



Lattice QCD and the Unitarity Triangle

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Abstract. Theoretical and computational advances in lattice calculations are reviewed, with focus on examples relevant to the unitarity triangle of the CKM matrix. Recent progress in semi-leptonic form factors for $B \rightarrow \pi l \nu$ and $B \rightarrow D^* l \nu$, as well as the parameter ξ in B^0 - \bar{B}^0 mixing, are highlighted.

INTRODUCTION

To test the CKM picture of CP and flavor violation, a combination of theory and experiment is needed. A vivid way to summarize the need for redundant information is the unitarity triangle (UT), sketched in Fig. 1. It depicts two triangles, $B\gamma A$, which can (in principle) be determined from tree processes (e.g., semi-leptonic decays), and $\alpha C\beta$, in which the amplitude for B^0 - \bar{B}^0 mixing is involved [1]. The “tree” triangle can, for the sake of argument, be taken as a measurement of the CKM matrix. Then, the “mixing” triangle tests whether new physics must be invoked to explain B^0 - \bar{B}^0 mixing.

To obtain the sides A , B , and C of these triangles one must calculate hadronic properties from first principles of QCD. Here “calculate” implies that a reliable error estimate is given: one that includes all sources of uncertainty. Lattice gauge theory is well suited to the task: semi-leptonic form factors and the mixing matrix elements are conceptually straightforward. Indeed, according to Martin Beneke [2], “[the] standard UT fit is now entirely in the hands of lattice QCD (up to, perhaps, $|V_{ub}|$).”

Until recently, lattice QCD has been burdened by something called the quenched approximation (explained below). A bit of good news is that the available computer power is now sufficient to get rid of this approximation [3, 4, 5]. Another bit of good news is that several lattice groups have used the quenched approximation in the spirit of a blind analysis: although quenching could change the central value, one can analyze all other uncertainties as if the underlying numerical data were real QCD. This exercise has left

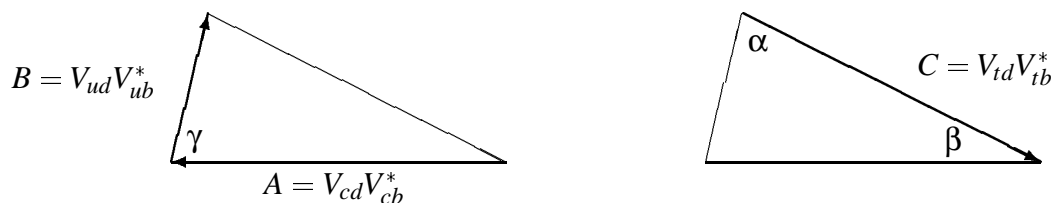


FIGURE 1. Unitarity triangles: on the left is the “tree” triangle; on the right, the “mixing” triangle.

us with several quantities, including those needed in the UT fits, with full error analyses, apart from quenching. In the next few years, we should have full QCD calculations with a complete assessment of the uncertainties, and thereafter the uncertainties can be incrementally reduced.

Because it is important to understand the theoretical uncertainties, this talk will start by sketching where they arise. Most reviews focus too much on central values, so, even when turning to numerical results, the focus here is on the error bars.

The paper ends with a short swim through the “Octopus’s Garden”, that is, some recent work by Nathan Isgur, who passed away a few weeks before Heavy Flavors 9 convened. The symposium, and this paper, are dedicated to his memory.

UNCERTAINTIES IN LATTICE CALCULATIONS

Lattice QCD calculates matrix elements by integrating the functional integral, using a Monte Carlo with importance sampling. The Monte Carlo leads to (correlated) statistical error bars. This part of the method is well understood for quenched QCD and, these days, rarely leads to controversy. When conflicting results arise, they originate in different treatments of the systematics. The consumer probably does not need to know how the Monte Carlo works, but should develop an intuition of how the systematics work.

The main tool for controlling systematics is effective field theory. In this talk, we are concerned mostly with three classes of effects: those connected with the lattice spacing a , the heavy quark mass(es) m_Q , and the light quark mass(es) m_q . Inside the computer there is a hierarchy of scales

$$m_q \ll \Lambda \ll m_Q, \pi/a, \quad (1)$$

although in practice π/a is only several GeV, and the “light” quarks are never as light as the up and down quarks. The QCD scale Λ gauges the size of power corrections; experience suggests it is 700 MeV, give or take a factor of $\sqrt{2}$. With familiar techniques of effective field theory, lattice theorists can control the extrapolation of artificial, numerical data to the real world, provided the data start “near” enough. Similarly, non-experts usually have an intuition of how effective field theories work, so they can check, on the back of the envelope, whether systematic errors have been treated sensibly.

The notable exception to the rule of effective field theory is the valence, or quenched, approximation. Consider the pictures in Fig. 2. The one on the left depicts a meson made of a valence quark and anti-quark, bound by a smear of gluons. The gluons can also create virtual quark-antiquark pairs, leading to the picture on the right. These are

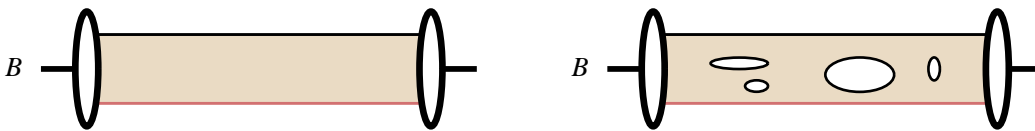


FIGURE 2. Quenched approximation: processes on the left are incorporated “exactly”, whereas processes on the right are not computed, but modeled with a shift in the bare couplings.

computationally very costly. The quenched approximation omits them but compensates the omission with a shift in the bare couplings. This is analogous to a dielectric, where $g_0^2 \rightarrow g_0^2 \epsilon$. Quenching retains many effects, such as retarded gauge potentials, that are omitted in, say, the quark model. It is also the first term in a systematic expansion [6].

The quenched approximation can fall short of reproducing chiral logarithms of the form $\ln(\Lambda^2/m_q^2)$ [7, 8, 9, 10]. Figure 3 shows some quark-line configurations that generate, at the hadronic level, meson loops. The quenched approximation includes (b), but not (a) or (c). As a consequence, some pion loops are omitted, and η' loops are mistreated. A better situation is a “partially quenched” theory, with dynamical quark loops, but, possibly, $m_{\text{valence}} \neq m_{\text{sea}}$. Then features like the η' - π splitting emerge, and it is possible to relate the partially quenched theory to full QCD using chiral perturbation theory. We shall return to chiral logs in the next section, when discussing B^0 - \bar{B}^0 mixing.

To understand lattice spacing effects Symanzik suggested matching lattice gauge theory to continuum QCD [12]:

$$L_{\text{lat}} \doteq L_{\text{QCD}} + \sum_i a^{s_i} K_i(g^2, m_q a; \mu) O_i(\mu). \quad (2)$$

The right-hand side is a local effective Lagrangian (LEL) renormalized in some continuum scheme; details of the scheme are not important. Discretization effects are, of course, short-distance effects, so as usual in an effective field theory, they are lumped into the coefficients K_i .

There are two key points to the Symanzik effective field theory. First, if Λa is small enough, the \sum_i can be treated as a perturbation. For example, for the proton mass

$$m_p(a) = m_p - a K_{\sigma F}(c_{\text{SW}}) \langle p | \bar{\psi} \sigma \cdot F \psi | p \rangle \quad (3)$$

using the leading term for Wilson fermions as an example. Here c_{SW} is the so-called clover coupling, and $K_{\sigma F} = (1 - c_{\text{SW}})/4 + O(\alpha_s)$. Second, to reduce lattice spacing effects, one can tune c_{SW} so that $K_{\sigma F}$ vanishes or, in practice, is $O(\alpha_s^l)$ or $O(a)$. Thus, the Symanzik effective field theory shows that lattice artifacts can be reduced through short-distance process-independent methods. Indeed, for light hadrons a combination of this Symanzik improvement and extrapolation in a^2 gives continuum QCD results with very well understood uncertainties (modulo quenching).

With the bottom and charmed quarks, the mass is large in lattice units: $m_b a \sim 1$ –2 and $m_{ch} a$ about a third of that. The split in the LEL between QCD and small correction breaks down, because there are terms in the \sum_i in Eq. (3) that go like $(m_Q a)^n$. It will not be possible to reduce a enough to make $m_b a \ll 1$ for a long time: Moore’s Law

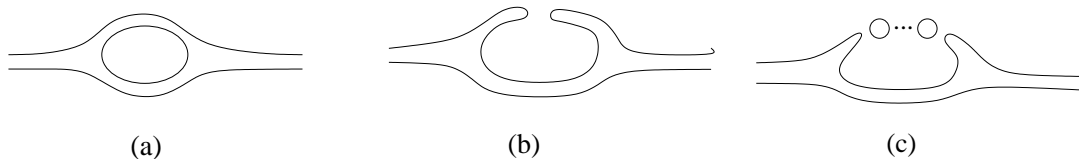


FIGURE 3. Quark line configurations that lead to meson loops. Adapted from Ref. [11].

TABLE 1. Widely used methods for heavy quarks in lattice gauge theory.

#	method	Ref.	how HQET enters
1.	static approximation	[13, 14]	$L_{\text{static}} = -\bar{h}D_4h$
2.	lattice NRQCD	[15, 16]	discretization of first few terms of HQET
3.	extrapolation from below charm	<i>ad hoc</i>	results with small m_Q fit to Taylor series in $1/m_Q$
3'	3 + 1	<i>ad hoc</i>	as in method 3
4.	“Fermilab”	[17, 18]	match lattice to QCD, term-by-term in HQET

suggests 15–25 years. There are, nevertheless, several ways to treat heavy-light hadrons in lattice calculations, all of which appeal to HQET in some way. Table 1 lists the most widely used methods. In lattice NRQCD and in the Fermilab method, it is possible to set $m_Q = m_b$, even when $a^{-1} \sim m_b$. There are, of course, uncertainties involved, but a grasp of the basics of heavy-quark theory is enough to understand them.

A convenient way to contrast the uncertainties in the various methods is to match lattice gauge theory to (continuum) HQET [18]. Instead of Eq. (3) one writes

$$L_{\text{lat}} \doteq L_{\text{HQET}} = \sum_n C_n^{\text{lat}} O_n = -m_1 \bar{h}h - \bar{h}D_4h + \frac{\bar{h}\mathbf{D}^2h}{2m_2} + \frac{\bar{h}i\boldsymbol{\Sigma} \cdot \mathbf{B}h}{2m_B} + \dots \quad (4)$$

The operators O_n on the right hand side are the same as in the HQET description of continuum QCD. The short-distance coefficients C_n^{lat} , on the other hand, are not the same, because there are two short distances, a and $1/m_Q$. Heavy-quark lattice artifacts are, thus, isolated into the mismatch $C_n^{\text{lat}} - C_n^{\text{cont}}$. In lattice NRQCD and in the Fermilab method, these mismatches can be reduced, along the same lines as reducing $K_{\sigma F}$ in the Symanzik program. In recent work, they are controlled to several percent or less.

Most work with method 3 has chosen normalization conditions for which the mismatch in the kinetic and chromomagnetic terms, although formally $O(m_Q a)^2$, is quite large in practice. It is also not well understood how this mismatch is amplified when extrapolating linearly or quadratically in $1/m_Q$ from $m_Q \sim 1$ GeV up to m_b .

LATTICE CALCULATIONS

We now turn our attention to some of the most interesting recent lattice calculations in B physics. The discussion focuses on the error bars, and central values are deferred to the next section. We consider the ratio ξ for B - \bar{B} mixing in unquenched QCD, and the semi-leptonic decays $B \rightarrow \pi l \nu$ and $B \rightarrow D^* l \nu$ for $|V_{ub}|$ and $|V_{cb}|$ in quenched QCD.

Neutral Meson Mixing

The mass difference of neutral B meson CP eigenstates is

$$\Delta m_{B_q^0} = \frac{G_F^2 m_W^2 S_0}{16\pi^2} |V_{tq}^* V_{tb}|^2 \eta_B \langle \bar{B}_q^0 | Q_q^{\Delta B=2} | B_q^0 \rangle + \text{new physics?} \quad (5)$$

where $q \in \{d, s\}$, S_0 is an Inami-Lim function, η_B is a short-distance QCD correction, and $Q_q^{\Delta B=2}$ is a four-quark operator in the $\Delta B = 2$ electroweak Hamiltonian. Note that new physics could compete with the Standard Model. The matrix element of $Q_q^{\Delta B=2}$ is usually written

$$\langle \bar{B}_q^0 | Q_q^{\Delta B=2} | B_q^0 \rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q}, \quad (6)$$

but lattice QCD gives $\langle \bar{B}_q^0 | Q_q^{\Delta B=2} | B_q^0 \rangle$ and f_{B_q} (from $\langle 0 | A_{bd}^\mu | B_q^0 \rangle$) individually. This traditional separation turns out to be useful when looking at chiral logs.

Conventional wisdom says that the uncertainties in B_{B_q} and in $\xi^2 = f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$ are “easy” to control, because they are ratios. At *Lattice '01*, Norikazu Yamada of JLQCD reported on new results [19] that suggest otherwise. The JLQCD collaboration is mounting a large project to carry out calculation with the loops of $n_f = 2$ flavors of quarks. Figure 4 compares their previous quenched work for f_{B_q} with their new (and still preliminary) work with $n_f = 2$. In the spirit of a blind analysis, the definition of $\Phi_{f_{B_q}}$ is not so important. The important matter is whether the curve is a straight line, or whether there is any curvature as a function of the light quark mass. (Because the pseudoscalar $m_\pi^2 \propto m_q$, the horizontal axis is, essentially, the light quark mass.)

The quenched approximation (diamonds in Fig. 4) shows no evidence of curvature. With $n_f = 2$ (triangles in Fig. 4), however, allowing for curvature obtains a better fit. In fact, curvature is expected from chiral logarithms. Meson loops with fixed spectator mass m_s and varying sea quark mass m_f contribute to f_{B_s} [20]

$$\Delta f_{B_s} = c_0 + c_1 m_{\eta_{ff}}^2 - \frac{1 + 3g^2}{(4\pi f_\pi)^2} m_{\eta_{sf}}^2 \ln \left(\frac{m_{\eta_{sf}}^2}{m_{\eta_{ss}}^2} \right), \quad (7)$$

where $\eta_{qq'}$ denotes a pseudoscalar meson with constituents q and q' . From the D^* width, $3g^2 \approx 1.04$ [21]. The second plot in Fig. 4 shows that Eq. (7) describes the points well.

It is worth making a few remarks about this kind of analysis. The formula for f_{B_d} (where the spectator and sea quarks have the same mass) is slightly different, so it is

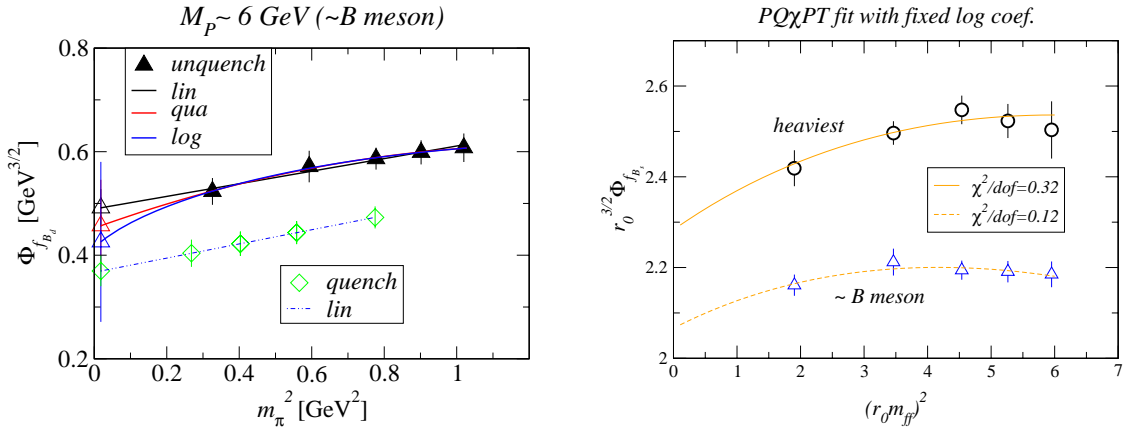


FIGURE 4. Recent calculations of the decay constant $\Phi_B = f_B \sqrt{m_B}$ from JLQCD.

difficult to display all chiral log effects on one plot. Nevertheless, one can see from Fig. 4 that the effect is to increase f_{B_s}/f_{B_d} . The chiral logs for B_{B_q} are multiplied by $1 - 3g^2 \approx 0.04$, so the B_{B_s}/B_{B_d} is almost insensitive to them. Therefore, the chiral logs in the ratio ξ come almost completely from f_{B_s}/f_{B_d} . One concludes that ξ may have been underestimated in the quenched approximation. A new estimate, based on JLQCD and previous work, is given below.

$B \rightarrow \pi l \nu$ and V_{ub}

The semi-leptonic decay $B \rightarrow \pi l \nu$ is mediated by a $b \rightarrow u$ transition. The decay rate is

$$\frac{d\Gamma}{dE} = \frac{G_F^2 m_B p^3}{12\pi^3} |V_{ub}|^2 |f_+(E)|^2, \quad (8)$$

where $v = p_B/m_B$ is the B meson's velocity, $E = v \cdot p_\pi$, and $p^2 = E^2 - m_\pi^2$. The form factor $f_+(E)$ is a linear combination of the form factors $f_\perp(E)$ and $f_\parallel(E)$, defined through the matrix element of the vector current

$$\langle \pi | V^\mu | B \rangle = \sqrt{2m_B} [v^\mu f_\parallel(E) + p_\perp^\mu f_\perp(E)]. \quad (9)$$

The pion energy E is related to $q^2 = m_B^2 + m_\pi^2 - 2m_B E$. Chiral symmetry and heavy-quark symmetry are simpler to follow with f_\parallel and f_\perp considered as functions of E , rather than f_+ considered as a function of q^2 . For example, heavy-quark symmetry suggests relations between B and D decays with the same E . In principle, one would like to compare the E dependence of experimental measurements with lattice calculations. Discretization uncertainties are smallest for low E , where phase space suppresses the event rate. Therefore, lattice calculations and experimental measurements will have to be combined in the way that minimizes the error on $|V_{ub}|$.

In the past year or so, four new quenched calculations of $f_+(E)$ for $B \rightarrow \pi l \nu$ appeared, using several different methods [22, 23, 24, 25]. It is, thus, timely to compare and contrast. In addition to discretization effects at large pa , there is evidence for considerable dependence on the light spectator quark mass [24].

The calculations of UKQCD [22] and APE [23] use method 3, so they have discretization errors of order $(m_Q a)^2$. They keep $m_Q < 1.2m_c$ and extrapolate up to m_b linearly or quadratically in $1/m_Q$. The quark masses used in Ref. [22] are shown in Table 2. Those in Ref. [23] are a bit larger, which is better for HQET but worse for the discretization errors. In these papers models are used to extrapolate to the full kinematic range of E . It is better to think of these calculations as computing the model parameters in D decays, and then invoking HQET to make predictions for

TABLE 2. Heavy quark masses used in Ref. [22]. Note that $m_{\text{pole}} < m^{\text{RI}}$.

κ	am_Q	m_Q^{RI} (GeV)	M_P (GeV)
0.1200	0.485	1.52	2.035(5)
0.1233	0.374	1.23	1.771(5)
0.1266	0.268	1.02	1.483(5)
0.1299	0.168	0.69	1.157(5)

B decays. The error associated with the assumptions are, at least to me, not transparent.

The calculation of El-Khadra *et al.* [24] uses the Fermilab method, and the calculation of JLQCD [25] uses lattice NRQCD. Since the matching procedures build in the heavy-quark expansion, both calculate directly at $m_Q = m_b$. To avoid introducing models of the E dependence, these two papers advocate comparing lattice and experiment in a region where the two overlap. For example, one can look at

$$T_B(E_{\max}) = \int_{m_\pi}^{E_{\max}} dE p^3 |f_+(E)|^2 = \frac{1}{|V_{ub}|^2} \frac{12\pi^3}{G_F^2 m_B} \int_{m_\pi}^{E_{\max}} dE \frac{d\Gamma}{dE}, \quad (10)$$

where E_{\max} is an upper kinematic cut. At present $E_{\max} \sim 1$ GeV, but, with future increases in computer power, it may be possible to raise the cut. This method is sketched in Fig. 5 using the form factor from Ref. [24]. Uncertainties are still about 15–20% on $|V_{ub}|$. A strategy for reducing them are given in Ref. [24].

Ref. [24] finds strong dependence on the light quark mass, as illustrated for $f_\perp(0.7 \text{ GeV})$ in Fig. 5. This is the only calculation to reduce the light quark down to $m_q = 0.4m_s$. The extrapolation to physical quark mass is the largest source of uncertainty quoted by Ref. [24], and a similar uncertainty is presumably present in all four calculations. One would like even lighter quarks, which take more CPU time.

Because of the discretization errors at higher E , the large light quark mass dependence, etc., a sanity check on $f_+(E)$ for $B \rightarrow \pi l \nu$ would be welcome. The calculation of similar form factors in D decays encounters many of the same issues. A compelling test would be to compare lattice and experiment for $f_+^{D \rightarrow K}/f_{D_s}$ and $f_+^{D \rightarrow \pi}/f_D$, as a function of E . New physics is unlikely to alter the underlying processes, and the CKM matrix drops out. The CLEO-c program [26] promises to measure these ratios with an uncertainty of a few percent.

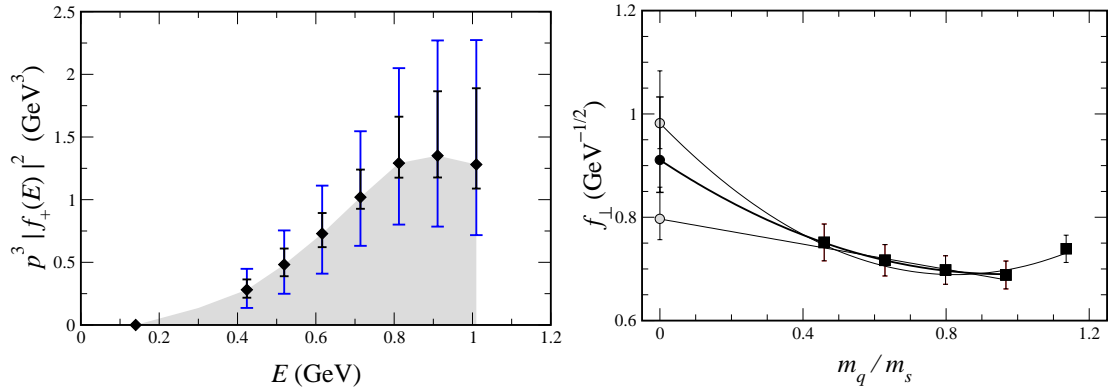


FIGURE 5. $B \rightarrow \pi l \nu$: (a) Integrand of Eq. (10). (b) Dependence of the form factor f_\perp on the light quark mass, in the range $0.4m_s < m_q < 1.2m_s$.

$B \rightarrow D^* lv$ and V_{cb}

The exclusive semi-leptonic decay $B \rightarrow D^* lv$ is a good way to determine $|V_{cb}|$, but an estimate of the hadronic transition is needed. The differential decay rate is

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} \sqrt{w^2 - 1} m_{D^*}^3 (m_B - m_{D^*})^2 G(w) |V_{cb}|^2 |F_{B \rightarrow D^*}(w)|^2, \quad (11)$$

where $w = v \cdot v'$ and $G(1) = 1$. At zero recoil ($w = 1$) heavy-quark flavor symmetry forbids terms of order $1/m_Q$. The exclusive determination of $|V_{cb}|$ therefore proceeds by extracting $|V_{cb} F_{B \rightarrow D^*}(1)|$ from a fit to the measurement of $d\Gamma/dw$, and then taking a theoretical estimate of $F_{B \rightarrow D^*}(1)$. One would prefer the latter not to depend on models.

To see what is needed from the theoretical calculation, let us review the anatomy of $F_{B \rightarrow D^*}(1)$. From HQET one can show that, at zero recoil,

$$F_{B \rightarrow D^*}(1) = \eta_A \left[1_{\text{Isgur-Wise}} + 0_{\text{Luke}/m} + \delta_{1/m^2} + \delta_{1/m^3} \right] \quad (12)$$

where η_A is a short-distance radiative correction, and the δ_{1/m^n} contain the long-distance properties of the bound states. The numerical values of η_A and the δ_{1/m^n} depend on how HQET is renormalized. It is less important *which* renormalization scheme is chosen than it is to use the *same* scheme for both. The $1/m_Q^n$ corrections take the form

$$\delta_{1/m^2} = -\frac{\ell_V}{(2m_c)^2} + \frac{2\ell_A}{(2m_c)(2m_b)} - \frac{\ell_P}{(2m_b)^2}, \quad (13)$$

$$\delta_{1/m^3} = -\frac{\ell_V^{(3)}}{(2m_c)^3} + \frac{\ell_A^{(3)}\Sigma + \ell_D^{(3)}\Delta}{(2m_c)(2m_b)} - \frac{\ell_P^{(3)}}{(2m_b)^3}, \quad (14)$$

where $\Sigma = 1/(2m_c) + 1/(2m_b)$ and $\Delta = 1/(2m_c) - 1/(2m_b)$.

With brute force alone, a sufficiently precise calculation of $F_{B \rightarrow D^*}(1)$ lies beyond reach. (See Ref. [27] for details.) Hashimoto *et al.* [27] have devised a method to extract all ℓ s in Eq. (13) and all but $\ell_D^{(3)}$ in Eq. (14). The key is the heavy-quark mass dependence of three double ratios of matrix elements. As in Eqs. (12)–(14), HQET implies

$$\frac{\langle D | \bar{c}\gamma^A b | B \rangle \langle B | \bar{b}\gamma^A c | D \rangle}{\langle D | \bar{c}\gamma^A c | D \rangle \langle B | \bar{b}\gamma^A b | B \rangle} = \eta_V^2 \left[1 - \Delta^2 \left(\ell_P + \ell_P^{(3)}\Sigma \right) \right]^2, \quad (15)$$

$$\frac{\langle D^* | \bar{c}\gamma^A b | B^* \rangle \langle B^* | \bar{b}\gamma^A c | D^* \rangle}{\langle D^* | \bar{c}\gamma^A c | D^* \rangle \langle B^* | \bar{b}\gamma^A b | B^* \rangle} = \eta_V^2 \left[1 - \Delta^2 \left(\ell_V + \ell_V^{(3)}\Sigma \right) \right]^2, \quad (16)$$

$$\frac{\langle D^* | \bar{c}\gamma^j \gamma_5 b | B \rangle \langle B^* | \bar{b}\gamma^j \gamma_5 c | D \rangle}{\langle D^* | \bar{c}\gamma^j \gamma_5 c | D \rangle \langle B^* | \bar{b}\gamma^j \gamma_5 b | B \rangle} = \frac{\eta_{Ac b} \eta_{A b c}}{\eta_{A c c} \eta_{A b b}} \left[1 - \Delta^2 \left(\ell_A + \ell_A^{(3)}\Sigma \right) \right]^2. \quad (17)$$

In particular, heavy-quark spin symmetry requires the same ℓ s to appear in Eqs. (15)–(17) as in Eqs. (13) and (14).

In a lattice calculation of these double ratios most of the statistical and systematic uncertainties cancel. The main difference from continuum QCD is that the short-distance

coefficients are different [18, 27]. But, just like in continuum QCD, the short-distance behavior can be computed in perturbation theory. Thus, the analysis proceeds by removing the lattice short-distance contribution, fitting to the prediction of HQET, and then reconstituting $F_{B \rightarrow D^*}(1)$ from the ℓ s and η_A . The result is

$$F_{B \rightarrow D^*}(1) = 0.97^{+0.024}_{-0.017} \pm 0.016^{+0.003+0.000+0.006}_{-0.014-0.016-0.014}. \quad (18)$$

To encourage the reader to think about the uncertainties, the central value is not revealed until the end. The error bars come from, in order, statistics and fitting, matching lattice gauge theory with HQET to QCD, lattice spacing dependence, light quark mass effects, and the quenched approximation. The secret to the small error on F is that they all scale as $F - 1$, by design. As a fraction of $F - 1$ the uncertainties are still sizable: 5–25%.

The most novel aspect of this analysis is how seriously it takes the idea of matching lattice gauge theory to QCD through HQET. A central part of the analysis is to calculate short-distance properties, which Ref. [27] does partly in perturbation theory. The second error bar reflects the associated uncertainty. It is reducible, but through traditional theoretical physics, rather than intensive computation.

AN OCTOPUS'S GARDEN

Although it does not bear directly the unitarity triangle, I would like to discuss some recent work by Nathan Isgur. I did not know Nathan well, but his enthusiasm for physics, especially the strong interactions, made a big impression on me. He struck me as the kind of person who had a hand all sorts of things. He must have needed eight arms to keep all his projects moving.

In the last year of his life, one of his projects was to understand whether instanton-like gauge fields or, on the other hand, disordered gauge fields play a larger role in QCD [28, 29, 11]. This work reflected Nathan's broad knowledge of the phenomenological and theoretical sides of the strong interactions, touching on the OZI rule, chiral zero modes, the η' mass, instantons, the quark model, the AdS/CFT correspondence, lattice QCD, and the large N_c limit. A particularly striking passage notes first how recent work by Witten on the AdS/CFT correspondence favors disordered, confining gauge fields (although those sentences were probably written by Nathan's collaborator Hank Thacker), and then how details of how to treat strong decays in the quark model favor the disorder also. That part was *certainly* written by Nathan.

These issues surround the quantum ground state, or vacuum, of QCD. Gauge theories have many classical ground states, one for each integer. Instantons are the semi-classical configurations that tunnel from one ground state to another. In a quantum mechanical situation, fluctuation-dominated configurations could also mediate tunneling. Nathan proceeded by devising tests that could distinguish whether the quantum vacuum is obtained via such disordered gauge fields or via a gas or liquid of instantons. He then used (quenched) lattice QCD to see which way the gauge fields behaved.

One test stemmed from the observation, from empirically successful quark models of strong decays, that quark pairs pop out of the vacuum with scalar (*i.e.*, 3P_0) quantum numbers. That implies that the OZI rule should fail for $J^{PC} = 0^{++}$ quantum numbers.

(The OZI rule says that decays, like $\phi \rightarrow \pi\pi$, in which quarks annihilate, are weaker than those, like $\phi \rightarrow KK$, in which they do not.) Empirically, the OZI rule fails in the 0^{-+} sector; that is how the η' mass is split away from the other pseudoscalar mesons. For the 0^{++} sector, however, there is a dearth of experimental data. Isgur and Thacker's idea to compute the quark annihilation process with lattice QCD, for various J^{PC} , which is possible at leading order in the OZI approximation even in quenched QCD [28]. They found a small amplitude where the OZI rule holds empirically, *e.g.*, for $J^{PC} = 1^{-+}$. But for scalar and pseudoscalar channels the amplitude is large. This is evidence against an instanton-dominated vacuum, because instantons couple to quarks through the axial anomaly, that is, preferentially to the pseudoscalar channel.

UNBLINDING

Instead of a paragraph of bland conclusions, the reward for readers who have made it this far consists of the most interesting numerical results. The preliminary results of JLQCD on B physics with $n_f = 2$ are [19]

$$f_{B_d} = 190(14)(7)(19) \text{ MeV}, \quad (19)$$

$$f_{B_s}/f_{B_d} = 1.184(26)(20)(15), \quad (20)$$

$$\xi = 1.183(27)(20)(15), \quad (21)$$

where ξ is a combination of their results for f_{B_s}/f_{B_d} and B_{B_s}/B_{B_d} . In her review at *Lattice '01* [30], Sinéad Ryan took stock of the ratio ξ , which enters into UT fits. Although JLQCD's result is still preliminary, it cannot be denied that the chiral logarithms could affect f_{B_s}/f_{B_d} and, hence, ξ . Ryan's average, which strikes me as reasonable, is

$$\xi = 1.15 \pm 0.06_{-0.00}^{+0.07} \quad (22)$$

which retains the central value in common usage, but allows for a future upward revision, once the chiral behavior is fully understood and controlled.

For the zero-recoil form factor needed to determine $|V_{cb}|$ from $B \rightarrow D^* l\nu$, Ref. [27] finds

$$F_{B \rightarrow D^*}(1) = 0.913_{-0.017-0.030}^{+0.024+0.017}, \quad (23)$$

with systematics added in quadrature. Note that this result does *not* include the QED correction of +0.007, which is included, for example, in the *BaBar* Physics Book. This result still relies on the quenched approximation. Nevertheless, it is probably still under better control than estimates based on the quark model.

A hallmark of these two results, shared by some work on $B \rightarrow \pi l\nu$ for $|V_{ub}|$ [24, 25], is that they attempt a full analysis of the uncertainties, including those of heavy quarks, the chiral behavior, and quenching. Thus, they are suitable templates for the next round of lattice calculations, from which one can expect serious *unquenched* calculations with a direct impact on our knowledge of the unitarity triangle.

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