arXiv:hep-lat/0110085v1 15 Oct 2001

One-loop renormalization of heavy-light currents*

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We calculate the mass dependent renormalization factors of heavy-light bilinears at one-loop order of perturbation theory, when the heavy quark is treated with the Fermilab formalism. We present numerical results for the Wilson and Sheikholeslami-Wohlert actions, with and without tree-level rotation. We find that in both cases our results smoothly interpolate from the static limit to the massless limit. We also calculate the mass dependent Brodsky-Lepage-Mackenzie scale q^*, with and without tadpole-improvement.

1. INTRODUCTION

Although lattice QCD offers a nonperturbative method of calculating weak matrix elements from first principles, in practice a perturbative renormalization is also required to extract the continuum quantities for heavy-light systems. In this talk we discuss the renormalization of heavy-light vector and axial vector currents. These currents are needed for heavy quark phenomenology, such as the calculation of the decay constants and semi-leptonic form factors of heavy-light mesons. Here we calculate explicitly the mass dependent renormalization factors of heavy-light currents at one-loop order, when the heavy quark is treated with the Fermilab formalism [1]. Results for the Wilson action have been obtained first in Ref. [2] and preliminary results for clover action have been reported in previous lattice conferences [3]. For tree-level improvement at order $1/m_Q$, we include so-called rotation term here. Tadpole-improved renormalization factors are also presented. We also calculate mass dependent Brodsky-Lepage-Mackenzie scale q^* [8], with and without tadpole-improvement. More details of this work will be given in Ref. [4].

2. ONE-LOOP RESULTS

The renormalization factors Z_{J_Γ} of heavy-light currents are simply the ratio of the lattice and continuum radiative corrections:

$$Z_{J_\Gamma} = \frac{[Z_{2h}^{1/2}\Lambda_\Gamma Z_{2l}^{1/2}]^{cont}}{[Z_{2h}^{1/2}\Lambda_\Gamma Z_{2l}^{1/2}]^{lat}}, \tag{1}$$

where Z_{2h} and Z_{2l} are wave-function renormalization factors of the heavy and light quarks, and the vertex function Λ_Γ is the sum of one-particle irreducible three-point diagrams. We calculate explicitly Z_A and Z_V at one-loop order of perturbation theory.

In view of the mass dependence, we write

$$e^{-m_1^{[0]}a/2} Z_{J_\Gamma} = 1 + \sum_{l=1}^{\infty} g_0^{2l} Z_{J_\Gamma}^{[l]}, \tag{2}$$

so that the $Z_{J_\Gamma}^{[l]}$ are only mildly mass dependent. Fig. 1 plots the full mass dependence of the renormalization factors for the axial vector current $Z_{A_4}^{[1]}$. These numerical results are for the SW action with and without rotation, and also for Wilson action without rotation. Our results agree with those previously obtained, for $c_{SW} = 0$ [2]

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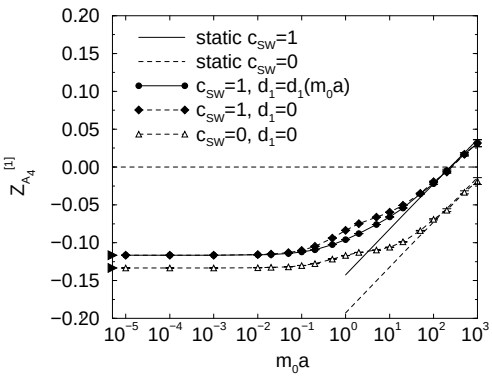

Figure 1. One-loop renormalization coefficient $Z^{[1]}_{A_4}$ as a function of am_0.

and for $c_{SW} = 1, d_1 = 0$ [3] [3]. We find that in both cases our results smoothly interpolate from the static to massless limit. The resulting analytical expressions are in Ref. [4]. Fig. 2 plots the tadpole-improved renormalization factor for $Z^{[1]}_{A_4}$. From this figure, we can see that tadpole-improvement significantly reduces the one-loop coefficients of renormalization factors. Results for Z_{A_i} and $Z_{V_{4,i}}$ are given in Ref. [4].

The slope of our mass-dependent renormalization factors in the massless limit is related to the improvement coefficients b_J and c_J [5]. We find

$$b^{[1]}_V = 0.153239(14), \quad (3)$$

$$b^{[1]}_A = 0.152189(14), \quad (4)$$

$$c^{[1]}_V = -0.016332(7), \quad (5)$$

$$c^{[1]}_A = -0.0075741(15). \quad (6)$$

These results agree perfectly with Ref. [6]. We also obtain by subtracting the integrands first,

$$b^{[1]}_V - b^{[1]}_A = 0.0010444(16) \quad (7)$$

which is more accurate than the difference of the two numbers quoted above. We find our one-loop result of $b_V - b_A$ are far from nonperturbative calculations [7].

3. SETTING THE SCALE

The typical gluon momentum q^* in the V-scheme, as suggested by Brodsky, Lepage and Mackenzie (BLM), is defined by [8]

$$\ln(q^{*2}) \equiv \frac{\int d^4q f(q) \ln(q^2)}{\int d^4q f(q)}, \quad (8)$$

where q is the momentum of gluon, and the form $\int d^4q f(q)$ is the one-loop integral for a particular renormalization constant, for example, $\int d^4q f(q) = Z^{[1]}_{J_\Gamma}$. Previously q^* has been calculated for the light-light current [9] [10] and the static-light current [11]. Here we calculate the mass dependent q^* for the heavy-light current. Results are plotted in Fig. 3. For Wilson action case, our results agree with Ref. [9] in the massless limit. From Fig. 3, we can see that the mass dependence of q^* is weak from massless limit to $m_0a \sim 1$, especially for clover with rotation case. The original BLM prescription of q^* breaks down at larger masses, because $Z^{[1]}_{J_\Gamma}$(denominator in Eq. (8)) goes through zero at there. A prescription for q^* in this case is given in Ref. [12]. We also calculate tadpole-improved q^* and results are plotted in Fig. 4. We can see that plaquette tadpole-improvement significantly reduces q^*, on the other hand, the reduction is rather small for κ_c tadpole-improvement. We summarize the results in the massless limit in Table. 1.

We can also obtain the BLM scale for improvement coefficients b_J and c_J [5]. Then it is interesting to compare BLM perturbation theory with non-perturbative calculations of these coefficients [7][13]. We will present these results for q^* and the mentioned comparison in another publication [14].

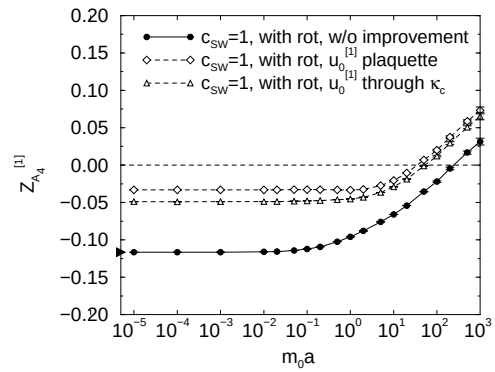

Figure 2. Tadpole-improved one-loop renormalization coefficient $Z^{[1]}_{A_4}$ as a function of am_0.

[3] The coefficient d_1 is field rotation parameter. See Ref. [1].

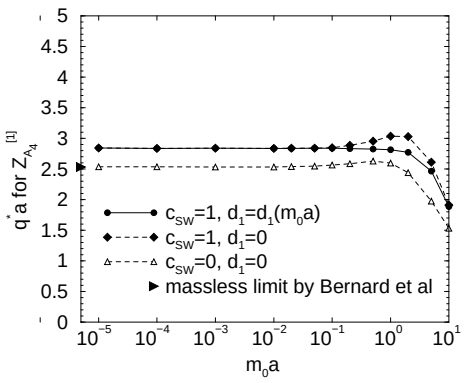

Figure 3. Brodsky-Lepage-Mackenzie scale q^* for $Z^{[1]}_{A_4}$ as a function of am_0.

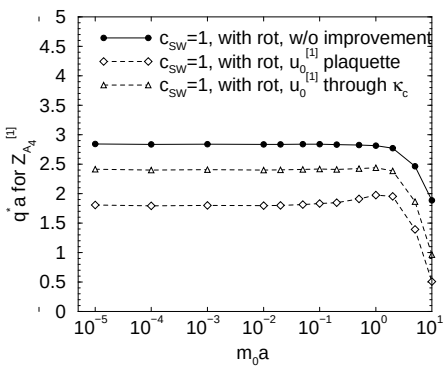

Figure 4. Tadpole-improved Brodsky-Lepage-Mackenzie scale q^* for $Z^{[1]}_{A_4}$ as a function of am_0.

4. CONCLUSIONS

We have obtained one-loop results of Z_A and Z_V with tree-level rotation, which should be useful for lattice calculations of f_B and of form factors for $B \to \pi l \nu$. We have also obtained the BLM scale q^* for arbitrary masses, which should reduce the uncertainty of one-loop calculations.

Acknowledgments

S.H. and T.O. are supported by the Grant-in-Aid of the Japanese Ministry of Education, (Nos.11740162, No.12640279). K.-I.I. and N.Y. are supported by the JSPS Research Fellowships. Fermilab is operated by Universities Research Association Inc., under contract with the U.S. Department of Energy.

Table 1
One-loop Z-factor and BLM scale q^* in the massless limit for clover (upper row) or Wilson (lower row) action with tadpole-improvement.

	no improvement	plaquette	through κ_c
$Z^{[1]}_A$	$-0.116457(2)$	$-0.033124(2)$	$-0.048938(2)$
	$-0.133375(2)$	$-0.050042(2)$	$-0.024803(4)$
$Z^{[1]}_V$	$-0.129430(2)$	$-0.046097(2)$	$-0.061911(2)$
	$-0.174086(2)$	$-0.090752(2)$	$-0.065514(4)$
$q^*_{Z_A} a$	2.839	1.802	2.408
	2.533	1.550	2.316
$q^*_{Z_V} a$	2.845	2.060	2.503
	2.370	1.700	2.052

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