

Extra dimensions present a new flatness problem

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Abstract

There is no known fundamental reason to demand as a cosmological initial condition that the bulk possess an $SO(3, 1)$ isometry. On the contrary, one expects bulk curvature terms that violate the $SO(3, 1)$ isometry at early epochs, leading to a violation of Lorentz invariance on our brane. Demanding that the Lorentz noninvariant terms are small leads to a new “flatness” problem, not solved by the usual formulation of inflation. Furthermore, unlike in four dimensions, the Lorentz violations induced from the bulk curvature cannot always be removed as the infrared cutoff is taken arbitrarily large. This means that the equivalence principle in higher dimensions does not guarantee the equivalence principle in dimensionally reduced theories. Near-future experiments are expected to severely constrain these Lorentz-violating “signatures” of extra dimensions.

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I. INTRODUCTION

Theories with extra dimensions have long played a role in attempts to unify other forces with gravity [1]. Traditional ideas about hiding extra dimensions involved making them compact and small [2] (generally assumed to be of the order of the Planck length [3]), so that propagation of standard model matter in the extra dimensions requires energy of the inverse of the size of the extra dimensions. Thus, if the extra dimensions are small enough, they effectively decouple from the low-energy theory.

The mechanism of confining standard-model fields on $(3 + 1)$ -dimensional subspaces (3-branes, or just branes) [4] of a higher-dimensional manifold leads to the possibility of scenarios with large extra dimensions. In models where the spacetime geometry is of a simple factorizable form, the space of extra dimensions (the bulk) may be compact and perhaps as large as a millimeter [5]. If the spacetime geometry has a nonfactorizable form, the extra dimensions may be warped and noncompact as in the work of Randall and Sundrum [6]. The possibility of warped noncompact extra dimensions has extended our intuition about how extra spatial dimensions are manifest in four-dimensional effective field theories by showing that even if gravity propagates in non-compact higher dimensional spaces, four-dimensional observers may still empirically deduce a four-dimensional Newton’s law.

There has been a great deal of recent activity studying various aspects of cosmology in large extra dimension scenarios. Nonetheless, model building is still in its infancy and general features are still being uncovered. As a contribution to this effort, we examine here whether these large extra dimension scenarios possess an analog of the “flatness problem” existing in four-dimensional Friedmann–Robertson–Walker (FRW) cosmology. We find that there is most likely a higher dimensional flatness problem of character significantly different from that of the FRW flatness problem. Furthermore, unlike the flatness problem in an FRW cosmology, we will argue this problem is not easily solvable by inflation.

We do not present our analysis in the context of any concrete realistic model. As there is no unified theory that can address the question of initial conditions, our conclusions necessarily must be based on certain (plausible) assumptions. It is impossible to know if the

fundamental theory will somehow naturally circumvent the difficulty we discuss. Also, we only address the issue of large (even infinitely extending) extra dimensions. We know that if the extra dimensions are macroscopic, effective field theories will be valid to describe the spacetime behavior and the calculations should be reliable.

In the rest of the Introduction, we shall lay out the assumptions under which our arguments apply. However, we first discuss the FRW flatness problem in a way that most closely parallels the flatness problem of large extra dimensions.

One way of viewing the FRW flatness problem is as a fine-tuning problem, associated with the fact that if the equation of state obeys $\rho + 3p > 0$, *any* deviation from spatial flatness in the early universe would, in a few expansion times, cause the universe either to collapse or to expand and become negative curvature dominated. Even an initial condition of exact flatness is problematic since spatially flat patches much larger than Planck size are unnatural because the Planck scale sets the energy scale associated with quantum gravitational fluctuations early in the universe. That is, even if the universe were initially globally FRW and perfectly spatially flat, any fluctuations would have destabilized spatial flatness early on, especially when the curvature of the spacetime was large enough for quantum gravitational fluctuations to be large.

Note that even without introducing the issue of quantum gravitational fluctuations there is the question of why the universe chose zero spatial curvature over a region that is much larger than the Planck scale to such a large degree of accuracy. There seem to be two options: (i) the FRW universe was nucleated by some quantum cosmological process to have exactly zero spatial curvature, or (ii) the FRW universe arose from a Planck-size flat patch which inflated to become our universe. The first of these options is possible, but not under computational control. The second of these options, usually referred to as inflation, is under computational control. The attraction of inflation, besides computational control, is that a physical mechanism to generate a large flat patch out of a tiny flat patch simultaneously solves other cosmological fine-tuning problems. It should be noted that even if inflation did not solve these other cosmological fine-tuning problems, the solution to the flatness problem alone would be enough reason to consider it seriously since there is no other known physical

mechanism that can solve the flatness problem.

One may understand the FRW flatness problem as due to the fact that there is no symmetry that prefers flat spatial geometries, and even if the spatial geometry was spatially flat, there is no symmetry to protect the flatness. If one states the FRW flatness problem in those terms, it is easy to appreciate the problem we discuss. We will point out that Lorentz invariance on our brane requires that the *entire* spacetime possess an approximate $SO(3, 1)$ isometry (not just our brane). Since bulk curvature would lead to $SO(3, 1)$ violation observed on our brane, in the absence of some symmetry there must be some cosmological mechanism to flatten the bulk.

The relation between bulk flatness and Lorentz invariance is easier to appreciate if one notes that even in four-dimensional FRW models, the reason we observe Lorentz invariance today can be connected to the fact that our universe is flat and old. One begins to understand this by noting that the equivalence principle of general relativity, which protects Lorentz invariance of the ultraviolet (UV) limit of a field theory living on a smooth Lorentzian manifold, does not protect *infrared* (IR) physics from obtaining what appears to be Lorentz violating terms. This statement is in some sense obvious from the fact that field solutions generically may break the symmetry of the underlying spacetime (a type of spontaneous symmetry breaking). In fact, the curvature of spacetime generated in a four-dimensional FRW universe breaks the Lorentz isometry of the zero-energy vacuum spacetime. Let us see this explicitly. If the FRW metric is written as usual as

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2, \quad (1)$$

we can write the action of a free scalar field in this background as

$$S = \int d^4x a^3(t) (\partial\phi)^2 = \int d^4x [1 + 3H_0\Delta t + \dots] [(\partial_0\phi)^2 - (1 - 2H_0\Delta t + \dots)(\partial_i\phi)^2], \quad (2)$$

where H_0 is the present ($t = t_0$) expansion rate and $\Delta t \equiv t - t_0$. The different coefficients of $(\partial_0\phi)^2$ and $(\partial_i\phi)^2$ signal Lorentz violation. Clearly if the IR cutoff for the field theory is taken to be $\Lambda_{IR} \gg H_0$, then $H_0\Delta t \ll 1$, and one can work with a Lorentz invariant field theory [7]. In other words, the field theory on the tangent space of the manifold is

Lorentz invariant only if the IR cutoff is taken to be larger than H_0 . Of course in this four-dimensional FRW case this field theory is really only valid on large scales (small UV energy cutoff), since the FRW metric is only valid on large scales during most of the history of the universe.

The only reason we ever deduced Lorentz invariance in the first place is because the spacetime curvature associated with the energy density in our universe is much smaller than the energy scales associated with our physics experiments involving the standard model of particle physics: we just can't probe energy scales smaller than H_0 (a similar argument applies to planetary curvature, etc.). The reason energy scales associated with our physics experiments are small compared to the FRW spacetime curvature scale (the Hubble expansion rate H_0) is because the universe is old, i.e., the flatness problem [8]. Since inflation solves the flatness problem, we can then say that our ability to observe Lorentz invariance today has much to do with the fact that inflation made our universe flat (neglecting any unaddressable metaphysical issues such as the anthropic principle).

Extending the analogy to extra dimension scenarios is not straightforward, however, because we do not really have a standard time-dependent model of brane/bulk cosmology including its "birth" [9]. What we first establish in the next section of this paper is that departures from the $SO(3,1)$ isometry for the *large scale* higher dimensional background geometry will result in Lorentz violations for *any* field living in the bulk [10]. We will find that the nature of Lorentz violations from bulk curvature is significantly different from the nature of Lorentz violations in a four-dimensional FRW universe. Namely, we will see that in some cases, the Lorentz symmetry is never recovered as the IR cutoff is taken to be arbitrarily large, unlike in the 4-D FRW case. This means that the four-dimensional equivalence principle does not necessarily result from a higher dimensional equivalence principle. This is what we will call the "inequivalence principle." Hence, if we assume that the observed four-dimensional gravitational theory is Lorentz invariant (obeys the equivalence principle) to a high accuracy, say to scales of order H_0^{-1} , an explanation is required.

As in the 4-D FRW model, two general classes of explanations exist. One is that for some reason the quantum gravitational dynamics is driven to initiate an $SO(3,1)$ invariant

universe, and the other is that a field theoretical dynamical mechanism exists to drive the system to approximate $SO(3,1)$ invariance. The first explanation cannot be meaningfully addressed because we do not know the fundamental unified theory. Indeed, if such a mechanism existed, then that will most likely allow an alternative to inflation for solving the flatness problem. We thus take the latter approach, and make the assumption that the universe initially was in an $SO(3,1)$ isometry violating state, which is generic since there is no symmetry to protect $SO(3,1)$ isometry. (For instance, $N = 1$ SUSY is compatible only with $SO(3,2)$ and $SO(3,1)$ isometries, but is generically broken with the presence of matter energy density.) Thus we will require spacetime expansion, which we will call bulk inflation, to eliminate the $SO(3,1)$ violating terms.

We find that in the most straightforward inflationary scenarios, the warp factor does not survive bulk inflation. Furthermore, we point out that for the large, flat, compact extra dimension scenarios, inflation at the effective field theory approximation cannot be used to flatten out any significant $SO(3,1)$ violating curvature because that would force the initial compact dimension size to be too small, i.e., smaller than the fundamental Planck length. This latter point has been addressed to a certain extent in Ref. [11]. Finally, we find that $SO(3,1)$ violating curvature terms in the bulk can be experimentally constrained.

The order of presentation will be as follows. We first discuss how apparent Lorentz violations arise in effective field theories in a curved spacetime in higher dimensions. We then discuss in detail why inflation generically would be necessary in noncompact warped extra dimension scenarios. Finally, we discuss the possibility of finding evidence for extra dimensions through experimental observation of Lorentz violations and conclude.

II. THE INEQUIVALENCE PRINCIPLE

It is well known that in four dimensions, the IR limit of any effective field theory is sensitive to the background spacetime curvature, which generically induces Lorentz violations. Due to the equivalence principle, this Lorentz violation effect generically vanishes in the limit that the IR cutoff is taken to be large. However, what is not as well known (at least it is new

to us) is that for any single dimensionally reduced field, increasing the IR cutoff will not lead to the recovery of Lorentz invariance if the $SO(3,1)$ violating curvature is due to warping in the extra dimensions. Hence, $SO(3,1)$ violating curvatures from extra dimensions induce a four-dimensional theory that violates the principle of equivalence. This is what we have referred to as the inequivalence principle. Another way of stating the inequivalence principle is that the equivalence principle of a higher dimensional gravitational field theory does not necessarily guarantee that the equivalence principle will be manifest for the dimensionally reduced effective field theory.

A perhaps noteworthy observation is that it is only an accident that we, as four-dimensional low energy observers, discovered Lorentz invariance as a fundamental symmetry of nature. If the total number spacetime dimensions is four and if we had been unlucky, spacetime curvature would have prevented us from ever deducing Lorentz invariance until our experiments reached an energy level above the curvature scale. As we will detail below, the situation in the brane scenario could have been worse. The inequivalence principle tells us that if we had been unlucky to be embedded in a higher dimensional spacetime with no $SO(3,1)$ isometry, it would have been very difficult to deduce that gravitons obey Lorentz invariance, *even at short distances* (short compared to the background curvature scale). This would be true irrespective of the energy probed by experiments, even if the energy were higher than the background curvature energy scale. This loss of Lorentz invariance in the UV limit is a signature of extra dimensions; it cannot be reproduced with any purely four-dimensional background curvature.

The loss of Lorentz invariance is connected to dimensional reduction. To see this, consider a higher dimensional spacetime. Finding a four-dimensional effective field theory description of a field propagating in higher dimensions usually requires introducing an infinite number of four-dimensional fields (a “KK tower”) and integrating over the coordinates of the extra dimensions. Let us denote these four-dimensional fields by X_n . The procedure for obtaining a four-dimensional effective field theory preserves the isometries, and thus the effective field theory for the X_n is Lorentz invariant if the underlying higher dimensional spacetime possesses an $SO(3,1)$ isometry with respect to the four-dimensional coordinates of interest.

Now, what about the condition on the IR cutoff? An interesting feature of the four-dimensional effective theory for X_n is that none of the *individual* kinetic terms for X_n will recover Lorentz invariance even if one takes the IR cutoff to be large. This is because the Lorentz invariant tangent space of the higher dimensional manifold is never four dimensional! In other words, the IR cutoff scale for any single X_n (for fixed n) is constrained to be less than $1/L$ where L is the length scale of the extra dimensions, because $1/L$ is the UV cutoff scale for X_n (the field theory has been coarse-grained over length scales of $1/L$). One can describe the propagation in a five-dimensional manifold up to resolution of Λ_{UV} only when all the fields up to mass $\Lambda_{UV} > 1/L$ are taken together. However, each individual X_n never recovers Lorentz invariance.

Hence, we deduce two useful sufficient conditions for the existence of Lorentz violation in the four-dimensional effective theory of a field propagating in higher dimensions. First, there must be no $SO(3,1)$ isometry in D dimensions, and second, the coarse-graining length scale (effective IR cutoff) must be much larger than the radius of curvature R : i.e., $L \gg R$.

We now present toy models to illustrate the nature of Lorentz violating terms induced from extra dimensions. Consider a metric of the form

$$ds^2 = A(t, U^M) dt^2 - B(t, U^M) d\vec{x}^2 + G_{MN}(U^M) dU^M dU^N, \quad (3)$$

where U^M are coordinates of the bulk and the U dependence of A and B makes the global geometry nonfactorizable as in the model of Ref. [6]. Since we are interested in cosmological solutions we have assumed that A and B are independent of the spatial coordinates of the brane and only depend on t , the comoving time coordinate of our universe. Furthermore, consider the situation in which we are interested in physics for energy scales much larger than $\partial_t \ln A$ and $\partial_t \ln B$. Then in the adiabatic approximation we can neglect the time dependence of A and B and set the time variable to a particular value t_0 . Under these conditions, our generic manifold is approximately described by

$$ds^2 = A(U^M) dt^2 - B(U^M) d\vec{x}^2 + G_{MN}(U^M) dU^M dU^N, \quad (4)$$

where in general,

$$A(U) \neq B(U), \quad (5)$$

because there is no fundamental symmetry imposing (or protecting) the condition $A(U) = B(U)$. This implies that there is no $SO(3, 1)$ isometry generated by the Lorentz group acting on the coordinates (t, \vec{x}) .

Although the Randall-Sundrum metric containing a single extra dimension with coordinate u ($ds^2 = e^{-2b|u|}(dt^2 - d\vec{x}^2) - du^2$) has an $SO(3, 1)$ isometry ($A(u) = B(u)$), the question to address is why did the metric evolve to this form. Brane solutions in which the bulk is not $SO(3, 1)$ isometric were recently constructed in Ref. [12], where the metric is given by

$$ds^2 = h(u) dt^2 - u^2 d\vec{x}^2 - h^{-1}(u) du^2, \quad (6)$$

with $h(u) = -\Lambda u^2/6 - \mu/u^2$, where Λ is a cosmological constant in the bulk and μ is a free constant parameter. If $h(u) \neq u^2$, then this metric breaks the $SO(3, 1)$ isometry [13].

Now consider a simple toy model that illustrates the breaking of the $SO(3, 1)$ isometry. Assume we live in five dimensions and the extra spatial dimension has the topology of S^1/Z_2 with the radius of S^1 equal to L/π . Suppose the background stress energy is arranged to give the line element

$$ds^2 = dt^2 - e^{-2bu} d\vec{x}^2 - du^2, \quad (7)$$

where again, u is the extra dimension coordinate [14]. Note that this metric is not diffeomorphic to that written by Randall and Sundrum [6]; notably, it is not a conformally flat spacetime. It is a static version of the cosmological spacetime presented in Ref. [15].

Now suppose that a free bulk scalar field ϕ lives in the background of this spacetime. We will assume that its contribution to the vacuum energy has been fine-tuned to zero (i.e., this is not the bulk field determining the background geometry of the spacetime), and consider what this field looks like to a four-dimensional observer living on our brane. The action for this bulk scalar field is

$$\Delta S_{\text{bulk}} = \int d^5x \sqrt{g} \frac{1}{2} (\partial\phi)^2, \quad (8)$$

where $g_{\mu\nu}(u)$ is the $(4 + 1)$ -dimensional metric of Eq. (7). We can consider what a four-dimensional observer will see by expanding this scalar field in a particular orthogonal basis

$$\phi = \sum_m Y_m(\vec{x}, t) h_m(u), \quad (9)$$

where the h_m satisfies

$$\frac{1}{\sqrt{g}} \partial_u [\sqrt{g} \partial_u h_n] = -m_n^2 h_n, \quad (10)$$

and the self-adjoint derivative condition implies

$$\partial_u h_m |_{\text{brane } i} = 0. \quad (11)$$

The basis functions h_n satisfying these conditions can be written down explicitly:

$$h_{n \neq 0} = N_n e^{3bu/2} \left[\frac{2\pi n}{3bL} \cos\left(\frac{\pi n}{L}u\right) - \sin\left(\frac{\pi n}{L}u\right) \right]. \quad (12)$$

If we insist on the normalization

$$\int du e^{-3bu} h_n h_m = \delta_{nm}, \quad (13)$$

we have the normalization constant

$$N_{n \neq 0}^{-1} = \sqrt{\frac{L}{2}} \sqrt{1 + \left(\frac{2\pi n}{3bL}\right)^2}. \quad (14)$$

Putting this expansion into Eq. (8), we find the effective action ($\vec{\nabla}^2 \equiv \sum_i \partial_i^2$)

$$\begin{aligned} \Delta S_{\text{bulk}} = & \int d^4x \frac{1}{2} Y_0 \left(-\partial_0^2 + \alpha_{00} \vec{\nabla}^2 - m_0^2 \right) Y_0 \\ & + \sum_{n \neq 0} \left[\int d^4x Y_n \left(-\partial_0^2 + \alpha_{nn} \vec{\nabla}^2 - m_n^2 \right) Y_n + \sum_{m \neq n} Y_m \alpha_{mn} \vec{\nabla}^2 Y_n \right], \end{aligned} \quad (15)$$

where we have defined an infinite dimensional matrix

$$\alpha_{rn} \equiv \int du e^{-bu} h_m(u) h_n(u) \neq \delta_{rn} \quad (16)$$

which characterizes the Lorentz noncovariant structure of the theory. Note that for a fixed index r and a given field Y_r , only the off-diagonal components of α_{rn} seem to be responsible for the Lorentz non-covariant structure, because for the diagonal component we can always rescale the coordinates to obtain the usual covariant form. However, because the standard model fields confined to the brane reveals the “true geometry” of the underlying spacetime, it is not true that the field redefinition completely hides the apparent Lorentz violation even for a given diagonal sector. For example, if $r = 0$ we can take $x \rightarrow x \sqrt{\alpha_{00}}$ in the first line of Eq. (15) to obtain

$$S_0^{\text{scaled}} = \int d^4x \alpha_{00}^{3/2} \frac{1}{2} Y_0 \left(-\partial_0^2 + \vec{\nabla}^2 - m_0^2 \right) Y_0, \quad (17)$$

apparently recovering Lorentz invariance for what we will call the 00 sector.

It is manifest that two points that are a distance d apart (as measured by the standard model physics of fields confined to our brane) are seen to be only a distance $d/\sqrt{\alpha_{00}}$ apart from the point of view of the effective four-dimensional field, which actually lives in higher dimensions. Of course there is no global coordinate transformation that one can make to have all diagonal nn sectors Lorentz covariant simultaneously. Hence, at least with the off-diagonal terms neglected, each field Y_n lives in a different apparent geometry; i.e., the effective distance that each field sees through its propagator is different even though the “true” distance in spacetime as measured by the standard model fields confined to the brane is the same. The distance $d/\sqrt{\alpha_{00}}$ is what an observer would deduce from an “inverse-square law” analysis, and hence we will refer to it as the “inverse-square distance.” The ratio of the inverse-square distance to the distance measured by fields confined to the brane is in this case just $1/\sqrt{\alpha_{00}}$. The fact that the ratio is not unity is nothing more than a consequence of the fact that the lightcone in the extra-dimensional spacetime is different from the lightcone of a field confined to the brane, as was discussed in Ref. [15]. In other words, causal signals can take a shortcut through the extra dimensions to get to a point on the brane that is farther than where a causal signal confined to the brane can go for a fixed time. From a $(3+1)$ -dimensional point of view, the higher dimensional signals seem acausal.

It is important to note that this noncovariant structure is independent of the basis chosen, and there is no coordinate redefinition nor field redefinition that will truly restore the Lorentz symmetry. Even more importantly, since the Lorentz violation structure is governed by the quantity bL (which is independent of Λ_{IR} , the IR cutoff of the $3+1$ dimensional momenta), increasing Λ_{IR} does not lead to the recovery of Lorentz invariance for any one field Y_n . Hence, the inequivalence principle is manifest. As argued before, the fact that $\alpha_{mn} \neq \delta_{mn}$ is a result of the fact that the underlying higher dimensional spacetime does not possess an $SO(3,1)$ isometry. Mathematically, this merely amounts to the fact that the partial differential equations governing the modes are not separable in the chosen coordinate directions.

Let us now examine the magnitude of these effects. The magnitude of the Lorentz-violating effects is characterized by the coefficients α_{ij} . First note that in this model the

Lorentz violating effects associated with the zero mode are not very large, because $1 \leq \alpha_{00} \leq 3$. In particular, the distances are only scaled by $d/\sqrt{\alpha_{00}}$. However, since the scalar field propagators behave approximately as the graviton propagator for Newton's law, one can see that the "inverse-square" distance vs. luminosity distance comparisons can discriminate such scalings [16]. We leave a more careful analysis of the observables to a future study. However, in the last section, we will explicitly show one possible experimental observable which is within the reach of upcoming gravitational experiments.

The effects for the higher mass mode can be extremely large, even if L^{-1} is much larger than electroweak energy scale. For instance, if $e^{bL} \gg 1$, we have

$$\alpha_{11} \sim e^{2bL} \frac{29\pi^2}{18b^3 L^3}, \quad (18)$$

which will be much larger than unity. Moreover, it is not always possible to treat the mixing of the zero-mode mass eigenstates with massive KK mass eigenstates as a perturbation because the mixing with massive modes can be equally large if e^{bL} is large, as can be seen by

$$\alpha_{01} \sim e^{bL/2} \frac{16\pi}{(bL)^{2/3}} \sqrt{\frac{2}{3}}. \quad (19)$$

In general, the zero mode truncation of the effective field theory in the $bL \gg 1$ limit is not valid because α_{mn} is much greater than unity if bL is much greater than unity, and the field theory must be considered from a higher dimensional point of view. This may be true even though the nonzero modes (the zero mode is massless) can be quite massive since L^{-1} may need to be large enough to hide the higher dimensional behavior of gravity. Explicitly, the mass spectrum of the nonzero modes is given by

$$m_{n \neq 0} = \frac{1}{2} \sqrt{9b^2 + \left(\frac{2\pi n}{L}\right)^2}, \quad (20)$$

which would naively justify decoupling if b or L^{-1} were sufficiently large. However, here in general, the mass eigenstates will not be momentum eigenstates, and there does not seem to be decoupling. What is clear, however, is that if we insist on a four-dimensional point of view, we have a theory in which the field labeled by different masses see a different effective geometry, i.e., the inverse-square distances corresponding to the same spacetime geometry distance are different for different four-dimensional effective fields.

Although the exact nature of the Lorentz violating effects characterized by α_{mn} is model dependent, its magnitude can be read off from the metric of the form given in Eq. (3). It is easy to show that in general whether α_{mn} is greater than or less than unity is roughly governed by the ratio

$$\alpha_{mn} \sim \frac{\langle B \rangle}{\langle A \rangle}, \quad (21)$$

where $\langle \dots \rangle$ denotes an average over the extra dimensions. Hence, for spacetimes with $\langle B \rangle / \langle A \rangle > 1$ we have an “acausal” effective theory while for $\langle B \rangle / \langle A \rangle < 1$, we have merely the momentum nonconserving Lorentz violating effects with respect to the off-diagonal terms of α_{mn} .

Note that the existence of Lorentz violation generalizes to higher spin fields. Consider the graviton field $h_{\mu\nu}$, which is defined to be the zero mode of the metric perturbation

$$ds^2 = A \left[(1 + h_{00}) dt^2 - \left(\frac{B}{A} \delta_{ij} - h_{ij} \right) dx^i dx^j \right] + G_{MN} dU^M dU^N, \quad (22)$$

where $h_{\mu\nu}$ only depends on $(3 + 1)$ -dimensional coordinates. The graviton kinetic term will contain a term

$$S \ni \int d^n x \sqrt{AB^3} \sqrt{G} \frac{1}{4} \left[A^{-1} (\partial_0 h^{\mu\nu} \partial_0 h_{\mu\nu}) - B^{-1} (\vec{\nabla} h^{\mu\nu} \cdot \vec{\nabla} h_{\mu\nu}) \right], \quad (23)$$

which is again Lorentz violating when integrated over the extra dimension coordinates U . Again, the metric implied by measurements of the brane-confined fields will be different from the constant metric obtained after integrating over U in Eq. (23).

III. A NEW FLATNESS PROBLEM

Since we have little control over the effective field theory in the context in which the extra dimensions are compactified to be Planck size, we will not discuss that scenario here. However, in the case in which the extra dimensions are large and flat [5] or in the case in which the extra dimensions are warped and noncompact [6], we can ask in the context of an effective field theory whether there may be a flatness problem as outlined in the Introduction. As we shall argue, in these large extra dimension scenarios there are additional flatness

TABLE I: The analogy between the FRW flatness problem and the extra-dimension flatness problem.

4D FRW	Extra dimensions
no reason for initial spatial flatness	no reason for initial $SO(3, 1)$ isometry
spatial curvature \rightarrow dynamical instability	?
observation: old age of the universe	observation: approximate $SO(3, 1)$ isometry

problem complications that did not arise in the four-dimensional FRW cosmology. In what follows, we shall first identify the flatness problem in the warped extra dimensions scenario and then discuss the case of the large extra dimension scenario.

As discussed more fully in the previous section, a higher dimensional spacetime described by Eqs. (4) and (5) implies that any dimensionally reduced effective theory, including gravity, will generically violate Lorentz invariance. In the view of treating gravity as a theory of vielbeins in Minkowski spacetime, this means that the VEV of the vielbeins spontaneously break Lorentz invariance. Hence, in warped extra dimensions scenarios, it is crucial to explain why there is an $SO(3, 1)$ isometry in the extra dimensions to an accuracy that allows four-dimensional gravity (or any other dimensionally reduced field) to be Lorentz invariant [17]. Hence, in analogy to the FRW flatness problem, if we assume that the bulk fields dimensionally reduced to four dimensions are observed to obey Lorentz invariance (as the IR cutoff is taken to be arbitrarily large), we have an extra-dimensional flatness problem. This analogy is summarized in Table 1.

If a typical initial condition of early cosmology contains $SO(3, 1)$ isometry violating curvature, the extra dimensional flatness problem is real and inflation may be required to eliminate it. Note that here we are making a crucial assumption that there is no fundamental reason (such as fundamental symmetry arguments or dynamical arguments) to choose an $SO(3, 1)$ isometric manifold as the initial condition. Indeed, if there were such a mechanism, then one may be able to modify it and utilize it to replace inflation altogether.

Given that we use inflation to solve the FRW flatness problem, let's see what normal inflation would do to solve the bulk flatness problem. Let's extend the toy model of the

previous section by allowing a brane scale factor $a(t)$:

$$ds^2 = dt^2 - a^2(t) e^{-2bu} d\vec{x}^2 - du^2, \quad (24)$$

where again, u is the extra dimension coordinate and our flat brane is located at $u = 0$. It is easy to see that no matter how much we inflate our brane by arranging $a(t)$ to grow exponentially, we will not recover the $SO(3,1)$ isometry.

One possible resolution to this problem is to inflate the u dimension. The difficulty lies in inflating the extra dimensions to smooth out the $SO(3,1)$ invariance violating curvature while preserving the large warping. To see this, introduce a bulk scale factor $c(t)$ for the extra dimension. The toy metric is then

$$ds^2 = dt^2 - a^2(t) e^{-2bu} d\vec{x}^2 - c^2(t) du^2. \quad (25)$$

Inflating $c(t)$ (“bulk inflation”) will render the curvature set by b harmless. One can see this by making the coordinate transformation $\tilde{u} = cu$, in which case the metric becomes

$$ds^2 = \left[1 - \left(\frac{\dot{c}}{c} \right)^2 \tilde{u}^2 \right] dt^2 - a^2(t) e^{-2b\tilde{u}/c} d\vec{x}^2 - d\tilde{u}^2 + 2 \left(\frac{\dot{c}}{c} \right) \tilde{u} d\tilde{u} dt. \quad (26)$$

Now imagine c inflates by large amount, after which $\dot{c}/c \rightarrow 0$ (or at least $\dot{c}/c < \dot{a}/a$). Then the factor b/c , which sets the spatial curvature scale in the bulk, can be made arbitrarily small while still recovering $SO(3,1)$ isometry. Of course the price one pays here is that inflation of the bulk has inflated away the warp factor!

An obvious loophole in the argument thus far is the possibility of a hierarchy between the Lorentz violating curvature and the warp factor curvature. Then one may be able to inflate away the Lorentz violating curvature without erasing the warp factor. For example, if one complicates the toy metric a bit further and takes it to be

$$ds^2 = e^{-2ku} \left[dt^2 - a^2(t) e^{-2bu} d\vec{x}^2 \right] - c^2(t) du^2 \quad (27)$$

with $k \gg b$, then bulk inflation may dilute away the curvature due to b while maintaining the warp factor from k . The challenge then is to come up with a physical scenario with a large hierarchy between k and b . Of course, there may be other solutions that involve

the bulk inflating and then shrinking in such a way that the $SO(3,1)$ violating curvature is removed, but the main point still stands: inflationary model building has a new problem to solve.

Finally, consider the large compact extra dimension scenarios [5] with r extra spatial dimensions. Imagine that an $SO(3,1)$ violating metric of the form Eq. (25) has been inflated as in Eq. (26) to end with an acceptably flat, compact extra dimension. Let us parameterize such a metric as

$$ds^2 = dt^2 - a^2(t)e^{-2bu}d\vec{x}^2 - c^2(t)\delta_{MN}dU^M dU^N \quad (28)$$

where U^M correspond to the compact extra dimensions where M runs through 4 to $3+r$. If the extra dimensions described by the coordinates U are compact, then inflation through $c(t)$ poses the danger of the compact dimensions being initially too small to be described by an effective field theory. More specifically, let us define the total expansion of the extra dimensions between some initial time t_i and today (t_0) as

$$E = \frac{c(t_0)}{c(t_i)} = \frac{b}{H_0}, \quad (29)$$

where the second equality is required by the requirement that the curvature not violate $SO(3,1)$ invariance out to scale of H_0^{-1} . Suppose the size of the extra dimensions is initially l_i . After inflation, the size of the extra dimensions is $l_0 = l_i E$. Then since the four-dimensional Planck scale requires $l_0^r M^{2+r} \approx M_{Pl}^2$ where M is the r -dimensional Planck scale, we find

$$\left(\frac{M^2 b}{M_{Pl}^2 H_0} \right)^{1/r} M = \frac{1}{l_i}. \quad (30)$$

If we require an effective field theory description to be valid by imposing $l_i^{-1} \leq M$, we find

$$b \leq H_0 \left(\frac{M_{Pl}}{M} \right)^2. \quad (31)$$

This means that if we require that the fundamental Planck scale M to be $M \geq 1$ TeV, we find that the curvature in the bulk before bulk inflation can be only very tiny, $b \leq 10^{-1}$ eV. Hence, we find that for large *compact* extra dimensions, only very tiny curvatures can be smoothed out by inflation. This suggests that a non-quantum-gravitational theory of inflation cannot smooth away the bulk curvature, and most likely some fundamental symmetry must play a

role in initially setting the $SO(3,1)$ violating curvature to zero. A related discussion can be found in Ref. [11].

Let us reiterate the main point of this section which is the main result of this paper. For *noncompact* warped extra dimension scenarios, one must invoke a mechanism such as inflation to smooth out the $SO(3,1)$ isometry violating curvature to obtain a Lorentz invariant effective field theory. However, the difficulty with this resolution is that in such an inflationary scenario, the bulk warp factor that one needs to localize gravity may be inflated away altogether. Hence, the new flatness problem is to inflate away the $SO(3,1)$ violating curvature selectively while preserving a large warp factor. In the case of large compact extra dimensions, we find that inflation at the level of an effective field theory generically cannot make the bulk flat.

IV. CONCLUSION

We have shown that curvature in the bulk leads to an apparent breaking of Lorentz symmetry with respect to an observer living on the brane observing a field propagating in higher dimensions. Unlike the $SO(3,1)$ violation in the four-dimensional world due to curvature, Lorentz violation in theories with fields propagating in higher dimensions persists as long as the spacetime does not possess an $SO(3,1)$ isometry, no matter how large of an energy the four-dimensional effective field theory is probed. This is in contrast with what is dictated by the equivalence principle in four dimensions. We called this apparent UV-limit-persisting violation of four-dimensional equivalence principle for dimensionally reduced theories the “inequivalence principle.” Note that Lorentz violation for the dimensionally reduced theory is true even when the brane has an $SO(3,1)$ isometry. Furthermore, the mismatch between the brane isometry group and the full spacetime isometry group results in an ambiguous geometrical picture from a four-dimensional empirical point of view regardless of the symmetry group.

This implies a new flatness problem for cosmological scenarios having large (possibly noncompact) extra dimensions. For warped noncompact extra dimensions scenarios, the

problem is to come up with a mechanism to flatten the $SO(3, 1)$ violating wrinkles while preserving the large warp factor necessary for graviton trapping. Inflation generically helps to smooth away the wrinkle, but it also eliminates the warp factor. For flat compact large extra dimensions scenarios, we demonstrated that inflation at the effective field theory level is not sufficient to smooth away any significant curvature in the bulk.

It is tempting to speculate that the first signatures for large extra dimensions may come from deducing the existence of Lorentz violations in the early universe. This would be possible only if the observable anomalies in the early universe arising from Lorentz violations are sufficiently distinct from the anomalies arising from other effects. Indeed, since in most popular scenarios only gravity and other extremely weakly interacting (with the SM fields) fields propagate in the bulk, it may be difficult to observe the Lorentz violations with respect to the zero modes of these fields unless the violation is extreme.

One Lorentz violating observable characteristic of the inequivalence principle is the wavelength independent deviation of signal propagation speed. For example, for the free scalar field of Eq. (8) propagating in a five-dimensional background of Eq. (7), one can solve the wave equation perturbatively, perturbing with the parameter bL where b characterized the Lorentz violating curvature and L is the characteristic size of the extra dimension. One can write one of the modes as

$$\phi \sim \frac{f(u)}{a} e^{-iE_k t} e^{i\vec{k}\cdot\vec{x}}, \quad (32)$$

where

$$f = c_0 \left[1 + b\vec{k}^2 \left(\frac{u^3}{3} - \frac{Lu^2}{2} \right) \right], \quad (33)$$

and

$$E_k = \left(1 + \frac{1}{2}bL \right) |\vec{k}|. \quad (34)$$

This implies that the group velocity is

$$\frac{\partial E_k}{\partial |\vec{k}|} = 1 + \frac{bL}{2}, \quad (35)$$

in agreement with Eq. (15) since there we have

$$\frac{\partial E_k}{\partial |\vec{k}|} = \sqrt{\alpha_{00}}. \quad (36)$$

As we have argued before, the propagation speed of gravitational waves will be similar. Hence, we conclude that one may be able to constrain the bulk-curvature violating $SO(3, 1)$ isometry by comparing the gravitational wave arrival time and the light arrival time.

Imagine measuring the time correlation of the arrival of the gravitational wave and light pulses from a gamma-ray burst. Taking the gamma-ray burst to be at a distance $D = 1000$ Mpc $\sim 10^{17}$ s, and assuming a resolution for the arrival times of the pulses of $\Delta t = 1$ s, we would be able to constrain the Lorentz violating curvature to be smaller than

$$\frac{bL}{2} < \frac{\Delta t}{D} = 10^{-17} \quad (37)$$

if no time lag is detected.

In conclusion, any realistic cosmological model with extra dimensions must account for a mechanism to generate approximate Lorentz symmetry for fields in the bulk, if the dimensionally reduced bulk field such as the graviton can be shown to be approximately Lorentz invariant. If we are lucky, perhaps nature will give us a clue regarding the extra dimensions through tests of Lorentz violations in the graviton sector and any other sector that may live in extra dimensions.

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