THEORIES OF NEUTRINO MASSES AND MIXINGS

CARL H. ALBRIGHT
Fermi National Accelerator Laboratory, P.O. Box 500
*Batavia, Illinois 60510, U.S.A.**
and
Department of Physics, Northern Illinois University
DeKalb, Illinois 60115, U.S.A.

Int. J. Mod. Phys. A 2001.16:697-700. Downloaded from www.worldscientific.com
by FERMI NATIONAL ACCELERATOR LABORATORY on 09/22/14. For personal use only.

An overview of neutrino models is presented with special emphasis on models with right-handed neutrinos which involve unifications of flavors and/or families. Some comments are made about the ease with which the various solar neutrino mixing scenarios can be obtained.

1. Types of Neutrino Models

Neutrino models can be divided into three broad categories: those involving only left-handed neutrinos and a left-handed Majorana mass matrix, those involving both left- and right-handed neutrinos from which both Dirac and Majorana mass matrices can be constructed, and those in which higher dimensions play a direct role. Examples of each type can be found in contributions to these proceedings.

With only left-handed neutrinos present, lepton number is violated by non-renormalizable mass terms, unless an isovector Higgs field is introduced which carries a lepton number two quantum number. In such models the scales of the ultralight neutrino masses are not easily understood. With both left- and right-handed neutrinos present and no contributions to the left-handed Majorana mass matrix present, ultralight neutrino masses are naturally obtained in GUT models with a unification scale of 2×10^{16} GeV by the well-known seesaw mechanism, provided the right-handed masses are in the range of $10^5 - 10^{14}$ GeV. On the other hand, with large extra dimensions and the compactification scale much lower than the string scale, a modified seesaw mechanism can generate ultralight neutrino masses. For this purpose, one must assume that the right-handed neutrinos are singlets under all gauge symmetries and can enter the bulk.

In the following, I shall mainly concentrate on standard seesaw models with both left- and right-handed neutrinos.

2. Reduction in the Number of Input Parameters

In the two standard models based on $SU(3)_c \times SU(2)_L \times U(1)_Y$, the SM with one Higgs doublet, and the supersymmetric MSSM with two Higgs doublets, as originally defined only left-handed neutrinos are present. Hence no renormalizable

*Present address

neutrino mass terms exist while many arbitrary Yukawa couplings are present. The number of parameters can be reduced, if one introduces some flavor (intra-family) symmetry as in the case of GUT's, whereby a unified treatment of quarks and leptons is obtained as all (or some) of them are placed in the same multiplet(s). Many examples such as $SU(5)$, $SO(10)$, E_6, ... can be found in the literature.

On the other hand, the introduction of a family (inter-family or horizontal) symmetry enables one to build in an apparent hierarchy for the different family masses corresponding to the same generic flavor. The family symmetry may be discrete such as Z_2, S_3, $Z_2 \times Z_2$, etc. which results in multiplicative quantum numbers or continuous such as $U(1)$, $U(2)$, $SU(3)$, etc. resulting in additive quantum numbers. The latter symmetry may be global or local and possibly anomalous. Models with a combination of both flavor and family symmetries reduce the number of model parameters even more effectively. Interestingly enough, unification of flavor and family symmetries into a single group such as $SO(18)$ or $SU(8)$ has generally not been successful, as too many extra states are present which must be made superheavy.

Int. J. Mod. Phys. A 2001.16:697-700. Downloaded from www.worldscientific.com
by FERMI NATIONAL ACCELERATOR LABORATORY on 09/22/14. For personal use only.

3. Examples of Unified Models

3.1. *MSSM with anomalous* $U(1)$ *family symmetry*

In 1979 Froggatt and Nielsen[1] added to the SM a scalar singlet "flavon" ϕ, which gets a VEV, together with heavy fermions (F, $\bar{F}$) in vector-like representations, all of which carry $U(1)$ family charges. With $\langle\phi\rangle/M_G \equiv \lambda \sim (0.01 - 0.2)$, the $U(1)$ symmetry is broken, and light and heavy fermions are mixed. λ can then serve as an expansion parameter for the mass matrix entries.

The λ^n structure of the mass matrices can be determined from the corresponding Wolfenstein λ structure of the CKM matrix and the quark and lepton mass ratios, where different $U(1)$ charges can be assigned to each quark and lepton field. However, the coefficients (prefactors) of the λ powers are not accurately determined. In this scenario, Ramond and many others[†] have shown that maximal mixing of $\nu_\mu \leftrightarrow \nu_\tau$ and the small mixing angle MSW solution (SMA) for $\nu_e \leftrightarrow \nu_\mu, \nu_\tau$ can be obtained, while the bimaximal mixing solutions are not obtained naturally.

3.2. *Minimal SUSY* $SU(5)$

Here the left-handed up and down quarks, left-handed conjugate up quarks and charged lepton of each family are placed in a **10** -dimensional representation of $SU(5)$, while the left-handed leptons and conjugate down quarks of each family are placed in a $\bar{\mathbf{5}}$ representation. The Higgs fields reside in $\mathbf{24}_H$, $\mathbf{5}_H$ and $\bar{\mathbf{5}}_H$ representations. The fermion mass matrices exhibit the symmetries: $M_U = M_U^T$, $M_D = M_L^T$, at the GUT scale from which one finds that $m_b = m_\tau$, but also the bad result $m_d/m_s = m_e/m_\mu$, since minimal $SU(5)$ is too simplistic with no family symmetry present. Proton decay is sufficiently suppressed, however, in the SUSY version.

[†]For reference listings in two more comprehensive reviews, see [2].

Int. J. Mod. Phys. A 2001.16:697-700. Downloaded from www.worldscientific.com
by FERMI NATIONAL ACCELERATOR LABORATORY on 09/22/14. For personal use only.

3.3. *SUSY* $SO(10)$ *with family symmetry*

While other models based on $SU(5)$ with a family symmetry are more satisfactory,[2] models based on SUSY $SO(10)$ are even more predictive. In the latter case, all fermions of one family are placed in a **16** supermultiplet and carry the same family charge assignment. Massive pairs of $(\mathbf{16},\ \overline{\mathbf{16}})$'s and **10**'s may also be present. The Higgs fields are typically placed in one or more $\mathbf{45}_H$'s, pairs of $(\mathbf{16}_H,\ \overline{\mathbf{16}}_H)$'s, and $\mathbf{10}_H$'s with the Majorana neutrino masses generated by a $\overline{\mathbf{126}}_H$ or an effective $\overline{\mathbf{16}}_H \cdot \overline{\mathbf{16}}_H$ field.

If the two electroweak-breaking VEV's, H_u and H_d, lie solely in a $\mathbf{10}_H$, $t-b-\tau$ Yukawa coupling unification is possible only for $\tan\beta \simeq 55$. However, if part of H_d also lies in a $\mathbf{16}_H$, Yukawa coupling unification is possible for $\tan\beta \ll 55$. Such an electroweak-breaking VEV can contribute asymmetrically to the down quark and charged lepton mass matrices, which can help to explain[3] why the neutrino mixing element $U_{\mu 3}$ yields maximal $\nu_\mu \leftrightarrow \nu_\tau$ mixing for atmospheric neutrinos, while the CKM element V_{cb} is so small in the quark sector. With $\langle \mathbf{45}_H \rangle$ pointing in the $B-L$ direction, the desirable Georgi-Jarlskog mass relations,[4] $m_s/m_b = m_\mu/3m_\tau$, $m_d/m_b = 3m_e/m_\tau$, can also be obtained.[3]

Various family symmetries appear in the literature in combination with the $SO(10)$ flavor group and include $U(1)$,[5] $U(1)\times Z_2\times Z_2$,[6] $U(2)$,[7] $SU(2)\times Z_2\times Z_2\times Z_2$,[8] $U(2)\times U(1)^n$,[9] and $SU(3)$.[10] The family charges under these various groups and the Higgs contents then impose certain texture zeros on the mass matrices which allows for various predictions of the masses and mixings of the quarks and leptons.

4. Mass and Mixing Matrices

Complex symmetric mass matrices are constructed for each flavor $f = U,\ D,\ N,\ L$ in the GUT flavor basis

$$\mathcal{B}_f = \{f_{iL},\ f^c_{iL}\},\ i = 1,2,3: \quad \mathcal{M}_f = \begin{pmatrix} 0 & M_f^T \\ M_f & M_R \end{pmatrix}, \tag{1}$$

where M_f is the 3×3 Dirac mass matrix for flavor f and M_R is the 3×3 superheavy right-handed Majorana neutrino mass matrix for $f = N$ which vanishes otherwise. The 3×3 effective light neutrino mass matrix is determined from the seesaw mechanism by $M_\nu = -M_N^T M_R^{-1} M_N$.

The CKM mixing matrix arises from the unitary matrices which diagonalize the up and down quark mass matrices, while the Maki-Nakagawa-Sakata (MNS) mixing matrix[11] arises from the unitary transformations of the charged lepton and effective light neutrino mass matrices:

$$V_{CKM} = U_U^\dagger U_D, \quad U_{MNS} = U_L^\dagger U_\nu. \tag{2}$$

Hence the neutrino mixing matrix results from an interplay of M_N, M_L and M_R.

Successful GUT models must essentially generate the CKM mixing matrix and one of the two MNS maximal (SMA MSW) or bimaximal (LMA MSW, LOW or Vacuum) mixing matrices corresponding to the still viable neutrino solutions where

$$
\begin{aligned}
V_{CKM} &\simeq \begin{pmatrix} 0.975 & 0.220 & 0.0032e^{-i65^o} \\ -0.220 & 0.974 & 0.040 \\ 0.0088 & -0.040 & 0.999 \end{pmatrix}, \\
U_{MNS}^{(SMA)} &\simeq \begin{pmatrix} 0.99 & 0.04 & 0.05 \\ -0.03 & 0.70 & -0.71 \\ -0.03 & 0.71 & 0.70 \end{pmatrix}, \\
U_{MNS}^{(bimax)} &\simeq \begin{pmatrix} 0.71 & -0.70 & \theta_{13} \\ 0.50 & 0.50 & -0.71 \\ 0.50 & 0.50 & 0.70 \end{pmatrix},
\end{aligned}
\tag{3}
$$

and $\theta_{13} < 0.2$ by the CHOOZ reactor experiment.[12]

The SMA MSW solar neutrino mixing solution is readily obtained in many unified models, since only one mixing - the atmospheric - need be large. Bimaximal mixing is obtainable in fewer models. Of the two, the vacuum solution with pseudo-Dirac neutrinos is somewhat more natural, while the large angle MSW solution requires a more careful tuning of the parameters of the mass matrices.

5. Summary

If a model builder restricts oneself to the lepton sector alone, considering simply the light left-handed neutrino matrix, it is possible to obtain many schemes which fit the neutrino mass and mixing data. A more challenging approach is to consider unified models which explain both the quark and lepton masses and mixings.

The most predictive models for the 12 "light" fermion masses and their 8 CKM and MNS mixing angles and phases are obtained in the framework of a Grand Unified Model with Family Symmetry. $SO(10)$ models are more tightly constrained than $SU(5)$ models and are more economical than larger groups like E_6, where more fields must be made supermassive. In fact, some $SO(10)$ models do very well in predicting the 20 "observables" with just 10 or more input parameters. It remains to be seen whether models based on neutrinos in higher dimensions can be as predictive and do as well.

References

1. C.D. Froggatt and H.B. Nielsen, *Nucl. Phys.* B 147 (1979) 277.
2. G. Altarelli, *Nucl. Phys. Proc. Suppl.* 87 (2000) 291; S.M. Barr and I. Dorsner, *Nucl. Phys.* B 585 (2000) 79.
3. C.H. Albright, K.S. Babu, and S.M. Barr, *Phys. Rev. Lett.* 81 (1998) 1167.
4. H. Georgi and C. Jarlskog, *Phys. Lett.* B 86 (1979) 297.
5. K.S. Babu, J.C. Pati, and F. Wilczek, *Nucl. Phys.* B 566 (2000) 33.
6. C.H. Albright and S.M. Barr, *Phys. Rev. Lett.* 85 (2000) 244; *Phys. Rev.* D 62 (2000) 093008.
7. R. Barbieri, L. J. Hall, S. Raby, and A. Romanino, *Nucl. Phys.* B 493 (1997) 3.
8. M.-C. Chen and K.T. Mahanthappa, *Phys. Rev.* D 62 (2000) 113007.
9. T. Blazek, S. Raby, and K. Tobe, *Phys. Rev.* D 62 (2000) 055001.
10. Z. Berezhiani and A. Rossi, hep-ph/0003084.
11. Z. Maki, M. Nakagawa, and S. Sakata, *Prog. Theor. Phys.* 28 (1962) 870.
12. CHOOZ Collab. (M. Apollonio *et al.*), *Phys. Lett.* B 420 (1998) 397.

Int. J. Mod. Phys. A 2001.16:697-700. Downloaded from www.worldscientific.com
by FERMI NATIONAL ACCELERATOR LABORATORY on 09/22/14. For personal use only.