

**Reanalysis of CP Violation in $K_L \rightarrow \pi^+ \pi^- \gamma$** **Jusak Tandean^{a*} and G. Valencia^{a,b†}**^a*Department of Physics and Astronomy, Iowa State University, Ames, IA 50011*^b*Fermi National Accelerator Laboratory, P.O.Box 500, Batavia, IL 60510***Abstract**

We present a reanalysis of direct CP violation in the decay $K_L \rightarrow \pi^+ \pi^- \gamma$. We point out an existing discrepancy between the theoretical and experimental definitions of $\epsilon'_{+-\gamma}$. Adopting the experimental definition of $\epsilon'_{+-\gamma}$, we estimate that $\epsilon'_{+-\gamma}/\epsilon$ could be as large as a few times 10^{-4} both within the standard model and beyond. We discuss these estimates in detail and we also show how a judicious choice of E_γ^* cuts can increase the sensitivity of the observable $\epsilon'_{+-\gamma}$ to the underlying CP violation.

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1 Introduction

The origin of CP violation remains an unsolved problem in particle physics. An important piece of this puzzle has been provided recently by the KTeV and NA48 collaborations [1] that experimentally confirmed the presence of direct CP violation in the $K \rightarrow \pi\pi$ neutral-kaon decays. Interestingly, the measured results were larger than most existing standard model estimates at the time. The difference could be attributed to new physics, although updated analyses of theoretical uncertainties indicate that there is no serious disagreement with the standard model [2]. This situation underscores the importance of observing CP violation in other reactions.

The decays $K_{L,S} \rightarrow \pi^+\pi^-\gamma$ have long been recognized [3, 4, 5, 6, 7] for their potential to test CP violation. The amplitude for each of these decays is conventionally divided into two contributions: inner bremsstrahlung (IB) and direct emission (DE). The IB is completely determined by the $K \rightarrow \pi^+\pi^-$ process that underlies it, whereas the DE part encodes additional dynamical features. The $K_S \rightarrow \pi^+\pi^-\gamma$ decay is known to be dominated by the IB, but in the K_L decay the IB is suppressed because the underlying $K_L \rightarrow \pi^+\pi^-$ process is CP -violating. This makes the more interesting DE term in the K_L decay more accessible, thereby raising the possibility of observing new direct CP violation in this mode.

The presence of CP violation in $K_L \rightarrow \pi^+\pi^-\gamma$ has been observed in recent experiments [8, 9], and this is characterized by the parameter $\eta_{+-\gamma}$, defined in analogy to the parameter η_{+-} in the $K \rightarrow \pi^+\pi^-$ case. However, the result has not yet reached the level of sensitivity needed for detecting direct CP violation in $K_L \rightarrow \pi^+\pi^-\gamma$. On the theoretical side, studies on direct CP violation in this decay have been performed by various authors [5, 7, 10, 11, 12]. In this paper, we revisit this subject for the following reasons. First, the existent experimental analysis of the CP -odd observable $\eta_{+-\gamma}$ is not consistent with the theoretical definitions because it assumes that this quantity is a constant over phase space. Second, KTeV will perform a new analysis of this mode with their recent data and it is therefore timely to update the theoretical expectations both within and beyond the standard model. Finally, we examine the possibility of measuring new, direct, CP violation in this mode from an analysis of the decay distribution [13] without $K_S - K_L$ interference.

In the following section we introduce our notation for the relevant decay amplitudes and the CP -violating parameters. In particular, we introduce the parameter, $\hat{\epsilon}$, to characterize the *new* direct CP violation in these modes. In Section 3, we relate these parameters to CP -violating observables occurring in the interference between the amplitudes for K_L and K_S decays into $\pi^+\pi^-\gamma$. By carefully treating the energy dependence in the amplitudes, we derive a relation between $\hat{\epsilon}$, and the experimental observable $\epsilon'_{+-\gamma}$. In this manner we extract the current limits on $\hat{\epsilon}$. In Section 4, we consider detecting direct CP violation in the interference between the inner bremsstrahlung and direct emission in the electric amplitude of $K_L \rightarrow \pi^+\pi^-\gamma$. Using a recent KTeV result, we extract

a bound on $\hat{\epsilon}$. In Section 5 we present an estimate for $\hat{\epsilon}$ in the standard model. Finally, in Section 6, we consider potentially large new physics contributions to $\hat{\epsilon}$ that may arise in left-right symmetric models [14] and in generic supersymmetric models [15].

2 Amplitudes and Direct CP -Violating Parameters

The amplitudes for $K^0 \rightarrow \pi^+\pi^-, \pi^0\pi^0$ are conventionally written as

$$\mathcal{M}_{K^0 \rightarrow \pi^+\pi^-} = A_0 e^{i\delta_0^0} + \frac{1}{\sqrt{2}} A_2 e^{i\delta_2^0}, \quad \mathcal{M}_{K^0 \rightarrow \pi^0\pi^0} = A_0 e^{i\delta_0^0} - \sqrt{2} A_2 e^{i\delta_2^0}, \quad (1)$$

where A_I is the component for the $\pi\pi$ state with isospin I , and δ_J^I is the strong-rescattering phase for the $\pi\pi$ state with angular momentum J . From $K \rightarrow \pi\pi$ data, one can extract in the isospin limit $\text{Re } A_0 \simeq 2.72 \times 10^{-7} \text{ GeV}$ and $\omega \equiv \text{Re } A_2 / \text{Re } A_0 \simeq 1/22.2$ [16].¹ The physical states K_L and K_S are given by

$$|K_L\rangle = \frac{(1 + \bar{\epsilon})|K^0\rangle + (1 - \bar{\epsilon})|\bar{K}^0\rangle}{\sqrt{2 + 2|\bar{\epsilon}|^2}}, \quad |K_S\rangle = \frac{(1 + \bar{\epsilon})|K^0\rangle - (1 - \bar{\epsilon})|\bar{K}^0\rangle}{\sqrt{2 + 2|\bar{\epsilon}|^2}}, \quad (2)$$

where $\bar{\epsilon}$ corresponds to CP violation in the kaon-mass matrix [18] and we use the convention $CP|K^0\rangle = -|\bar{K}^0\rangle$. The CP -violating parameters in $K \rightarrow \pi\pi$ decays are

$$\epsilon = \bar{\epsilon} + \frac{i \text{Im } A_0}{\text{Re } A_0}, \quad \epsilon' = \frac{\omega}{\sqrt{2}} \left(\frac{\text{Im } A_2}{\text{Re } A_2} - \frac{\text{Im } A_0}{\text{Re } A_0} \right) e^{i(\delta_2^0 - \delta_0^0 + \pi/2)}, \quad (3)$$

corresponding to indirect and direct CP violation, respectively. It is also conventional to define the ratio of amplitudes

$$\eta_{+-} \equiv \frac{\mathcal{M}_{K_L \rightarrow \pi^+\pi^-}}{\mathcal{M}_{K_S \rightarrow \pi^+\pi^-}} = \epsilon + \epsilon'. \quad (4)$$

Experimentally it is found that $|\eta_{+-}| = (2.285 \pm 0.019) \times 10^{-3}$ and that its phase is $\phi_{+-} = 43.5^\circ \pm 0.6^\circ$ [16]. The recent measurements of direct CP violation [1] lead to a new world average [19] $\epsilon'/\epsilon = (19.3 \pm 2.4) \times 10^{-4}$.

In the $K \rightarrow \pi\pi\gamma$ decay, the amplitude is generally decomposed into electric and magnetic terms. The electric part E receives contributions from both inner-bremsstrahlung and direct-emission processes, whereas the magnetic part M arises exclusively from the direct-emission. Our notation for the amplitude for $K \rightarrow \pi^+(p_+) \pi^-(p_-) \gamma(q)$, where K is any neutral kaon (K^0, \bar{K}^0, K_L, K_S), is

$$\begin{aligned} \mathcal{M}_{K \rightarrow \pi^+\pi^-\gamma} &= [E_{\text{IB}}(K) + E_{\text{DE}}(K)] \frac{\varepsilon_\mu^* [(p_+ + p_-)^\mu \nu - (p_+ - p_-)^\mu z]}{m_K} \\ &+ M(K) \frac{4 \epsilon_{\lambda\mu\rho\sigma} \varepsilon^{\lambda*} p_+^\mu p_-^\rho q^\sigma}{m_K^3}, \end{aligned} \quad (5)$$

¹Isospin violation significantly complicates this analysis [17].

with the kinematic variables

$$\nu = \frac{2k \cdot (p_+ - p_-)}{m_K^2}, \quad z = \frac{2k \cdot q}{m_K^2}. \quad (6)$$

In the kaon rest-frame, $\nu = 2(E_{\pi^+} - E_{\pi^-})/m_K$ and $z = 2E_\gamma^*/m_K$, with E_γ^* being the usual notation for the photon energy in this frame.

For $K^0 \rightarrow \pi^+\pi^-\gamma$, the IB term,

$$E_{\text{IB}}(K^0) = \frac{4e \mathcal{M}_{K^0 \rightarrow \pi^+\pi^-}}{m_K (z^2 - \nu^2)}, \quad (7)$$

is completely determined [20] by the amplitude for the underlying nonradiative decay $K^0 \rightarrow \pi^+\pi^-$. Experimentally, it is separated by fitting the characteristic bremsstrahlung spectrum, which behaves as $d\Gamma_{\text{IB}}/dE_\gamma^* \sim 1/E_\gamma^*$ as $E_\gamma^* \rightarrow 0$. Following Ref. [6], we write, guided by dimensional analysis, the DE terms

$$E_{\text{DE}}(K^0) = e |G_8| f_\pi^2 \xi_E(\nu, z), \quad M(K^0) = e |G_8| f_\pi^2 i\xi_M(\nu, z), \quad (8)$$

so that the dimensionless form-factor ξ_E (ξ_M) in the electric (magnetic) amplitude is expected to be of order one. Also, $f_\pi \simeq 92.4 \text{ MeV}$ is the pion-decay constant and G_8 is defined by

$$A_0 = \sqrt{2} G_8 f_\pi (m_K^2 - m_\pi^2). \quad (9)$$

The corresponding amplitudes for $\bar{K}^0 \rightarrow \pi^+\pi^-\gamma$ are obtained by requiring CPT invariance, and they are

$$E_{\text{IB}}(\bar{K}^0) = \frac{-4e \mathcal{M}_{K^0 \rightarrow \pi^+\pi^-}^*}{m_K (z^2 - \nu^2)}, \quad (10)$$

$$E_{\text{DE}}(\bar{K}^0) = -e |G_8| f_\pi^2 \xi_E^*(-\nu, z), \quad M(\bar{K}^0) = +e |G_8| f_\pi^2 i\xi_M^*(-\nu, z),$$

where in $\mathcal{M}_{K \rightarrow \pi\pi}^*$ and $\xi_{E,M}^*$ the complex conjugation refers only to the weak CP -violating phases and not to the strong final-state interaction phases.

The form factors $\xi_{E,M}$ can be expressed in a multipole expansion [4]. There is almost no experimental information on the electric form-factor, as it generates small corrections to the amplitudes in both K_S and K_L decays. For this reason, it will be sufficient for us to assume that ξ_E is saturated by the leading multipole, E1, and use

$$\xi_E(\nu, z) = F_E e^{i\delta_1^1}, \quad (11)$$

where F_E is a dimensionless complex constant, expected to be of order 1 by dimensional analysis. The final state interaction phase is δ_1^1 , reflecting the fact that in this amplitude the two pions are

in an $I = J = 1$ state. The magnetic form-factor, on the other hand, has been experimentally studied in some detail [21, 22] and found to depend mostly on z . For the remainder of this paper, we will take

$$\xi_M(\nu, z) = \xi_M(z) e^{i\delta_1^1}, \quad (12)$$

corresponding to the leading multipole, M1, with an E_γ^* dependent form-factor. In what follows we neglect any $\Delta I = 3/2$ contribution to $\xi_{E,M}$.

For the physical kaon states K_S and K_L the IB amplitudes are given by

$$E_{\text{IB}}(K_S) = \frac{4e \mathcal{M}_{K_S \rightarrow \pi^+ \pi^-}}{m_K (z^2 - \nu^2)}, \quad E_{\text{IB}}(K_L) = \eta_{+-} E_{\text{IB}}(K_S). \quad (13)$$

The electric DE amplitudes for $K_{L,S} \rightarrow \pi^+ \pi^- \gamma$ are

$$E_{\text{DE}}(K_{L,S}) = e |G_8| f_\pi^2 \xi_E^{L,S}, \quad (14)$$

where

$$\xi_E^L = \sqrt{2} \text{Re } F_E \left[\epsilon + i \left(\frac{\text{Im } F_E}{\text{Re } F_E} - \frac{\text{Im } A_0}{\text{Re } A_0} \right) \right] e^{i\delta_1^1}, \quad (15)$$

$$\xi_E^S = \sqrt{2} \text{Re } F_E e^{i\delta_1^1}. \quad (16)$$

We have, as usual, dropped terms quadratic in weak phases. Similarly, for the magnetic amplitudes we have

$$M(K_{L,S}) = e |G_8| f_\pi^2 i \xi_M^{L,S}(z), \quad (17)$$

where

$$\xi_M^L = \sqrt{2} \text{Re } \xi_M e^{i\delta_1^1} \quad (18)$$

$$\xi_M^S = \sqrt{2} \text{Re } \xi_M \left[\epsilon + i \left(\frac{\text{Im } \xi_M}{\text{Re } \xi_M} - \frac{\text{Im } A_0}{\text{Re } A_0} \right) \right] e^{i\delta_1^1}. \quad (19)$$

It is then possible to define two CP -violating quantities associated with the electric and magnetic amplitudes. For the electric amplitude, one has the usual ratio [5, 7]²

$$\tilde{\eta}_{+-\gamma} \equiv \frac{E_{\text{IB}}(K_L) + E_{\text{DE}}(K_L)}{E_{\text{IB}}(K_S) + E_{\text{DE}}(K_S)} = \frac{\eta_{+-} + \frac{|G_8| f_\pi^2 m_K}{4 \text{Re } A_0} \frac{\xi_E^L}{\sqrt{2}} (z^2 - \nu^2) e^{-i\delta_0^0}}{1 + \frac{|G_8| f_\pi^2 m_K}{4 \text{Re } A_0} \frac{\xi_E^S}{\sqrt{2}} (z^2 - \nu^2) e^{-i\delta_0^0}}. \quad (20)$$

²Notice that we call it $\tilde{\eta}_{+-\gamma}$ instead of $\eta_{+-\gamma}$ to differentiate it from the experimental observable. This is an important distinction that has not been appreciated in the literature.

Noting that

$$\frac{|G_8| f_\pi^2 m_K}{4 A_0} \simeq \frac{f_\pi}{4\sqrt{2} m_K} \sim 0.03 \quad (21)$$

and that $z^2 - \nu^2$ is always less than 1, we write

$$\begin{aligned} \tilde{\eta}_{+-\gamma} &\simeq \eta_{+-} + \frac{|G_8| f_\pi^2 m_K (\xi_E^L - \eta_{+-} \xi_E^S)}{4 \operatorname{Re} A_0 \sqrt{2}} (z^2 - \nu^2) e^{-i\delta_0^0} \\ &\simeq \eta_{+-} + \frac{|G_8| f_\pi^2 m_K \operatorname{Re} F_E}{4 \operatorname{Re} A_0} i \left(\frac{\operatorname{Im} F_E}{\operatorname{Re} F_E} - \frac{\operatorname{Im} A_0}{\operatorname{Re} A_0} \right) (z^2 - \nu^2) e^{i(\delta_1^1 - \delta_0^0)}. \end{aligned} \quad (22)$$

It is, therefore, convenient to define the parameter

$$\hat{\epsilon} \equiv \frac{|G_8| f_\pi^2 m_K \operatorname{Re} F_E}{4 \operatorname{Re} A_0} \left(\frac{\operatorname{Im} F_E}{\operatorname{Re} F_E} - \frac{\operatorname{Im} A_0}{\operatorname{Re} A_0} \right) \quad (23)$$

to characterize *new* direct CP violation in $K \rightarrow \pi^+ \pi^- \gamma$. Thus, we have

$$\tilde{\epsilon}'_{+-\gamma} \equiv \tilde{\eta}_{+-\gamma} - \eta_{+-} = \hat{\epsilon} e^{i\delta_\epsilon} (z^2 - \nu^2), \quad \delta_\epsilon \equiv \delta_1^1 - \delta_0^0 + \pi/2. \quad (24)$$

The quantity $\tilde{\epsilon}'_{+-\gamma}$ varies across the Dalitz plot, and for this reason we prefer $\hat{\epsilon}$ as a more natural measure of direct CP violation in this mode.

Similarly, for the magnetic amplitudes, we can define

$$\eta_{+-\gamma}^M \equiv \frac{M(K_S)}{M(K_L)} = \epsilon + \epsilon_{+-\gamma}^M, \quad (25)$$

where in this case the direct- CP -violating parameter is

$$\epsilon_{+-\gamma}^M = i \left(\frac{\operatorname{Im} \xi_M}{\operatorname{Re} \xi_M} - \frac{\operatorname{Im} A_0}{\operatorname{Re} A_0} \right) \quad (26)$$

and we have again dropped terms quadratic in weak phases. It appears that it is not possible to measure this parameter in an experiment that does not detect the photon polarization, as we will see in the next two sections.

It was argued in the past [5, 10] that $\tilde{\epsilon}'_{+-\gamma}$ could be several times larger than ϵ' because it predominantly arises from the interference between $\Delta I = 1/2$ components of the IB and DE terms and, therefore, it is not suppressed by the factor $\omega = \operatorname{Re} A_2 / \operatorname{Re} A_0$. Rather, since the IB and DE contributions are generated at orders p^2 and p^4 , respectively, in chiral perturbation theory, $\tilde{\epsilon}'_{+-\gamma}$ has a suppression factor of $p^2 / \Lambda^2 \sim m_K^2 / \Lambda^2$, where $\Lambda \sim 1 \text{ GeV}$. With the additional assumption that the weak phases in $\tilde{\epsilon}'_{+-\gamma}$ and ϵ' are comparable, the enhancement was expected to be³ $\hat{\epsilon} / \epsilon' \sim$

³The original argument was done in terms of $\tilde{\epsilon}'_{+-\gamma}$ which is related to $\hat{\epsilon}$ by Eq. (24). The kinematic dependence of $\tilde{\epsilon}'_{+-\gamma}$ is such, however, that a considerable suppression results when it is integrated over phase space, as we will see in the next section.

$(m_K^2/\Lambda^2)/\omega \sim 5$. Using Eqs. (3) and (23), we find

$$\left| \frac{\hat{\epsilon}}{\epsilon'} \right| \sim \frac{|G_8| f_\pi^2 m_K |F_E|}{2\sqrt{2} \omega \text{Re} A_0} \sim \frac{f_\pi |F_E|}{4\omega m_K}, \quad (27)$$

From this result, one can see that if F_E is of order one as we have speculated, then $\hat{\epsilon} \sim \epsilon'$ and there is no enhancement in this reaction. Equivalently, for the original estimate $\hat{\epsilon} \sim 5\epsilon'$ to be true, one would need $F_E \sim 5$. It is really impossible to distinguish between these two scenarios solely on the grounds of dimensional analysis. As we will see in Section 5, an estimate of F_E based on factorization of the leading current-current operator in the effective weak Hamiltonian supplemented with vector-meson saturation of the p^4 strong counterterms suggests that $F_E \sim 1.7$ and $\hat{\epsilon}$ is not much larger than ϵ' within the standard model.

A limit for $\text{Re} F_E$ can be obtained from the measured $K_S \rightarrow \pi^+\pi^-\gamma$ rate, as a nonzero $\text{Re} F_E$ would give rise to a difference between the measured rate and the calculated IB rate [23]. Including the DE electric-amplitude from Eqs. (14) and (16), and neglecting the magnetic contribution, we derive⁴

$$\text{BR}(K_S \rightarrow \pi^+\pi^-\gamma) \simeq 1.75 \times 10^{-3} + 1.07 \times 10^{-5} \text{Re} F_E + 2.81 \times 10^{-8} (\text{Re} F_E)^2 \quad (28)$$

for $E_\gamma^* > 50 \text{ MeV}$. Since measurements [21] give $\text{BR}(K_S \rightarrow \pi^+\pi^-\gamma, E_\gamma^* > 50 \text{ MeV}) = (1.76 \pm 0.06) \times 10^{-3}$, we extract $\text{Re} F_E \simeq 1$ or -380 using the central value. Dropping the second solution, which is unnaturally large, we obtain

$$\text{Re} F_E = 1 \pm 6. \quad (29)$$

If the PDG's number [16] $\text{BR}(K_S \rightarrow \pi^+\pi^-\gamma, E_\gamma^* > 50 \text{ MeV}) = (1.78 \pm 0.05) \times 10^{-3}$ is used instead, the result is similar, $\text{Re} F_E = 3 \pm 5$. Therefore, experimentally the question of the natural size of $\hat{\epsilon}/\epsilon'$ has not been resolved, and a value $\hat{\epsilon}/\epsilon' \sim 5$ as in the original dimensional analysis estimate is possible.

3 Interference between $K_{L,S} \rightarrow \pi^+\pi^-\gamma$ amplitudes

The parameter $\hat{\epsilon}$ may be measured in an experiment studying the interference between the amplitudes for K_L and K_S decaying into $\pi^+\pi^-\gamma$. Such an experiment typically [8, 9] employs two K_L beams, one of which is passed through a ‘‘regenerator’’, which coherently converts some K_L to K_S . It follows that the initial kaon state can be expressed as a coherent mixture

$$|K_L\rangle + \rho |K_S\rangle, \quad (30)$$

⁴ In the first term (IB only) we have incorporated the complete $K_S \rightarrow \pi^+\pi^-$ amplitude as extracted from data [16], and in the second term we have used an energy-dependent $\delta_0^0 - \delta_1^1$ from Ref. [24].

up to a normalization constant, where $\rho \equiv |\rho| e^{i\phi_\rho}$ is the regeneration parameter. Then the number of decays into $\pi^+\pi^-\gamma$ per unit proper time τ is given by [3]

$$\frac{dN}{d\tau} \propto \left\{ |\rho|^2 \Gamma_{K_S \rightarrow \pi\pi\gamma} e^{-\tau/\tau_S} + \Gamma_{K_L \rightarrow \pi\pi\gamma} e^{-\tau/\tau_L} + 2 \operatorname{Re} \left[\rho \gamma_{LS}^* e^{i\Delta m \tau} \right] e^{-(1/\tau_L + 1/\tau_S)\tau/2} \right\}, \quad (31)$$

where τ_L (τ_S) is the K_L (K_S) lifetime, Δm is the K_L - K_S mass difference, $\Gamma_{K \rightarrow \pi\pi\gamma}$ is the partial width of $K \rightarrow \pi^+\pi^-\gamma$, and γ_{LS} is an integral containing the interference between the K_L and K_S amplitudes. It is important to keep in mind that the quantities $\Gamma_{K \rightarrow \pi\pi\gamma}$ and γ_{LS} are *not* integrated over all phase space. They depend implicitly on the cuts that define the region of phase space under study, and in the following discussion it is implied that all the terms in Eq. (31) are subject to the *same* set of kinematic cuts.

We now proceed to examine the three terms in Eq. (31) in detail. To this aim, we will make use of the following relations:

$$\Gamma_{K_L \rightarrow \pi^+\pi^-\gamma}^{\text{IB}} = |\eta_{+-}|^2 \Gamma_{K_S \rightarrow \pi^+\pi^-\gamma}^{\text{IB}}, \quad (32)$$

which follows from Eq. (13); the ratio [22]

$$f \equiv \frac{\Gamma_{K_L \rightarrow \pi^+\pi^-\gamma}^{\text{M}}}{\Gamma_{K_L \rightarrow \pi^+\pi^-\gamma}} \equiv \frac{r}{1+r} \simeq 0.685, \quad (33)$$

which is determined experimentally from a fit to the decay spectrum that **assumes** that the interference between the IB and an E1-DE is negligible; and Eqs. (20), (25). Consequently,

$$r = \frac{\Gamma_{K_L \rightarrow \pi^+\pi^-\gamma}^{\text{M}}}{\Gamma_{K_L \rightarrow \pi^+\pi^-\gamma}^{\text{E}}} \simeq 2.16. \quad (34)$$

If the photon polarization is not observed, there is no interference between the electric and magnetic amplitudes. In this case the $K_S \rightarrow \pi^+\pi^-\gamma$ rate can be decomposed into the sum of electric and magnetic rates

$$\Gamma_{K_S \rightarrow \pi^+\pi^-\gamma}^{\text{E}} + \Gamma_{K_S \rightarrow \pi^+\pi^-\gamma}^{\text{M}}.$$

Since the second term is CP -violating, it is convenient to rewrite it in terms of Eq. (25) as⁵

$$\begin{aligned} \Gamma_{K_S \rightarrow \pi^+\pi^-\gamma} &= \Gamma_{K_S \rightarrow \pi^+\pi^-\gamma}^{\text{E}} + |\eta_{+-}^{\text{M}}|^2 \Gamma_{K_L \rightarrow \pi^+\pi^-\gamma}^{\text{M}} \\ &\simeq \left(1 + |\eta_{+-}|^2 |\eta_{+-}^{\text{M}}|^2 r \right) \Gamma_{K_S \rightarrow \pi^+\pi^-\gamma}^{\text{E}} \simeq \Gamma_{K_S \rightarrow \pi^+\pi^-\gamma}^{\text{E}}. \end{aligned} \quad (35)$$

The interference term in Eq. (31) can be written in the kaon rest-frame as

$$\gamma_{LS} = \int d[\text{PS}] \left\{ [E_{\text{IB}}(K_L) + E_{\text{DE}}(K_L)] [E_{\text{IB}}^*(K_S) + E_{\text{DE}}^*(K_S)] + M(K_L) M^*(K_S) \right\}, \quad (36)$$

⁵We have here assumed η_{+-}^{M} to be a constant, which is appropriate for our purposes.

where

$$d[\text{PS}] \equiv d \cos \theta dE_\gamma^* \frac{(\beta E_\gamma^*)^3 \sin^2 \theta}{32\pi^3 m_K^3} \left(1 - \frac{2E_\gamma^*}{m_K}\right), \quad (37)$$

with θ being the angle between \mathbf{p}_+ and \mathbf{q} in the $\pi\pi$ rest-frame, and $\beta = \sqrt{1 - 4m_\pi^2/(m_K^2 - 2E_\gamma^* m_K)}$. We remark that $d[\text{PS}]$ contains not only the phase-space factor, but also factors that resulted from summing over the photon polarizations and from contraction of tensor forms. Making use of the definitions in Eqs. (20), (25), (33) and defining⁶

$$\epsilon'_{+-\gamma} \equiv \frac{1}{\Gamma_{K_S \rightarrow \pi^+ \pi^- \gamma}} \int d[\text{PS}] \check{\epsilon}'_{+-\gamma} |E_{\text{IB}}(K_S) + E_{\text{DE}}(K_S)|^2, \quad (38)$$

one finds that

$$\gamma_{\text{LS}} = \left\{ \eta_{+-} + \epsilon'_{+-\gamma} + \eta_{+-}^{\text{M}*} r \left[|\eta_{+-}|^2 + 2 \text{Re}(\eta_{+-}^* \epsilon'_{+-\gamma}) \right] \right\} \Gamma_{K_S \rightarrow \pi^+ \pi^- \gamma}, \quad (39)$$

having dropped terms suppressed by additional powers of ϵ or $\hat{\epsilon}$.

Turning now to the second term in Eq. (31), we have

$$\Gamma_{K_L \rightarrow \pi^+ \pi^- \gamma} = \int d[\text{PS}] \left(|E_{\text{IB}}(K_L) + E_{\text{DE}}(K_L)|^2 + |M(K_L)|^2 \right), \quad (40)$$

where the photon polarizations have been summed over. Then, from Eqs. (20), (38), and (33), neglecting terms of orders $\hat{\epsilon}^2$ and ϵ^6 , we find

$$\Gamma_{K_L \rightarrow \pi^+ \pi^- \gamma} = (1+r) \left[|\eta_{+-}|^2 + 2 \text{Re}(\eta_{+-}^* \epsilon'_{+-\gamma}) \right] \Gamma_{K_S \rightarrow \pi^+ \pi^- \gamma}. \quad (41)$$

Collecting all these results, we can now rewrite Eq. (31) as

$$\begin{aligned} \frac{dN}{d\tau} \propto & \Gamma_{K_S \rightarrow \pi^+ \pi^- \gamma} \left\{ |\rho|^2 e^{-\tau/\tau_S} + \left[|\eta_{+-}|^2 + 2 \text{Re}(\eta_{+-}^* \epsilon'_{+-\gamma}) \right] (1+r) e^{-\tau/\tau_L} \right. \\ & \left. + 2 \left| \eta_{+-} + \epsilon'_{+-\gamma} \right| |\rho| \cos(\Delta m \tau + \phi_\rho - \phi_\eta) e^{-\tau(1/\tau_L + 1/\tau_S)/2} \right\}, \end{aligned} \quad (42)$$

where ϕ_η is the phase of $(\eta_{+-} + \epsilon'_{+-\gamma})$. This rate equation can be used to extract $\hat{\epsilon}$ from measurements. The most recent experimental study [9] on these decays starts from the definition

$$\begin{aligned} \frac{dN}{d\tau} = & \frac{N_S \Gamma_{K_S \rightarrow \pi^+ \pi^- \gamma}}{|\rho|^2} \left\{ |\rho|^2 e^{-\tau/\tau_S} + |\eta_{+-\gamma}|^2 (1+r) e^{-\tau/\tau_L} \right. \\ & \left. + 2 |\eta_{+-\gamma}| |\rho| \cos(\Delta m \tau + \phi_\rho - \phi_\eta) e^{-\tau(1/\tau_L + 1/\tau_S)/2} \right\}, \end{aligned} \quad (43)$$

⁶Notice that *both* $\Gamma_{K_S \rightarrow \pi^+ \pi^- \gamma}$ in the denominator and the integral in the numerator of Eq. (38) depend on the E_γ^* cut.

where N_S is the number of K_S regenerated. Comparing Eqs. (42) and (43), we see that

$$\eta_{+-\gamma} \equiv \eta_{+-} + \epsilon'_{+-\gamma} \quad (44)$$

if terms of order $(\hat{\epsilon}/\epsilon)^2$ are neglected.

With a minimum photon energy, $E_\gamma^* > 20 \text{ MeV}$, we integrate over phase space ($-1 < \cos \theta < 1$ and $20 \text{ MeV} < E_\gamma^* < m_K/2 - 2m_\pi^2/m_K$), and use $E(K_S) \sim E_{\text{IB}}(K_S)$, to find

$$\epsilon'_{+-\gamma} \simeq 0.041 \hat{\epsilon} e^{i\delta_\epsilon}, \quad (45)$$

where we have also used $\text{BR}(K_S \rightarrow \pi^+\pi^-\gamma, E_\gamma^* > 20 \text{ MeV}) = 4.87 \times 10^{-3}$ from Ref. [21] and $e^2/(4\pi) = 1/137$, and assumed δ_ϵ to be constant. We have checked that a phase which varies with energy would not alter this result in a significant way. With an energy-dependent formula for $\delta_0^0 - \delta_1^1$ from Ref. [24], we get $\epsilon'_{+-\gamma} \simeq (0.014 + 0.039 i) \hat{\epsilon}$, which is not different from Eq. (45) if $\delta_0^0 - \delta_1^1$ takes its average value of 17.4° .

We see that the observable $\epsilon'_{+-\gamma}$ is suppressed by a factor of 0.041 with respect to $\hat{\epsilon}$. This factor can be understood as the ratio of $\int d(\text{phase space})/(z^2 - \nu^2)$ to $\int d(\text{phase space})/(z^2 - \nu^2)^2$, which are the corresponding forms for the interference and IB terms in the rate. It is interesting to notice that the suppression factor decreases as the cut in E_γ^* is increased. This is due to the fact that with higher cuts the ratio of the IB contribution to that of the electric DE becomes smaller. For example, we find

$$\epsilon'_{+-\gamma} \simeq 0.090 \hat{\epsilon} e^{i\delta_\epsilon} \quad \text{for } E_\gamma^* > 50 \text{ MeV}, \quad (46)$$

by means of $\text{BR}(K_S \rightarrow \pi^+\pi^-\gamma, E_\gamma^* > 50 \text{ MeV}) = 1.76 \times 10^{-3}$ from Ref. [21]. Clearly if the E_γ^* cut is increased much further, one has to reconsider the assumption of IB dominance in the K_S amplitude. Experimentally, of course, an increased E_γ^* cut results in a smaller event sample.

The best measurement ($E_\gamma^* > 20 \text{ MeV}$) to date [25],

$$\left| \frac{\epsilon'_{+-\gamma}}{\epsilon} \right|_{\text{exp}} = 0.041 \pm 0.035, \quad (47)$$

translates into the one-sigma limit

$$\left| \frac{\hat{\epsilon}}{\epsilon} \right| < 1.9. \quad (48)$$

Then, if we assume that there is no cancellation between the two phases in Eq. (23), this implies that $\text{Im } F_E$ could have a magnitude as large as

$$|\text{Im } F_E| \sim 0.1. \quad (49)$$

The KTeV experiment is expected [25] to reduce the uncertainties in $|\epsilon'_{+-\gamma}/\epsilon|$ to the 0.4% level, which would improve these bounds by a factor of 10.

To end this section we comment on the observability of the quantity $\eta_{+-\gamma}^M$ for CP violation in the K_S magnetic-amplitude. It is evident from Eqs. (35) and (39) that $\eta_{+-\gamma}^M$ occurs amongst terms of order ϵ^3 or higher. Therefore, it does not seem possible to measure the direct- CP -violation parameter $\epsilon_{+-\gamma}^M$, defined in Eq. (26), in this kind of experiment.

4 Interference between IB and DE in electric amplitude

The direct CP violating parameter $\hat{\epsilon}$ can also be detected in principle by analyzing in detail the decay distribution in $K_L \rightarrow \pi^+\pi^-\gamma$. Let us first consider the photon-energy (E_γ^*) spectrum used to separate IB from DE contributions. The E_γ^* distribution is given by

$$\frac{d\Gamma_{K_L \rightarrow \pi^+\pi^-\gamma}}{dE_\gamma^*} = \frac{(\beta E_\gamma^*)^3}{32\pi^3 m_K^3} \left(1 - \frac{2E_\gamma^*}{m_K}\right) \int_{-1}^1 \sin^2\theta d\cos\theta \left(|E_{\text{IB}}(K_L) + E_{\text{DE}}(K_L)|^2 + |M(K_L)|^2\right). \quad (50)$$

Recent experimental studies [21, 22] of the E_γ^* spectrum assume that the DE process is purely magnetic and parameterize it with a form-factor-modified M1 contribution. By parameterizing the CP -violating E1 component in the DE electric amplitude as in Eq. (11) we can calculate its interference with the IB contribution. The presence of this term modifies the shape of the (E_γ^*) spectrum, and this generates a limit on $\hat{\epsilon}$.

We present in Fig. 1 an example of such a deviation. For this figure, the magnetic amplitude is parameterized in the form used by recent experiments [21, 22],

$$\xi_M^L = \left(\frac{a_1}{m_\rho^2 - m_K^2 + 2E_\gamma^* m_K} + a_2\right) e^{i\delta_1^1}, \quad (51)$$

where m_ρ is the ρ -meson mass, and $a_{1,2}$ are parameters obtained from the experimental fit. The solid curve describes the combination of the IB and magnetic DE processes, without any electric DE contribution. For this curve, we employ $a_1/a_2 = -0.729 \text{ GeV}^2$ from Ref. [22] and adjust the value of a_2 in order to get a $K_L \rightarrow \pi^+\pi^-\gamma$ rate that matches the measured one (thus here $a_2 \simeq 3.09$). The upper dashed-curve includes the IB, the magnetic DE with a_1/a_2 as before and an IB–E1–DE interference using $\text{Re } F_E = 0$ and $\text{Im } F_E = 0.5$ for illustration. In this case we have used $a_2 \simeq 2.40$ to keep the total rate fixed. The lower dashed-curve corresponds to the interference between the IB and E1 DE only.

From Eq. (50), we find the interference between the IB and DE components of the electric amplitude to be

$$\Gamma_{K_L \rightarrow \pi^+\pi^-\gamma}^{\text{E,int}} = \int d[\text{PS}] 2 \text{Re} [E_{\text{IB}}^*(K_L) E_{\text{DE}}(K_L)], \quad (52)$$

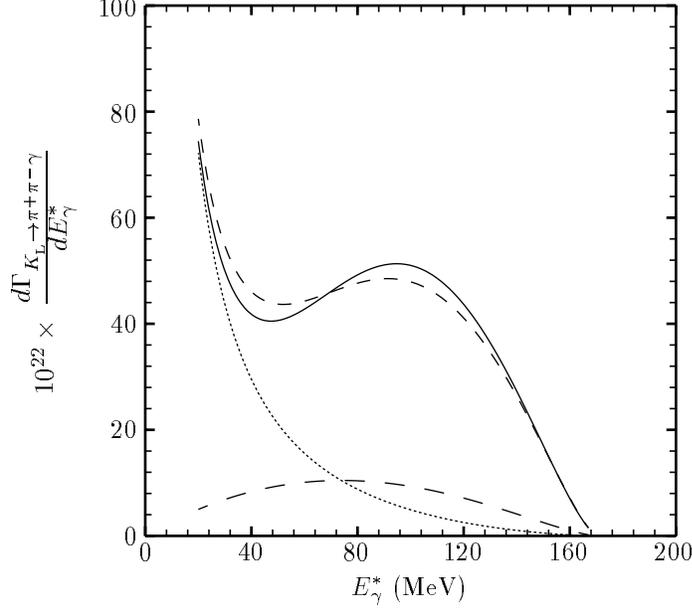


Figure 1: Photon-energy (E_γ^*) distribution in $K_L \rightarrow \pi^+ \pi^- \gamma$. The solid curve represents the sum of the inner bremsstrahlung and the magnetic direct-emission contributions. The dotted curve is the IB alone. The upper dashed-curve corresponds to IB plus M1-DE contributions as in the solid curve, plus an IB–E1-DE interference with $F_E = 0.5i$, as explained in the text. The lower dashed-curve shows the interference between the IB and E1 DE only.

with the integration ranges being $-1 < \cos \theta < 1$ and $20 \text{ MeV} < E_\gamma^* < m_K/2 - 2m_\pi^2/m_K$. Using

$$\Gamma_{K_L \rightarrow \pi^+ \pi^- \gamma}(E_\gamma^* > 20 \text{ MeV}) = 5.86 \times 10^{-19} \text{ MeV} \quad (53)$$

from Ref. [16], Eqs. (13) and (15), as well as $\delta_0^0 - \delta_1^1 = 17.4^\circ$ in δ_ϵ , we obtain

$$\begin{aligned} \frac{\Gamma_{K_L \rightarrow \pi^+ \pi^- \gamma}^{\text{E,int}}}{\Gamma_{K_L \rightarrow \pi^+ \pi^- \gamma}} &\simeq 5.0 \times 10^3 |\eta_{+-}| \hat{\epsilon} \cos(\delta_\epsilon - \phi_{+-}) + 1.8 \times 10^2 |\epsilon \eta_{+-}| \text{Re } F_E \cos(\delta_\epsilon - \pi/2) \\ &\simeq 0.023 \frac{\hat{\epsilon}}{|\epsilon|} + 8.9 \times 10^{-4} \text{Re } F_E . \end{aligned} \quad (54)$$

An analysis of KTeV data with the same E_γ^* cut, gives [13],

$$\frac{\Gamma_{K_L \rightarrow \pi^+ \pi^- \gamma}^{\text{E,int}}}{\Gamma_{K_L \rightarrow \pi^+ \pi^- \gamma}} < 0.30 \quad (90\% \text{ C.L.}) . \quad (55)$$

Taken literally, this bound implies that,

$$\left| \frac{\hat{\epsilon}}{\epsilon} + 0.04 \text{Re } F_E \right| < 13 , \quad (56)$$

and assuming that the two terms do not cancel,

$$|\text{Im } F_E| \lesssim 1, \quad |\text{Re } F_E| \lesssim 350. \quad (57)$$

These limits are much weaker than Eqs. (48), (49), and (29). However, it is important to notice that the bound, Eq. (55), is not particularly strong. In fact, it is amazing that this interference between the IB and E1-DE contributions, which is expected to be small, could account for as much as one third of the experimental rate. This is due in part to the fact that an analysis of the full E_γ^* distribution is not the optimal observable to isolate this term as we can see in Fig. 1.

It may be possible to improve the bound, Eq. (55) by restricting the analysis to the region of the E_γ^* distribution where this term is most important. A glance at the dashed curves in Fig. 1 suggests that the cuts $50 \text{ MeV} < E_\gamma^* < 90 \text{ MeV}$, for example, would improve the bound. With these cuts, we find

$$\frac{\Gamma_{K_L \rightarrow \pi^+ \pi^- \gamma}^{\text{E,int}}}{\Gamma_{K_L \rightarrow \pi^+ \pi^- \gamma}} \simeq 0.029 \frac{\hat{\epsilon}}{|\epsilon|} + 1.1 \times 10^{-3} \text{Re } F_E, \quad (58)$$

where we used $\delta_0^0 - \delta_1^1 = 21.3^\circ$ (the average phase difference in this region) and

$$\Gamma_{K_L \rightarrow \pi^+ \pi^- \gamma}(50 \text{ MeV} < E_\gamma^* < 90 \text{ MeV}) \simeq 1.84 \times 10^{-19} \text{ MeV}, \quad (59)$$

obtained by including only the IB and magnetic contributions (since these should dominate the rate).⁷ These numbers suggest that this simple set of cuts could improve the bound on $\hat{\epsilon}$ by about 30%. However, this procedure depends on the assumed E_γ^* dependence of ξ_M^L , Eq. (51), and of F_E (independent of E_γ^*).

A more detailed analysis of the decay distribution is desirable in order to separate all the contributions. Unfortunately, we find that the decay distributions for the M1 term and for the IB-E1 interference term are remarkably similar when the photon polarization is not observed. We illustrate this by presenting a $\cos \theta$ distribution in Fig. 2. We can see in this figure that the shapes of the M1 distribution (solid line), and IB-E1 interference term (dashed line) are very similar. The only significant difference between the two is that the interference term could be negative, depending on the phase of F_E .

5 Standard-model contribution

In this section we review the estimate of F_E that exists in the literature and we apply it to an estimate of $\hat{\epsilon}$ within the standard model.

⁷ For the magnetic part, $a_1/a_2 = -0.729 \text{ GeV}^2$ and $a_2 \simeq 3.09$ have been used.

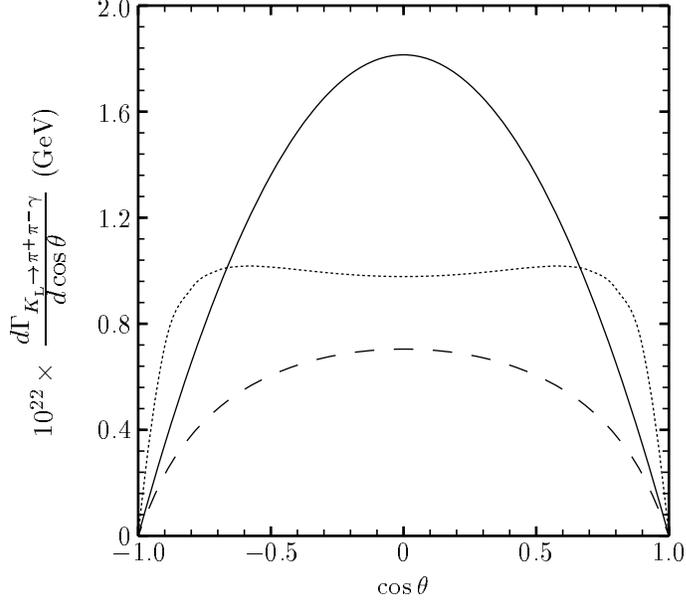


Figure 2: Distributions of various components of the $K_L \rightarrow \pi^+ \pi^- \gamma$ rate in terms of $\cos \theta$, where θ is the angle between the π^+ and γ momenta in the $\pi\pi$ rest-frame. The solid curve represents only the magnetic contribution. The dotted curve is the IB alone. The dashed-curve corresponds to the IB-E1-DE interference with $F_E = 0.5i$.

The SM contribution to the DE electric-amplitude in $K_L \rightarrow \pi^+ \pi^- \gamma$ is generated at short distance by both four-quark and two-quark ($s \rightarrow d\gamma, dg$) operators with known coefficients. The matrix elements of these operators arise at $\mathcal{O}(p^4)$ in chiral perturbation theory. They are presumably dominated by the chiral realizations of the four-quark operators that transform as $(8_L, 1_R)$ under chiral rotations, as well as by $\mathcal{O}(p^4)$ loop diagrams [7, 26]. The contribution from the $s \rightarrow d\gamma, dg$ operators in the SM starts at order p^6 , and since the coefficients of these two-quark operators are quite small, it is appropriate to neglect them in the standard model estimate.

The $\mathcal{O}(p^4)$ weak chiral Lagrangian transforming as $(8_L, 1_R)$ is given by [26, 27]

$$\mathcal{L}_w^{(4)} = G_8 f_\pi^2 \sum_{i=14}^{17} N_i W_i + \text{H.c.}, \quad (60)$$

where N_i are dimensionless coupling constants and

$$\begin{aligned} W_{14} &= i \langle \lambda \{ F_{\mu\nu}^L + U^\dagger F_{\mu\nu}^R U, L^\mu L^\nu \} \rangle, & W_{15} &= i \langle \lambda L^\mu (F_{\mu\nu}^L + U^\dagger F_{\mu\nu}^R U) L^\nu \rangle, \\ W_{16} &= i \langle \lambda \{ F_{\mu\nu}^L - U^\dagger F_{\mu\nu}^R U, L^\mu L^\nu \} \rangle, & W_{17} &= i \langle \lambda L^\mu (F_{\mu\nu}^L - U^\dagger F_{\mu\nu}^R U) L^\nu \rangle. \end{aligned} \quad (61)$$

In these formulas, $\langle \dots \rangle \equiv \text{Tr}(\dots)$,

$$\lambda \equiv \frac{1}{2}(\lambda_6 - i\lambda_7), \quad F_{\mu\nu}^L = F_{\mu\nu}^R = -eQF_{\mu\nu}, \quad U = e^{i\phi/f_\pi}, \quad L_\mu = iU^\dagger D_\mu U, \quad (62)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the photon field-strength tensor, $Q = \text{diag}(2, -1, -1)/3$ is the quark-charge matrix, ϕ is a 3×3 matrix containing the octet of pseudo-Goldstone bosons, and $D_\mu U = \partial_\mu U + ie[Q, U]A_\mu$. Under chiral rotations, these fields transform as

$$F_{\mu\nu}^L \rightarrow V_L F_{\mu\nu}^L V_L^\dagger, \quad F_{\mu\nu}^R \rightarrow V_R F_{\mu\nu}^R V_R^\dagger, \quad U \rightarrow V_R U V_L^\dagger, \quad L_\mu \rightarrow V_L L_\mu V_L^\dagger. \quad (63)$$

In terms of this effective Lagrangian, the SM contribution to F_E can be written as,

$$F_E = \frac{-m_K^3}{\sqrt{2} f_\pi^3} (N_{14} - N_{15} - N_{16} - N_{17}) + \mathcal{O}(p^4) \text{ loop terms}. \quad (64)$$

This combination of constants (N_i) is not known from experiment so one needs to resort to model calculations as in Ref. [28]. The loop terms, on the other hand, are known [29].

The standard assumption is that the real parts of the N_i ($i = 14, \dots, 17$) arise mostly from the dominant octet quark-operator $Q_1 - Q_2$ (in the notation of Ref. [30]). They have been calculated in a couple of models in Ref. [28] with the result,

$$\text{Re}(N_{14} - N_{15} - N_{16} - N_{17}) = \frac{-f_\pi^2}{2m_\rho^2} k_f, \quad (65)$$

where $k_f = 1$ corresponds to the naive factorization model and $k_f = 1/2$ corresponds to the so-called ‘‘weak deformation model’’. Taking $k_f = 1$ for illustration, and including the loop terms calculated by D’Ambrosio and Isidori [7] (the 0.9 term), one obtains the estimate,

$$|\text{Re } F_E|_{\text{SM}} = \frac{m_K^3}{2\sqrt{2} f_\pi m_\rho^2} + 0.9 = 1.7. \quad (66)$$

In order to estimate $\hat{\epsilon}$ one also needs an estimate for the imaginary parts of the matrix elements, or equivalently, of the N_i . Within the standard model these imaginary parts are expected to be dominated by the gluonic penguin operator. Unfortunately a detailed estimate of the bosonization of the penguin operator at order p^4 does not exist, and we have to resort to dimensional analysis arguments which indicate that $\text{Im } F_E / \text{Re } F_E$ is of the same order as $\text{Im } A_0 / \text{Re } A_0 \sim \sqrt{2} \epsilon' / \omega \sim 1 \times 10^{-4}$. Taking $\text{Re } F_E = 1$, and assuming that there is no large cancellation between the two terms in Eq. (23), we arrive at the estimate

$$\left| \frac{\hat{\epsilon}}{\epsilon} \right|_{\text{SM}} \lesssim \frac{f_\pi}{2\sqrt{2} |\epsilon| m_K} \frac{\text{Im } A_0}{\text{Re } A_0} \sim 3 \times 10^{-3}. \quad (67)$$

This number is almost three orders of magnitude below the current experimental limit in Eq. (48). It should be clear, however, that this is no more than an order of magnitude estimate.

6 New physics and short-distance $s \rightarrow d\gamma, dg$ transitions

Physics beyond the standard model modifies the short distance coefficients of the two and four-quark operators that contribute to the DE electric-amplitude in $K_L \rightarrow \pi^+\pi^-\gamma$, and may also generate further operators. In this section we discuss the $s \rightarrow d\gamma, dg$ operators, as they can be significantly enhanced in certain models [12, 31, 32, 33].

The effective Hamiltonian responsible for the short-distance $s \rightarrow d\gamma, dg$ transitions can be written, following the notation of Ref. [31], as

$$\mathcal{H}_{\text{eff}} = C_\gamma^+ Q_\gamma^+ + C_\gamma^- Q_\gamma^- + C_g^+ Q_g^+ + C_g^- Q_g^- + \text{H.c.}, \quad (68)$$

where $C_{\gamma,g}^\pm$ are the Wilson coefficients and

$$\begin{aligned} Q_\gamma^\pm &= \frac{eQ_d}{16\pi^2} \left(\bar{s}_L \sigma_{\mu\nu} d_R \pm \bar{s}_R \sigma_{\mu\nu} d_L \right) F^{\mu\nu}, \\ Q_g^\pm &= \frac{g_s}{16\pi^2} \left(\bar{s}_L \sigma_{\mu\nu} t_a d_R \pm \bar{s}_R \sigma_{\mu\nu} t_a d_L \right) G_a^{\mu\nu} \end{aligned} \quad (69)$$

are the so-called electromagnetic- and chromomagnetic-dipole operators, respectively, with $eQ_d = -e/3$ being the d -quark charge and $G_a^{\mu\nu}$ being the gluon field-strength tensor. We notice that $Q_{\gamma,g}^\pm$ transform as $(\bar{3}_L, 3_R) \pm (3_L, \bar{3}_R)$ under $SU(3)_L \times SU(3)_R$ transformations, and $Q_{\gamma,g}^+$ ($Q_{\gamma,g}^-$) are even (odd) under parity. We further notice that $Q_{\gamma,g}^+$ ($Q_{\gamma,g}^-$) are also even (odd) under a CPS transformation (a CP operation followed by interchanging the s and d quarks). These magnetic-dipole operators are expected to produce the most important contributions to the $s \rightarrow d\gamma, dg$ transitions. In this section we estimate the impact of these operators by employing a chiral-Lagrangian approach combined with naive dimensional-analysis [34, 35] to evaluate the necessary hadronic matrix elements.

The chiral Lagrangian that represents the $s \rightarrow d\gamma, dg$ operators is constructed as follows. We begin by defining the objects

$$\lambda_{LR} = \lambda_{RL} \equiv \lambda, \quad (70)$$

with which we can rewrite Q_γ^\pm in the form

$$Q_\gamma^\pm = \frac{eQ_d}{16\pi^2} \left(\bar{q}_L \lambda_{LR} \sigma_{\mu\nu} q_R \pm \bar{q}_R \lambda_{RL} \sigma_{\mu\nu} q_L \right) F^{\mu\nu}, \quad (71)$$

where $q = (u \ d \ s)^T$. If we imagine that under a chiral rotation

$$\lambda_{LR} \rightarrow V_L \lambda_{LR} V_R^\dagger, \quad \lambda_{RL} \rightarrow V_R \lambda_{RL} V_L^\dagger, \quad (72)$$

where $V_{L,R} \in SU(3)_{L,R}$, then the quark operators inside the brackets in Eq. (71) would be chirally invariant. Consequently, we construct the effective Lagrangian by writing chiral invariant terms

with one power of λ_{LR} or λ_{RL} assumed to transform as in Eq. (72), and with an explicit photon field strength tensor $F^{\mu\nu}$. We also require the effective Lagrangian to have the same parity and *CPS* structure of Q_γ^\pm . In this regard, λ_{LR} and λ_{RL} interchange places under parity, but remain unchanged under charge conjugation.

There are many possible chiral realizations of Q_γ^\pm . As relevant examples at leading order, p^4 , we write down

$$\mathcal{L}_\gamma^{(\pm)} = i\beta_\gamma^\pm e Q_d \left\langle \lambda_{\text{LR}} U L_\mu L_\nu \pm \lambda_{\text{RL}} L_\mu L_\nu U^\dagger \right\rangle F^{\mu\nu} + \text{H.c.}, \quad (73)$$

where $\mathcal{L}_\gamma^{(\pm)}$ have the same symmetry properties as those of Q_γ^\pm . Employing naive dimensional-analysis [34, 35], we obtain the order-of-magnitude estimate

$$\beta_\gamma^\pm = \frac{C_\gamma^\pm f_\pi}{16\pi^2} \frac{f_\pi}{\Lambda}, \quad (74)$$

where $\Lambda = 4\pi f_\pi$. Notice that $\mathcal{L}_\gamma^{(\pm)}$ are not suppressed by light-quark masses, as is appropriate for the new-physics interactions of interest in Eq. (68). As was noted before, within the SM the short-distance operator is suppressed by light-quark masses, and this results in a different chiral Lagrangian (of order p^6 at least) involving the usual chiral-symmetry breaking factor [7].

The contribution of the $s \rightarrow d\gamma$ operators to the direct-emission electric-amplitude of $K^0 \rightarrow \pi^+\pi^-\gamma$ comes from $\mathcal{L}_\gamma^{(-)}$ and is given by

$$(F_E)_\gamma = \frac{\sqrt{2}\beta_-}{3G_8 f_\pi^2} \frac{m_K^3}{f_\pi^3} = \frac{m_K^3}{96\sqrt{2}\pi^3 G_8 f_\pi^4} C_\gamma^-. \quad (75)$$

Then, if the new-physics contribution to F_E is such that $\text{Im } F_E / \text{Re } F_E \gg \text{Im } A_0 / \text{Re } A_0$ (i.e. *CP* violation is dominated by new physics) in $\hat{\epsilon}$, the contribution of C_γ^- to $\hat{\epsilon}/\epsilon$ is

$$\left| \frac{\hat{\epsilon}}{\epsilon} \right| \simeq \frac{m_K^4}{384\sqrt{2}\pi^3 |\epsilon| f_\pi^2 \text{Re } A_0} \left| \text{Im } C_\gamma^- \right| \simeq 6.9 \times 10^5 \text{ GeV} \left| \text{Im } C_\gamma^- \right|. \quad (76)$$

This and Eq. (48) imply the current limit

$$\left| \text{Im } C_\gamma^- \right| < 2.7 \times 10^{-6} \text{ GeV}^{-1}. \quad (77)$$

Future measurements [25] may improve this value by about a factor of 10.

Turning now to the chiral realization of the $s \rightarrow dg$ operators Q_g^\pm , we rewrite them as

$$Q_g^\pm = \frac{g_s}{16\pi^2} \left(\bar{q}_L \lambda_{\text{LR}} \sigma_{\mu\nu} t_a G_a^{\mu\nu} q_R \pm \bar{q}_R \lambda_{\text{RL}} \sigma_{\mu\nu} t_a G_a^{\mu\nu} q_L \right). \quad (78)$$

Since Q_g^\pm do not contain the photon field, in constructing the corresponding chiral Lagrangian involving an $F_{\mu\nu}$, we employ the chiral field-strength tensors $F_{\mu\nu}^{\text{L}}$ and $F_{\mu\nu}^{\text{R}}$, which were defined in

Eq. (62). These tensors transform under parity as $F_{L,R}^{\mu\nu} \rightarrow (F_{R,L})_{\mu\nu}$ and under charge conjugation as $F_{L,R}^{\mu\nu} \rightarrow -(F_{R,L}^T)^{\mu\nu}$.

Thus, examples of chiral Lagrangians at order p^4 that correspond to Q_g^\pm are

$$\mathcal{L}_g^{(\pm)} = i\beta_g^\pm \left\langle \lambda_{LR} U L^\mu \left(F_{\mu\nu}^L + U^\dagger F_{\mu\nu}^R U \right) L^\nu \pm \lambda_{RL} L^\mu \left(F_{\mu\nu}^L + U^\dagger F_{\mu\nu}^R U \right) L^\nu U^\dagger \right\rangle + \text{H.c.}, \quad (79)$$

where, from naive dimensional-analysis [34, 35],

$$\beta_g^\pm = \frac{C_g^\pm f_\pi}{16\pi^2} \frac{f_\pi}{\Lambda} g_s. \quad (80)$$

For numerical estimates we use $g_s \sim \sqrt{4\pi}$, corresponding to a strong coupling $\alpha_s \sim 1$. The contribution to F_E arises from $\mathcal{L}_g^{(-)}$ and is given by

$$(F_E)_g = \frac{g_s m_K^3}{96\sqrt{2} \pi^3 G_8 f_\pi^4} C_g^-. \quad (81)$$

We again assume that CP violation is dominated by the new physics, in such a way that $\text{Im}F_E/\text{Re}F_E \gg \text{Im}A_0/\text{Re}A_0$, to obtain

$$\left| \frac{\hat{\epsilon}}{\epsilon} \right|_g \simeq \frac{g_s m_K^4}{384\sqrt{2} \pi^3 |\epsilon| f_\pi^2 \text{Re}A_0} \left| \text{Im}C_g^- \right| \simeq 2.4 \times 10^6 \text{ GeV} \left| \text{Im}C_g^- \right|, \quad (82)$$

which, with Eq. (48), implies the limit

$$\left| \text{Im}C_g^- \right| < 7.8 \times 10^{-7} \text{ GeV}^{-1}. \quad (83)$$

Since it is known [31, 36, 37] that Q_g^- also contributes to ϵ' , we can use the measured value of ϵ' to derive another limit on the contribution of Q_g^- to $\hat{\epsilon}$. The leading chiral realization of Q_g^- that contributes to the $K_L \rightarrow \pi\pi$ amplitude is of order p^2 , as the $\mathcal{O}(p^0)$ realization does not contribute once tadpole diagrams have been properly taken into account [36]. An example that we can construct is⁸

$$\mathcal{L}_g^{(2)} = \gamma_g^- f_\pi^2 \left\langle \lambda (U - U^\dagger) \right\rangle \left\langle L^\mu L_\mu \right\rangle + \text{H.c.}, \quad (84)$$

where γ_g^- is a dimensionless coupling constant. Using the naive dimensional-analysis prescribed in Ref. [35], we find the order-of-magnitude estimate

$$\gamma_g^- = \frac{C_g^- f_\pi g_s}{16\pi^2}. \quad (85)$$

⁸A similar Lagrangian has been given in Ref. [38].

The resulting amplitude is

$$(A_0)_g = \frac{C_g^- g_s}{2\sqrt{2}\pi^2} (m_K^2 - 2m_\pi^2), \quad (86)$$

leading to

$$\left| \frac{\epsilon'}{\epsilon} \right|_g = \frac{\omega}{|\epsilon| \text{Re} A_0} \frac{g_s}{4\pi^2} (m_K^2 - 2m_\pi^2) |\text{Im} C_g^-| \simeq 1.4 \times 10^6 |\text{Im} C_g^-|. \quad (87)$$

Assuming that the current value $|\epsilon'/\epsilon| \sim 2 \times 10^{-3}$ is saturated by the Q_g^- contribution yields

$$|\text{Im} C_g^-| < 1.6 \times 10^{-9}, \quad (88)$$

which is a much better constraint than Eq. (83). Furthermore, combining Eqs. (87) and (82) results in

$$\left| \frac{\hat{\epsilon}}{\epsilon} \right|_g \simeq \frac{m_K^4}{96\sqrt{2}\pi\omega e f_\pi^2 (m_K^2 - 2m_\pi^2)} \left| \frac{\epsilon'}{\epsilon} \right|_g \simeq 1.8 \left| \frac{\epsilon'}{\epsilon} \right|_g < 4 \times 10^{-3}. \quad (89)$$

This is twice as large as the standard model result, but the two should be considered equivalent within the uncertainties of our estimates. From this we conclude that improved measurements of $\hat{\epsilon}$ are more important to place bounds on Q_γ^- . Bounds on Q_g^- from $\hat{\epsilon}$ are not likely to be competitive with bounds from ϵ'/ϵ in the foreseeable future. We now turn our attention to two specific models for Q_γ^- .

6.1 Left-right symmetric models

In left-right symmetric models the coefficients of Q_γ^- and Q_g^- can be enhanced because the mixing of left- and right-handed W -bosons removes the helicity suppression present in the standard model. Variations of this model have been studied in the context of $b \rightarrow s\gamma$ in detail [39, 40].

We start from the effective Lagrangian that results from integrating out the heavy right-handed W . This can be written down directly by following the formalism of Ref. [41]. In the unitary gauge,

$$\mathcal{L}_{\text{RH}} = -\frac{g_2}{\sqrt{2}} (\bar{u}_R \quad \bar{c}_R \quad \bar{t}_R) \gamma^\mu \tilde{V} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} W_\mu^+ + \text{h.c.}, \quad (90)$$

where \tilde{V} is a 3×3 unitary matrix having elements $\tilde{V}_{qq'} = V_{qq'} \kappa_{qq'}^{\text{R}}$, with $V_{qq'}$ being CKM-matrix elements and $\kappa_{qq'}^{\text{R}}$ complex numbers. In writing \mathcal{L}_{eff} above, we have ignored modifications to the

left-handed W -couplings which do not lead to enhanced effects. Using the results of Refs. [39, 42] we write,

$$\begin{aligned} C_{\gamma,\text{RH}}^-(m_W) &= \frac{G_F}{\sqrt{2}} \sum_{q=c,t} V_{qd} V_{qs}^* (\kappa_{qs}^{\text{R}*} - \kappa_{qd}^{\text{R}}) m_q \frac{F_{\text{RH}}(x_q)}{Q_d}, \\ C_{g,\text{RH}}^-(m_W) &= \frac{G_F}{\sqrt{2}} \sum_{q=c,t} V_{qd} V_{qs}^* (\kappa_{qs}^{\text{R}*} - \kappa_{qd}^{\text{R}}) m_q G_{\text{RH}}(x_q), \end{aligned} \quad (91)$$

where $x_q = m_q^2/m_W^2$ and

$$F_{\text{RH}}(x) = \frac{-3x^2 + 2x}{(x-1)^3} \ln x - \frac{5x^2 - 31x + 20}{6(x-1)^2}, \quad G_{\text{RH}}(x) = \frac{6x \ln x}{(x-1)^3} - \frac{3+3x}{(x-1)^2} - 1. \quad (92)$$

For our numerical estimates, we will use $\alpha_s(m_Z) = 0.119$, $m_c = 1.25 \text{ GeV}$, $m_t = 173.8 \text{ GeV}$, and the CKM-matrix elements in the Wolfenstein parameterization from Ref. [44]: $\lambda = 0.22$, $A = 0.82$, $\rho = 0.16$, and $\eta = 0.38$. This gives,

$$C_{\gamma,\text{RH}}^- \simeq \left[1 \times 10^3 V_{cd} V_{cs}^* (\kappa_{cs}^{\text{R}*} - \kappa_{cd}^{\text{R}}) + 7 \times 10^4 V_{td} V_{ts}^* (\kappa_{ts}^{\text{R}*} - \kappa_{td}^{\text{R}}) \right] \times 10^{-7} \text{ GeV}^{-1}, \quad (93)$$

$$C_{g,\text{RH}}^- \simeq \left[-4 \times 10^2 V_{cd} V_{cs}^* (\kappa_{cs}^{\text{R}*} - \kappa_{cd}^{\text{R}}) - 2 \times 10^4 V_{td} V_{ts}^* (\kappa_{ts}^{\text{R}*} - \kappa_{td}^{\text{R}}) \right] \times 10^{-7} \text{ GeV}^{-1}. \quad (94)$$

It then follows that

$$\left| \frac{\hat{\epsilon}}{\epsilon} \right|_{\gamma,\text{RH}} \simeq \left| 15 \text{Im}(\kappa_{cs}^{\text{R}*} - \kappa_{cd}^{\text{R}}) + \text{Im}[(1.3 - 0.6i)(\kappa_{ts}^{\text{R}*} - \kappa_{td}^{\text{R}})] \right|. \quad (95)$$

These same couplings contribute to ϵ'/ϵ in this model through Q_g ,

$$\left| \frac{\epsilon'}{\epsilon} \right|_{g,\text{RH}} \simeq \left| 12 \text{Im}(\kappa_{cs}^{\text{R}*} - \kappa_{cd}^{\text{R}}) + \text{Im}[(0.8 - 0.4i)(\kappa_{ts}^{\text{R}*} - \kappa_{td}^{\text{R}})] \right|. \quad (96)$$

From these results, we see that the contribution of Q_γ^- to $\hat{\epsilon}$ is also constrained by the contribution of Q_g^- to ϵ' . For example, if all $\text{Im}(\kappa_{qs}^{\text{R}*} - \kappa_{qd}^{\text{R}})$ are of the same order, then the c -quark contributions dominate both observables and $|\hat{\epsilon}| \sim 1.3 |\epsilon'|$. Similarly, if the t -quark intermediate-state dominates, $|\hat{\epsilon}| \sim 1.7 |\epsilon'|$. If one includes perturbative-QCD running of the Wilson coefficients from the W -mass scale down to a scale near the charm-quark mass [30, 43], these numbers slightly change to $|\hat{\epsilon}| \sim 1.5 |\epsilon'|$ and $1.9 |\epsilon'|$, respectively. With $|\epsilon'/\epsilon|_{g,\text{RH}} \sim 2 \times 10^{-3}$, this results in

$$\left| \frac{\hat{\epsilon}}{\epsilon} \right|_{\gamma,\text{RH}} \sim 4 \times 10^{-3}, \quad (97)$$

which is comparable to the contribution of Q_g as in Eq. (89). In full generality, $\hat{\epsilon}/\epsilon$ and ϵ'/ϵ are proportional to different combinations of the κ 's, so that it is possible for $\hat{\epsilon}$ to be significantly larger than ϵ' , although this does not seem likely.

6.2 Supersymmetric Models

In certain supersymmetric models, one can generate the $s \rightarrow d\gamma$ operators at one-loop via intermediate squarks and gluinos resulting in large $C_{\gamma,g}$. The enhancement is due both to the strong coupling constant and to the removal of chirality suppression present in the standard model. We follow Ref. [15] and work in the so-called mass-insertion approximation. The full expressions for $C_{\gamma,\text{susy}}^-$ can be found in Ref. [15]. Here we are interested only in the terms enhanced by $m_{\tilde{g}}/m_s$, with $m_{\tilde{g}}$ being the average gluino-mass, and they are

$$\begin{aligned} C_{\gamma,\text{susy}}^-(m_{\tilde{g}}) &= \frac{\pi \alpha_s(m_{\tilde{g}})}{m_{\tilde{g}}} \left[(\delta_{21}^d)_{\text{LR}} - (\delta_{21}^d)_{\text{RL}} \right] F_{\text{susy}}(x_{gq}), \\ C_{g,\text{susy}}^-(m_{\tilde{g}}) &= \frac{\pi \alpha_s(m_{\tilde{g}})}{m_{\tilde{g}}} \left[(\delta_{21}^d)_{\text{LR}} - (\delta_{21}^d)_{\text{RL}} \right] G_{\text{susy}}(x_{gq}), \end{aligned} \quad (98)$$

where the δ 's are the parameters of the mass-insertion formalism, $x_{gq} = m_{\tilde{g}}^2/m_{\tilde{q}}^2$, with $m_{\tilde{q}}$ being the average squark-mass, and

$$\begin{aligned} F_{\text{susy}}(x) &= \frac{4x(1+4x-5x^2+4x\ln x+2x^2\ln x)}{3(1-x)^4}, \\ G_{\text{susy}}(x) &= \frac{x(22-20x-2x^2-x^2\ln x+16x\ln x+9\ln x)}{3(1-x)^4}. \end{aligned} \quad (99)$$

For our estimates, it is sufficient to approximate $F_{\text{susy}}(x) \sim F_{\text{susy}}(1) = 2/9$ and $G_{\text{susy}}(x) \sim G_{\text{susy}}(1) = -5/18$. This approximation introduces an error smaller than a factor of two.

Then, by means of Eqs. (76) and (87), we obtain

$$\begin{aligned} \left| \frac{\hat{\epsilon}}{\epsilon} \right|_{\gamma,\text{susy}} &\simeq 92 \left(\frac{\alpha_s(m_{\tilde{g}})}{\alpha_s(500 \text{ GeV})} \right) \frac{500 \text{ GeV}}{m_{\tilde{g}}} \left| \text{Im} \left[(\delta_{12}^d)_{\text{LR}} - (\delta_{12}^d)_{\text{RL}} \right] \right| \\ &\simeq 0.4 \left| \frac{\epsilon'}{\epsilon} \right|_{g,\text{susy}}. \end{aligned} \quad (100)$$

We find that the Q_g^- contribution to ϵ' constrains the Q_γ^- contribution to $\hat{\epsilon}$ as it also happened in the left-right model. Once again with $|\epsilon'/\epsilon|_{g,\text{susy}} \sim 2 \times 10^{-3}$, the contribution of $C_{\gamma,\text{susy}}$ to $\hat{\epsilon}$ can reach

$$\left| \frac{\hat{\epsilon}}{\epsilon} \right|_{\gamma,\text{susy}} \sim 8 \times 10^{-4}. \quad (101)$$

If we incorporate the running of the coefficients down to a low-energy scale [31], we will instead have $|\hat{\epsilon}/\epsilon|_{\gamma,\text{susy}} \sim 1.7 \times 10^{-3}$, which is roughly comparable to the contribution of Q_g in Eq. (89). Our estimate, translated into $\tilde{\epsilon}'_{+-\gamma}$, is three times smaller than the estimate of this parameter in Ref. [12]. The factor of three can be traced back to the use in Ref. [12] of a hadronic matrix element three times larger than ours. These differences are a good indication of the level of precision that can be expected from this type of estimates.

7 Summary and Conclusions

We have reanalyzed direct CP violation in the electric amplitude for the decay $K_L \rightarrow \pi^+\pi^-\gamma$. The dominant CP violating observable arises from an interference between the IB (inner bremsstrahlung) and the E1 direct emission amplitudes. The previous theoretical definition of $\eta_{+-\gamma}$ results in a quantity that varies with kinematic variables and that differs from the previously used experimental definition of $\eta_{+-\gamma}$. To clarify this situation we have introduced a new theoretical quantity, $\hat{\epsilon}$, to parameterize *new* direct CP violation in this decay. This quantity is a constant, and by simple dimensional analysis is expected to be between a few and five times larger than ϵ' .

The experimental observable $\epsilon'_{+-\gamma}$ is related to $\hat{\epsilon}$ by a normalized integration over phase space. We find that the very different E_γ^* dependence of the IB and E1-DE amplitudes introduces a large kinematic suppression into $\epsilon'_{+-\gamma}$. In particular, for $E_\gamma^* > 20$ MeV, $|\epsilon'_{+-\gamma}| = 0.041|\hat{\epsilon}|$, and thus much smaller than ϵ' . This kinematic suppression can be reduced by increasing the minimum E_γ^* accepted at the cost of diminished statistics.

From presently available data we have extracted the bound $|\hat{\epsilon}/\epsilon| < 1.9$, significantly larger than the theoretical expectation $|\hat{\epsilon}/\epsilon| < 0.01$.

We have estimated the value of $\hat{\epsilon}$ in several models. The estimates are hindered by unknown hadronic matrix elements and must be considered order of magnitude estimates. The general conclusions from these estimates are

- If the CP -violating phase of the E1-DE amplitude is similar in size to the CP violating phase of A_0 , and the two do not cancel, then $\hat{\epsilon}$ is about as large as ϵ' and could be as much as five times larger. This is a refined form of the naive dimensional analysis estimate. This is the case in the standard model estimate of $\hat{\epsilon}$ where the CP -violating phase cannot be computed and is simply assumed to be of the same order as the phase of A_0 .
- Beyond the standard model, in models where the chromomagnetic dipole operator is enhanced, the maximum value of $\hat{\epsilon}$ is directly constrained by the fraction of ϵ' that is attributed to the new physics. Without invoking a fine-tuned cancellation in ϵ' , this implies that in these models $\hat{\epsilon}$ is also at most a few to five times larger than ϵ' . This happens, of course, because the bound from ϵ' is equivalent to the simple assumption that the CP violating phases of the E1-DE amplitude and A_0 are of similar size.
- In principle, $\hat{\epsilon}$ could be larger in models in which it is induced primarily by an electromagnetic dipole operator which does not contribute significantly to ϵ' . In the specific models that we studied, however, the coefficients of the electromagnetic and chromomagnetic dipole operators are highly correlated. For this reason, in these models, the maximum $\hat{\epsilon}$ is, once again, limited by the fraction of ϵ' that we attribute to the new physics.

To conclude, $\hat{\epsilon}$ is expected to be comparable to ϵ' although it can be as much as five times larger both in the standard model and beyond. Unfortunately, the experimental observable $\epsilon'_{+-\gamma}$ is kinematically suppressed, calculated to be $\sim 0.04 - 0.09\hat{\epsilon}$ for $E_\gamma^* > 20 - 50$ MeV.

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