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SO(10) SUSY GUT Model with a $U(1) \times Z_2 \times Z_2$ Flavor Symmetry*

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An SO(10) SUSY GUT model which leads to maximal atmospheric neutrino mixing and the LMA solar neutrino solution, developed in collaboration with S.M. Barr, is briefly described. Since the model is quantitatively predictive, it can be used to assess the need for a neutrino factory, as shown in collaboration with S. Geer.

The GUT model in question [1] is based on the grand unified group $SO(10)$ with a $U(1) \times Z_2 \times Z_2$ flavor symmetry. The model involves a minimum set of Higgs fields which solves the doublet-triplet splitting problem. This requires just one $\mathbf{45}_H$ whose VEV points in the $B-L$ direction, two pairs of $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$'s which stabilize the solution, along with several Higgs in the $\mathbf{10}_H$ representations and Higgs singlets [2]. The Higgs superpotential exhibits the $U(1)\times Z_2\times Z_2$ symmetry which is used for the flavor symmetry of the GUT model. The combination of VEVs, $\langle \mathbf{45_H}\rangle_{B-L}$, $\langle 1(\mathbf{16_H})\rangle$ and $\langle 1(\overline{\mathbf{16}}_\mathbf{H})\rangle$ break $SO(10)$ to the Standard Model. The electroweak VEVs arise from the combinations $v_u = \langle 5(\mathbf{10_H})\rangle$ and $v_d = \langle \overline{5}(\mathbf{10_H})\rangle \cos\gamma + \langle \overline{5}(\mathbf{16'_H})\rangle \sin\gamma$, while the combination orthogonal to v_d gets massive at the GUT scale. As such, Yukawa coupling unification can be achieved at the GUT scale with $\tan\beta \sim 2-55$, depending upon the $\overline{5}(\mathbf{10_H}) - \overline{5}(\mathbf{16_H})$ mixing present for the v_d VEV. In addition, matter superfields appear in the following representations: $\mathbf{16_1}$, $\mathbf{16_2}$, $\mathbf{16_3}$; $\mathbf{16}$, $\overline{\mathbf{16}}$, $\mathbf{16}'$, $\overline{\mathbf{16}}'$, $\mathbf{10_1}$, $\mathbf{10_2}$, and $\mathbf{1}$'s, where all but the $\mathbf{16_i}$ $(i = 1, 2, 3)$ get superheavy and are integrated out.

The Dirac mass matrices for the up quarks, down quarks, neutrinos and charged leptons are found to be

$$U = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} M_U, \quad D = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix} M_D,$$
$$N = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} M_U, \qquad L = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma + \epsilon & 1 \end{pmatrix} M_D. \tag{1}$$

The above textures were obtained by imposing the Georgi-Jarlskog relations [3] at Λ_{GUT}, $m_s^0 \simeq m_\mu^0/3$, $m_d^0 \simeq 3m_e^0$ with Yukawa coupling unification holding for $\tan\beta \sim 5$. The matrix element contributions can be understood in terms of Froggatt-Nielsen diagrams [4] as explained in [1].

All nine quark and charged lepton masses, plus the three CKM angles and CP phase, are well-fitted with the eight input parameters

$$\begin{aligned} M_U &\simeq 113 \text{ GeV}, & M_D &\simeq 1 \text{ GeV}, \\ \sigma &= 1.78, & \epsilon &= 0.145, \\ \delta &= 0.0086, & \delta' &= 0.0079, \\ \phi &= 126^\circ, & \eta &= 8 \times 10^{-6}, \end{aligned} \tag{2}$$

defined at the GUT scale to fit the low scale observables after evolution downward from Λ_{GUT}:

$$\begin{aligned} m_t(m_t) &= 165 \text{ GeV}, & m_\tau &= 1.777 \text{ GeV}, \\ m_u(1 \text{ GeV}) &= 4.5 \text{ MeV}, & m_\mu &= 105.7 \text{ MeV}, \\ V_{us} &= 0.220, & m_e &= 0.511 \text{ MeV}, \\ V_{cb} &= 0.0395, & \delta_{CP} &= 64^\circ. \end{aligned} \tag{3}$$

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These lead to the following predictions:

$$\begin{aligned} m_b(m_b) &= 4.25 \text{ GeV}, & m_c(m_c) &= 1.23 \text{ GeV}, \\ m_s(1 \text{ GeV}) &= 148 \text{ MeV}, & m_d(1 \text{ MeV}) &= 7.9 \text{ MeV}, \\ |V_{ub}/V_{cb}| &= 0.080, & \sin 2\beta &= 0.64. \end{aligned} \tag{4}$$

With no extra phases present, aside from the one appearing in the CKM mixing matrix, the vertex of the CKM unitary triangle occurs near the center of the presently allowed region with $\sin 2\beta \simeq 0.64$. The Hermitian matrices $U^\dagger U$, $D^\dagger D$, and $N^\dagger N$ are diagonalized with small left-handed rotations, while $L^\dagger L$ is diagonalized by a large left-handed rotation. This accounts for the small value of $V_{cb} = (U_U^\dagger U_D)_{cb}$, while $|U_{\mu 3}| = |(U_L^\dagger U_\nu)_{\mu 3}|$ will turn out to be large for any reasonable right-handed Majorana mass matrix, M_R [5].

The effective light neutrino mass matrix, M_ν, is obtained from the seesaw mechanism once the right-handed Majorana mass matrix, M_R, is specified. While the large atmospheric neutrino mixing $\nu_\mu \leftrightarrow \nu_\tau$ arises primarily from the structure of the charged lepton mass matrix, the solar and atmospheric mixings are essentially decoupled in the model, so the structure of the right-handed Majorana mass matrix determines the type of $\nu_e \leftrightarrow \nu_\mu,\ \nu_\tau$ solar neutrino mixing. Any one of the recently favored four solar neutrino mixing solutions can be obtained. The LMA solution relevant to this discussion requires some fine-tuning and a hierarchical structure, but this can also be explained in terms of Froggatt-Nielsen diagrams. The most general form for the right-handed Majorana mass matrix considered in [1] is

$$M_R = \begin{pmatrix} c^2\eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_R, \tag{5}$$

where the parameters ϵ and η are those introduced in Eq.(1) for the Dirac sector. With $a \neq b = c$, the structure of M_R arises from one Higgs singlet which induces a $\Delta L = 2$ transition and contributes to all nine matrix elements while, by virtue of its flavor charge assignment, a second Higgs singlet breaks lepton number but modifies only the 13 and 31 elements of M_R.

As a numerical example, with just three additional input parameters: $a = 1,\ b = c = 2$ and $\Lambda_R = 2.4 \times 10^{14}$ GeV, where the latter is used to scale Δm^2_{32},

$$M_\nu = \begin{pmatrix} 0 & -\epsilon & 0 \\ -\epsilon & 0 & 2\epsilon \\ 0 & 2\epsilon & 1 \end{pmatrix} M_U^2/\Lambda_R \tag{6}$$

leads to

$$\begin{aligned} & m_1 = 5.6 \times 10^{-3}, \quad m_2 = 9.8 \times 10^{-3}, && m_3 = 57 \times 10^{-3} \text{ eV}, \\ & M_1 = M_2 = 2.8 \times 10^8 \text{ GeV}, && M_3 = 2.5 \times 10^{14} \text{ GeV}, \\ & \Delta m^2_{32} = 3.2 \times 10^{-3} \text{ eV}^2, && \sin^2 2\theta_{\text{atm}} = 0.994, \\ & \Delta m^2_{21} = 6.5 \times 10^{-5} \text{ eV}^2, && \sin^2 2\theta_{\text{sol}} = 0.88, \\ & U_{e3} = -0.01395 - 0.00085i, && \sin^2 2\theta_{\text{reac}} = 0.0008, \\ & J = 2.0 \times 10^{-4}, \quad \delta_{CP} = -3.5^\circ, && \chi_1 = -0.2^\circ, \quad \chi_2 = 0.1^\circ, \end{aligned} \tag{7}$$

to be compared with the present S-K atmospheric data [6] and best-fit point in the LMA region [7]

$$\begin{aligned} \Delta m^2_{32} &\simeq 3.2 \times 10^{-3} \text{ eV}^2, & \sin^2 2\theta_{23} &= 1.0, (\geq 0.89 \text{ at } 90\% \text{ c.l.}), \\ \Delta m^2_{21} &= 7 \times 10^{-5} \text{ eV}^2, & \sin^2 2\theta_{\text{sol}} &= 0.87. \end{aligned} \tag{8}$$

In fact, the whole presently-allowed LMA region can be covered with

$$1.0 \lesssim a \lesssim 2.4, \quad 1.8 \lesssim b = c \lesssim 5.2. \tag{9}$$

The viable region of GUT model parameter space consistent with the LMA solar solution is shown in Fig. 1. Superimposed on the allowed region, Fig. 1 shows lines of constant $\sin^2 2\theta_{12}$ and contours of constant $\sin^2 2\theta_{13}$. For the fully allowed parameter space, we see that $\sin^2 2\theta_{13} < 0.006$. While a new generation of upgraded conventional neutrino beams is being considered [8], and is expected to be able to probe the region $\sin^2 2\theta_{13} > 0.003$, the "superbeams" will be able to measure the parameter θ_{13} if the solution lies in the upper part of the

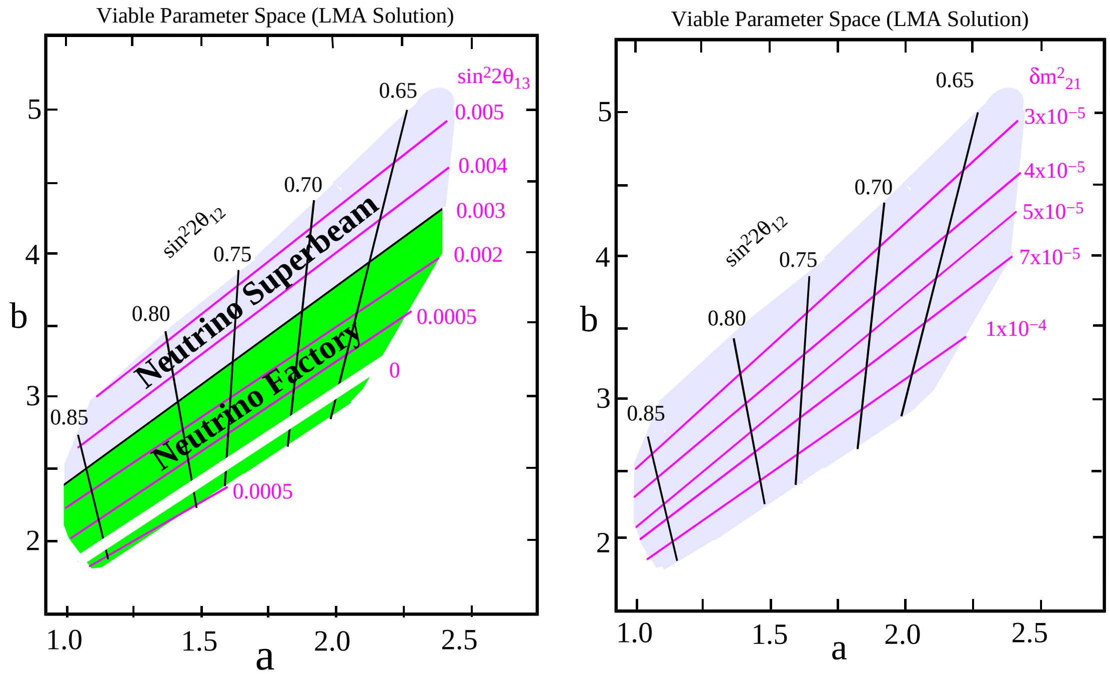

FIG. 1: The viable region of GUT parameter space consistent with the present bounds on the LMA MSW solution. Contours of constant $\sin^2 2\theta_{13}$ and lines of constant $\sin^2 2\theta_{12}$ are shown on the left. The region above $\sin^2 2\theta_{13} = 0.003$ can be explored with Neutrino Superbeams, while the region below this can be explored with Neutrino Factories, down to $\sin^2 2\theta_{13} \sim 0.0001$. Similarly, contours of constant Δm^2_{21} along with lines of constant $\sin^2 2\theta_{12}$ are shown on the right.

allowed (a, b)-plane indicated in the figure. A Neutrino Factory [9], on the other hand, is expected to be able to probe down to values of $\sin^2 2\theta_{13}$ as low as $O(10^{-4})$, which will therefore cover the entire allowed (a, b)-plane, except for a narrow band in which $\sin^2 2\theta_{13} \to 0$ as $\sin^2 2\theta_{23}$ becomes maximal.

Also in Fig. 1, contours of constant Δm^2_{21} are displayed within the viable region of parameter space consistent with the LMA solar solution. These contours are almost parallel to the contours of constant $\sin^2 2\theta_{13}$ also shown in Fig. 1. This implies a remarkable correlation between the predicted values of Δm^2_{21} and $\sin^2 2\theta_{13}$. If the LMA solution is indeed the correct solution to explain the solar neutrino deficit observations, KamLAND [10] is expected to provide measurements of Δm^2_{21} and $\sin^2 2\theta_{12}$. Hence the GUT model we are considering will be able to give a precise prediction for $\sin^2 2\theta_{13}$ once Δm^2_{21} and $\sin^2 2\theta_{12}$ are known.

A more detailed study of the correlations predicted by the model for points within the allowed LMA region has been carried out in collaboration with S. Geer [11]. An additional source of CP violation in the leptonic sector has also been studied there, whereby the two Higgs singlets breaking lepton number and leading to a and b contributions are assigned a relative phase. Although maximal atmospheric $\nu_\mu - \nu_\tau$ mixing tends to favor a small leptonic δ_{CP} phase, the phase can be as large as $|\delta_{CP}| \lesssim 50°$ in the presently allowed LMA region. We conclude that a precise test of all the model predictions for the solar and atmospheric neutrino oscillations will require a Neutrino Factory.

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