H_{i-1} \qquad (i \geq 1) \tag{4.55}$$

which has the solution $H_{i+1} = \epsilon H_i$ where:

$$\epsilon = \frac{\Lambda^2 - \sqrt{\Lambda^4 - 4M_0^4}}{2M_0^2} \tag{4.56}$$

If substitute the solution back into the action of eq.(**??**) we see that we obtain:

$$-V_{Higgs} = -\tilde{M}^2 H_0^\dagger H_0 - \frac{\lambda}{2}(H_0^\dagger H_0)^2 \tag{4.57}$$

and we can thus minimize the potential on the zeroth brane as:

$$\left\langle \tilde{H}_0 \right\rangle = \left(\begin{array}{c} v_0 \\ 0 \end{array} \right) \tag{4.58}$$

where $v_0^2 = \tilde{M}^2/\tilde{\lambda}$.

We can substitute the full dynamical Higgs field into this expression,

$$\tilde{H}_n = \left(\begin{array}{c} v_{0n} + h_n/\sqrt{2} \\ 0 \end{array} \right) \tag{4.59}$$

and again we have:

$$v_{0n} = \epsilon^n v_0 \qquad h_n = \epsilon^n h_0 \tag{4.60}$$

Now, we substiture into the kinetic terms of eq.(??) to obtain the dynamical Higgs field kinetic term:

$$\sum_{n=0}^{N}(D_\mu H_i)^\dagger(D^\mu H_i) \to \frac{1}{2}[1+\sum_{n=1}^{N}\epsilon^{-2n}](\partial h)^2 \tag{4.61}$$

We see that the dynamical Higgs field has a wave-function renormalization constant:

$$Z = [1+\sum_{n=1}^{N}\epsilon^{-2n}] = \frac{1-\epsilon^{-2N-2}}{1-\epsilon^{-2}} \tag{4.62}$$

Thus, the physical mass of the Higgs field becomes:

$$m_H^2 = 2\tilde{M}^2/Z \tag{4.63}$$

The Higgs is strongly localized in the limit $\Lambda/M_0 \to 0$. In this limit $\epsilon \to 0$ and the only Higgs field receiving the VEV is effectively H_0. Then the zero-mode gauge masses are given by $\propto \tilde{g}^2 v_0^2/\sqrt{N+1} \sim g^2 v_0^2$ and we see that v_0 is indeed the electroweak VEV. Since $v_0^2 \sim \tilde{M}^2/\tilde{\lambda}$ we see that $\tilde{M} \lesssim 1$ TeV, by perturbative unitarity, $\tilde{\lambda} \lesssim 16\pi^2$. We furthermore see that the physical Higgs is heavy, as $2\tilde{M}^2/Z \sim 2\tilde{M}^2 \sim$ TeV.

On the other hand, we can delocalize the Higgs with $\epsilon \to 1-\eta$ and $\eta << 1$. Then we see that $Z \to (N+1)$. Now the zero-mode gauge masses are given by $\propto \tilde{g}^2 v_0^2 \sim (N+1) g^2 v_0^2$ and we see that $\sqrt{N+1}v_0$ is the electroweak VEV.

4.1 Fermion Masses

4.2 Localization and the Dvali-Shifman Model

Restoring the link-Higgs fields for gauge covariance, the nearest neighbor interactions generates a profile for the Higgs field of the form $H_j = \Pi_{i=0}^{j}(\epsilon\Phi_i{}'\phi_i{}'/v_2 v_1)H_0$, which is the discretized version of the exponential attenuation in x^5 away from the source $H(x^5) \sim \exp(-M|x^5|)H(0)$.

For diagonal masses we consider only the fermions placed on a given brane. If there is a complete family of fermions on the jth brane, it is charged under $SU(3)_j \times SU(2)_j \times U(1)_j$ only. We postulate a coupling to the Higgs field H_j as:

$$\mathcal{L}_{Yukawa} = g_{\ell j}\overline{\ell}_{j,L} H_j^c \ell_{j,uR} + g_{uj}\overline{q}_{j,L} H_j q_{j,uR} + g_{dj}\overline{q}_{j,L} H_j^c q_{j,dR} + h.c. \tag{4.64}$$

(H^c is the charge-conjugated Higgs field). These fermions thus acquire masses as $\langle H_j \rangle$ becomes non-zero,

$$\to \mathcal{L}_{mass} = g_{\ell j} v_0 \epsilon^{|k-j|} \overline{\ell}_j \ell_j + g_{uj} v_0 \epsilon^{|k-j|} \overline{u}_j u_j + g_{dj} v_0 \epsilon^{|k-j|} \overline{d}_j d_j \tag{4.65}$$

If we place the three fermion generations on different branes $j_1 \neq j_2 \neq j_3$, the diagonal hierachy between the families is generated through the suppression factors $\epsilon^{|k-j_i|}$ [**?**].

The off-diagonal terms in the mass matrix must be generated to give a nontrivial CKM matrix. We specialize to quarks. This mixing now arises through higher dimensional operators corresponding to the overlap of the wave-functions of the chiral zero-mode fermions localized on different branes:

$$\mathcal{L}_{mixed} = g_{u,il} \overline{q}_{j_i,L} \left(\Pi_{l=j_i}^{j_n} \frac{\tilde{\Phi}_l}{M_f} \right) H_{j_l} q_{j_l,uR} + g_{d,il} \overline{q}_{j_i,L} \left(\Pi_{l=j_i}^{j_n} \frac{\tilde{\Phi}_l}{M_f} \right) H_{j_l}^c q_{j_l,dR} \tag{4.66}$$

where the fields $\tilde{\Phi}_l$ are composites defined as $\Phi_l' \phi_l'$. We emphasize that the mass scale M_f is new, and is related to the masses of the decoupled vectorlike fermions. The above expression effectly mimics the overlaping of fermion wave functions in the set-up of split fermions [**?**]. The suppressed off-diagonal mass terms are therefore:

$$\mathcal{L}_{mixed} = g_{u,il} v_0 (\epsilon')^{|j_n - j_i|} \epsilon^{|k-j_l|} \overline{u}_{j_i,L} u_{j_l,R} + g_{d,il} v_0 (\epsilon')^{|j_n - j_i|} \epsilon^{|k-j_l|} \overline{d}_{j_i,L} d_{j_l,R} + h.c., \tag{4.67}$$

where $\epsilon' = v/M_f$. In this manner a model of the CKM matrix can be generated.

Of course, at the end of the day we view this as a $3+1$ dimensional model in which there are many mixing interactions and higher dimension operators giving the hierarchy.

5 Discussion and Conclusion

In conclusion, we have constructed a description of the Standard Model in the bulk as a pure $3+1$ dimensional effective theory. One can discard the notion of an extra-dimension and view this as an extension of the Standard Model within $3+1$ dimensions. The connection to extra dimensions is made through the transverse lattice, and this may be viewed as a manifestly gauge invariant low energy effective theory for an extra-dimensional Standard Model.

The expanded gauge invariance needed to describe KK modes in $3+1$ may be viewed as a consequence of hidden local symmetries required to make renormalizeable theories of spin-1 objects [**?**], or alternatively, as the expanding gauge invariance that appears as an extra dimension opens up.

Many issues remain to be addressed in the context of this transverse lattice approach to describing extra dimensions [?]. For example, how does a dynamical electroweak symmetry reaking scheme emerge in this description [?]? One thing we see immediately in this approach is the emergence of an imbedding of QCD as in $SU(3) \to SU(3) \times SU(3)$, etc. This is remniscent of Topcolor, [?], and suggests the models in which the electroweak symmetry is broken dynamically [?].

We view the transverse lattice approach as a powerful new insight into the construction of new extensions beyond the Standard Model within $3+1$ model building.

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