



# The Physics and Symmetry of The Periodic Table of the Elements

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## Abstract

This is a high school module on the scientific basis of the Periodic Table of the Elements. We give a brief review of introductory Quantum Mechanics and the basis of the quantum numbers of electrons in an atom. We build up the atoms emphasizing the role of the symmetry of identical particles and the Pauli exclusion principle. We argue that symmetry is a powerful tool in all core science disciplines and that an early introduction of this topic is valuable.

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# 1 Elements of the Quantum Theory

It is generally agreed that the ubiquitous chart, the Periodic Table of the Elements (or the Mendele'ev Chart), is the starting point to the subject of Chemistry. We have discovered that very few high school teachers understand why the Periodic Table is what it is. This unit is an “arm waving” effort to explain, from fundamentals, how the Periodic Table gets to be the way it is, and, in so doing, touches on some quantum theory and the all important, but also all too unreasonably ignored, concepts of symmetry.

We'll assume that a preliminary course in conceptual physics or chemistry [1] gives students a sense of the structure of atoms: atoms are a central, very small nucleus, carrying almost the entire mass and the total positive charge of the atom. Outside the nucleus are the negatively charged electrons in cloud-like orbits about the nucleus. The laws of quantum theory teach us — and we transmit this to the chemistry students — that only certain special orbits of the electron motion, or orbitals, are allowed. The orbits are “quantized,” and hence the term “quantum theory.”

From Louis de Broglie's 1923 insight, we learn that electrons behave like waves, and each electron is associated with a “wavelength.” Experiments carried out by G. P. Thompson in England in 1928, and by C. Davisson and L. H. Germer at Bell Laboratories in New Jersey confirmed DeBroglie's theory that the wavelength “associated” with an electron is given by:

$$\lambda = \frac{h}{mv} \quad (1.1)$$

Here  $h$  is Planck's constant, the “logo” of quantum theory, and  $m$  is the electron mass and  $v$  the electron velocity. Indeed,  $mv$  is just the electron's “momentum.”

In the old quantum theory of Bohr we assume that when an electron is bound in an atom it moves in a circular orbit of radius  $R$ . The allowed orbits are only those in which the circumference of the orbit,  $2\pi R$ , matches to an exact integer number of wavelengths. This is much like the sound vibrations produced by a musical instrument, e.g., the “standing waves” that occur on a guitar string or in an organ pipe have wavelengths that are (inverse integer) multiples of the length of the string or pipe. Putting the statement mathematically, we can squeeze  $n$  wavelengths of the electron into one full circumference, or:

$$n\lambda = 2\pi R = \frac{nh}{mv} \quad (1.2)$$

where  $n$  is an integer: 1, 2, 3, etc., which enumerates orbits and  $\lambda$  is the wavelength of the electron, given by eq.(1.1).

We can now derive a formula for the allowed radii of the orbits in terms of other known quantities. Our delicious mixture of quantum and classical ideas leads to an expression for the orbit radius “ $R$ ” in the atomic orbit that depends only upon  $n$  (i.e., we eliminate the unknowns,  $\lambda$ ,  $v$ , etc.).

We start with Newton’s law of motion  $F = ma$ , and we assume a circular orbit. For a particle moving in a circular orbit of radius  $R$  with velocity  $v$  there is a “centripetal” (pointing toward the center) acceleration of  $v^2/R$ . Thus, to produce the motion in the circular orbit we need to apply a force of  $F = ma = mv^2/R$  directed toward the center of the orbit.

Our centripetal force is provided by the electrostatic attraction, the “Coulomb force,” between the negatively charged electron and the positively charged nucleus of the atom. This is given by  $Ze^2/R^2$ , for the force between the nucleus of charge  $Ze$  and the electron of charge  $e$ , separated by a distance  $R$ . (Note: Here, the charges are measured in the cgs system of units in terms of “esu” or “electrostatic units;” The electron has a charge of  $4.8 \times 10^{-10}$  esu )

So, we have:

$$\frac{mv^2}{R} = \frac{Ze^2}{R^2} \quad (1.3)$$

but eq.(1.2) tells us that  $v$  is related to  $R$ :

$$v = \frac{nh}{2\pi mR} \quad (1.4)$$

We substitute the expression for  $v$  in eq.(1.4) into eq.(1.3) and solve for  $R$ :

$$R = \frac{n^2 h^2}{4\pi^2 mZe^2} \quad (1.5)$$

Voila! This is the desired expression for the atomic orbit radius in terms of known quantities and the “quantum number”  $n$ , where  $n = 1, 2, 3, \dots$ . For the special case  $n = 1$  we define the expression of eq.(1.5) to be the symbol  $a_0$ :

$$a_0 = \frac{h^2}{4\pi^2 mZe^2} \quad (1.6)$$

The quantity  $a_0$  has dimensions of length, and is called the “Bohr radius.” When we put in Planck’s constant, the  $\pi$ ’s, etc., we get:

$$a_0 = 5.25 \times 10^{-9} cm \quad (1.7)$$

So we see how we get only certain allowed radii for the orbits, given by:

$$R = n^2 a_0 \quad (1.8)$$

This is completely different than the case in classical physics, where we could have any radius or any orbit we desire by simply choosing the velocity of the electron at will. We say that the orbits are “quantized.” For the special case of the Hydrogen atom we have  $Z = 1$ , and the orbital radii are then:  $a_0$ ,  $4a_0$ ,  $9a_0$  ...

What about the energy of the orbiting electron? The total energy of the electron is the sum of the kinetic energy, the “energy of motion,”  $\frac{1}{2}mv^2$ , and the potential energy,  $-Ze^2/R$ . Note that the potential energy is negative here because the force is attractive between the nucleus and electron; thus the electron is pulled down into the negative potential; a positive potential energy would represent a repulsive force, such as between pairs of electrons. Hence, for the total energy we have:

$$E = \frac{1}{2}mv^2 - \frac{Ze^2}{R} \quad (1.9)$$

However, from equation (1.3), we see that  $mv^2/2 = Ze^2/2R$ , i.e., the kinetic energy equals half the (positive) magnitude of the potential energy. So, upon substituting into eq.(1.9), the formula for the total energy, we have:

$$E = -\frac{Ze^2}{2R} \quad (1.10)$$

Now, using our formula for  $R$  in eq.(1.5) we get:

$$E = -\frac{2\pi^2 Z^2 e^4}{n^2 h^2} = -(13.6 \text{ electron volts}) \times Z^2/n^2 \quad (1.11)$$

Thus, each allowed orbit has an energy that is associated with with the quantum number  $n$ . For the special case of the Hydrogen atom, we have  $Z = 1$  and therefore the energies are:

$$\begin{aligned} n = 1 & \quad E_1 = -13.6 \text{ eV} \\ n = 2 & \quad E_2 = -3.4 \text{ eV} \\ n = 3 & \quad E_3 = -1.52 \text{ eV} \\ & \quad \text{etc.} \end{aligned} \quad (1.12)$$

What is the meaning of negative energy? This means that the electron is bound to the nucleus in an atom. Thus, if we want to free the electron from the “groundstate”

(the state of lowest energy, or most negative energy), we must give it at least 13.6 eV of energy. Then it can escape. So  $-13.6$  eV is the lowest possible energy of an electron bound in a Hydrogen atom, and therefore the most stable state.

We learn that once it is in an excited state, an electron can jump, or fall, down to a lower energy orbit, releasing a quantum of light energy: a “photon.” Conversely, an electron can jump from a lower energy to a higher energy orbit, if it absorbs the required energy from some outside source of photons. The quantum theory predicts the exact values of these photon energies to be the exact differences between the energies of the levels of the hopping electrons. Thus, Bohr’s simple theory explains the discrete spectral lines of ionized gases, such as in a discharge tube in the lab, or in the corona of the sun. This had been a complete mystery for the preceding half century of physics.

We apologize that here we used the “old quantum theory” of N. Bohr, rather than the modern (and complete) version based upon the Schroedinger Equation, with the electron described by its wave function  $\psi(x, t)$ . We all know that the Bohr theory we have outlined above gives the right answer and is a quick study that every student (and teacher) should know. In the full quantum theory, however, the electron does not circle the nucleus in a well defined orbit, but rather is described by a wave function,  $\psi_{n,\dots}(x, t)$ . The wave-function determines the probability of finding the electron in the  $n$ th orbital, at the location,  $x$ , at any time,  $t$ , given by:  $|\psi_{n,\dots}(x, t)|^2$ . All statements in the old quantum theory are essentially sort-of “averages” as defined by the more complete theory. We’ll have more to say about this below.

## 2 More Quantum Numbers

As we have seen, the quantum theory typically selects only certain discrete energies or orbits as contrasted with Newtonian (classical) theory where arbitrary energies and orbits would be allowed. Discrete values of the orbit are enumerated in the quantum theory by integers (or half-integers) called quantum numbers. Thus in eq.(1.5),  $n$  is a quantum number which can take on integer values 1, 2, 3, ... for the different radii.  $n$  is special, and is called the “principal quantum number.”

It turns out, however, that other properties of the atom, which would have classically a continuum of values, are also “quantized” and are allowed only certain values. One important property is angular momentum which in Newton’s mechanics is  $\vec{L} = \vec{r} \times \vec{p}$  or the magnitude,  $|\vec{L}| = mvR$  for the electron in a circular orbit in classical physics. In

Quantum Mechanics, the magnitude of  $\vec{L}$ , too, is quantized with a new quantum number  $\ell$ , and the formula for  $\vec{L}$  is somewhat more complicated:

$$|\vec{L}| = \hbar\sqrt{\ell(\ell + 1)} \quad (\ell = \text{an integer}) \quad (2.13)$$

where we introduce the more conventional quantity,  $\hbar = h/2\pi$ . To understand the orbital angular momentum in the atom, and the above formula, one really has to work through the full solution of the Schrodinger equation, which is beyond the scope of this article (we refer you to the standard book by L. Schiff [2] or any other reasonable book on Quantum Mechanics; it requires a basic understanding of ordinary and partial differential equations, and their special function solutions). We'll just state the results presently. The angular momentum quantum number,  $\ell$ , in an orbital of principle quantum number  $n$ , can take on values from zero to a maximum value which is  $n - 1$ . one less than the principal quantum number. For example, if  $n = 1$ , then  $\ell = 0$ . If  $n = 2$ , the  $\ell$  can take on values of 0 and 1; if  $n = 3$  then  $\ell$  can take on values of 0, 1 and 2, and so on.

Angular momentum is further complicated by the fact that it is a vector, and therefore can point in some direction in space. In Quantum Mechanics the question of measuring the direction of angular momentum is subtle and is posed somewhat philosophically, i.e., in any experiment the experimentalist chooses a direction in space, e.g., the “ $z$ ” direction, and asks: What is the value of the angular momentum pointing in this direction? Again, the answer is always quantized: For any value of  $\ell$ , the measured value of angular momentum along, e.g., the  $z$ -direction is

$$L_z = m\hbar \quad \text{where} \quad (m = -\ell - \ell + 1, -\ell + 2, \dots, \ell - 1, \ell) \quad (2.14)$$

For example, an orbit can have angular momentum  $\ell = 1$ , hence the magnitude of the total angular momentum is  $|\vec{L}| = \hbar\sqrt{1(1 + 1)} = \hbar\sqrt{2}$ . Then the experimentalist picks a direction, e.g., the  $y$ -axis, and she will measure a value of either  $\hbar$ , 0 or  $-\hbar$  along this axis, corresponding to the three allowed values of  $m$ , of (1, 0,  $-1$ ). Only when  $\ell$  becomes very large do we approach the classical limit. For example, with  $\ell = 1000$ , we can have a measured value (a “projection”) in the  $z$ -direction of  $m = 5$  (this is like having a classical angular momentum vector nearly perpendicular to the  $z$ -direction); alternatively we can have  $\ell = 1000$   $m = -998$  (this is like having the classical vector nearly pointing exactly in the  $-z$  direction), etc.

Finally, of quintessential importance, the electron itself has internal angular momentum, called spin. Spin, which sounds like what a top does, is really spooky; it is intrinsic

angular momentum which is simply a part of the electron. We can never stop an electron from spinning. We know the value of the electron spin angular momentum has  $\ell_e = 1/2$ , which is always a fixed value, and never changes. Then, the measured value of electron spin along the  $z$ -direction can be only one of two values (which we denote by  $s$ , the analogue of  $m$ ): either  $s = +1/2$  (often called “spin up”), or  $s = -1/2$  (often called “spin down”). If all this is somewhat unsettling to you, join the quantum crowd. We physicists become comfortable (eventually) *using* Quantum Mechanics, because it is true and it works, but we never really become comfortable that we fully *understand* it.

Thus, in the quantum world of an electron in an atom, there are four quantum numbers:

$$n, \ell, m, s \tag{2.15}$$

Let’s hold the many questions and accept that these four quantum numbers completely define the state of motion of an electron in an atom. From these quantum numbers and the mathematical solution of the quantum equations, we get a complete description of where the electron is in its various states.

As we mentioned above, in the full quantum theory, we can only compute the wave-function,  $\psi$ , which, when squared, gives the probability of finding an electron in some small volume somewhere at some time in the space of the atom.  $\psi$  is a complicated function of position,  $\vec{x} = (x, y, z)$  and time,  $t$ . This is written as  $\psi(x, y, z, t)$ . Each orbital is specified by  $n, \ell, m, s$ , so we write the wave-function for any given orbital as  $\psi_{n,\ell,m,s}(x, y, z, t)$ . In general, in quantum mechanics, the description of any physical system e.g. a quark, atom, molecule, two particles undergoing a collision, etc., are described by some kind of “wave function” in analogy to  $\psi$ . This function will in general depend on all the coordinates of all the particles in the system and will contain whatever other relevant properties exist of the particles, e.g., charges, spins, etc. To obtain predictions as to the results of measurements, we must take the absolute value squared of  $\psi$  which gives the probability of obtaining results from the measurements.

This is half of what we need to know to construct the Periodic Table. There’s more, and it revolves around the fact that all electrons are exactly identical in all respects. Let’s proceed.

### 3 Symmetry

Now that we have given a very limited view of how quantum theory works, we want to use the concept of symmetry to understand how nature builds up the chemical elements, atoms, from the simplest: hydrogen with one nuclear charge ( $Z = 1$ ) and one electron, all the way to uranium with ( $Z = 92$ ) 92 electrons and beyond.

We have noted that there are states of discrete energy, the lowest energy ( $n = 1$ ) being the most stable. For an atom of nuclear charge  $Z$ , e.g.,  $Z = 2, 3, 4...$  (i.e. helium, lithium, boron, ...) we must add electrons in order to balance the nuclear charge. Like a marble on a sloping floor which rolls down to the potential energy minimum, the electron will roll down to the lowest energy state of allowed motion — so too will the next 10 marbles. Where do they go?

We now have to understand the rules. The rules emerge from the concept of symmetry. We define symmetry as follows: a system exhibits symmetry when it does not change even though you perform some operation on it. A perfect cylinder can be rotated through any angle around the symmetry axis leaving the system identical to how it was before the rotation. A sphere exhibits an even more perfect symmetry; it doesn't change if you rotate it about any diameter through any angle. We call this rotational symmetry. A 3-bladed propeller, or an equilateral triangle, has a more restricted symmetry, it doesn't change if you rotate the propeller through  $120^\circ$  about its symmetry axis.

The fancy language is that the system is “invariant” (doesn't change) when subject to some symmetry operation [3]. Any physical system is described by certain mathematical equations, e.g., we have  $x^2 + y^2 + z^2 = R^2$  for the points defining a sphere of radius  $R$  in an (x,y,z) coordinate system. We can perform a rotation on the coordinate system about the origin. After the operation of rotation, each of the coordinates change  $x \rightarrow x'$ ,  $y \rightarrow y'$ , and  $z \rightarrow z'$ , but the symmetry of a sphere implies that after the rotation we still get:  $x'^2 + y'^2 + z'^2 = R^2$ . The mathematical description of the sphere hasn't changed, or is “invariant” under the rotation [3].

The symmetry we want to discuss presently has to do with two electrons, one located at  $\vec{x}$  and the other located at  $\vec{y}$ . There is a hypothesis that all electrons are identical in every detail. Thus, if we interchange the locations of the two electrons in space,  $\vec{x} \leftrightarrow \vec{y}$  the system looks the same, indeed, it is the same! This is a symmetry: it is called “exchange symmetry.”

How do we describe exchange symmetry mathematically? The description of our

two electrons is given by a wave function  $\psi(\vec{x}, \vec{y}, t)$  (we'll henceforth not write  $t$ ). If we interchange the two electrons, we interchange  $\vec{x}$  and  $\vec{y}$  we will get the new wave-function  $\psi(\vec{y}, \vec{x})$ . *But*, The result of this interchange, according to our hypothesis, must be an identical system! *So*, there must be no change in the probability of observing the two electrons, one at  $x$  and one at  $y$ , i.e.,

$$|\psi(\vec{x}, \vec{y})|^2 = |\psi(\vec{y}, \vec{x})|^2 \quad (3.16)$$

At the wave-function level there are two ways of satisfying this equation:

$$\text{either (A): } \psi(\vec{x}, \vec{y}) = \psi(\vec{y}, \vec{x}) \text{ or (B): } \psi(\vec{x}, \vec{y}) = -\psi(\vec{y}, \vec{x}) \quad (3.17)$$

(this is no deeper than the fact that  $\sqrt{4}$  has two solutions,  $+2$  and  $-2$ ). Solution (A) says that  $\psi$  is an even function under the swapping of the positions of the two particles, sometimes called a symmetrical function. Solution (B) says that  $\psi$  is an odd function under swapping positions, called anti-symmetric. So what?

It turns out that nature makes a really big fuss about even vs. odd behaviors under exchange of identical particles. A broad class of particles, including electrons, protons, neutrino's and quarks, obey Solution (B), i.e., they are anti-symmetric under interchange of any two particles. Yet, another class of particles, photons, pions, W's, Z's, . . . , behave as in Solution (A) and are even functions under exchange. There is a very deep connection: the first list all have a spin equal to  $1/2, 3/2, 5/2, . . .$  i.e. odd half integer spins. The second class have spins  $0, 1, 2, 3 . . .$  integral spins. These two groups behave very differently. The half integer particles are collectively called "fermions," the integer spin objects are called "bosons."

Now here is how they behave differently. The odd particles have the property that if all of their quantum numbers are the same (charges, spins, etc.) and we attempt to set  $\vec{x} = \vec{y}$ , i.e. the two particles are pushed to the same point in space at the same time, then (4) tells us that

$$\psi(\vec{x}, \vec{x}) = -\psi(\vec{x}, \vec{x}) \quad (3.18)$$

which can only be satisfied by  $\psi(\vec{x}, \vec{x}) = 0$ . Therefore, the probability of identical electrons co-existing at the same place in space at the same time, is zero (their spins must be aligned in this case, i.e.,  $s_1 = s_2$ )! It is as if some powerful force prevents them from co-existing at the same point in space. Lest you say: "of course, it is just the electrical repulsion of negative charges," we remind you that the prohibition against co-existing in the same point in space simultaneously also applies to neutrons and neutrinos!

The particles of the boson group, on the other hand, have no problem co-existing at the same point in space (again, for identical spin states, charges, etc.) because  $\psi(x, y)$  doesn't have to vanish at  $x = y$  for the even symmetry. In fact, when many many particle states of bosons are created there is an enhanced probability of all of the bosons going into exactly the same state of position, or momentum. You may have read about Einstein-Bose condensates, i.e. clusters of boson particles that form a very compact and dense coherent state; laser beams are coherent states of many photons in the same exact state of motion, while superfluids are coherent states of bosonic  $\text{He}^4$  marching together in bosonic lock-step.

The apparent attraction of bosons to each other and the apparent repulsion of fermion particles give rise to the descriptive term “exchange force,” even though there really is no force acting here, only a curious behavior of the wave functions respecting the symmetry of identical particles. The exchange force, which is not a real force, requires some justification. The wave function and its square, the probability of a given arrangement of two electrons, gives us some feeling for this. If we consider a situation which has a high probability, e.g. throwing a seven in a pair of dice, the behavior is as if the “3” and the “4” have an attraction for each other, or, in general, the dice are much more “attracted” to the “7”. Similarly, they are least attracted or even repelled from the “2” and the “12”, the least probable numbers.

So very probable outcomes i.e.,  $|\psi(x, x)|^2$  close to unity, appears to act like an attractive force, whereas  $|\psi(x, x)|^2$  close to zero appears as a repulsive force. Physicists call this the exchange force (which is not a force!).

## 4 Wolfgang Pauli and the Elements

Now comes the question of building up the elements in the Periodic Table. We can discuss this by considering a nucleus of  $Z = 2$  (Helium). We need to add a second electron to the atom whose nucleus has charge  $+2$ . One would naturally assume that the electron would go to the lowest energy orbit,  $n = 1$ . However, we know that two negatively charged electrons repel each other and thus, it may come out that the second electron is happier (the total energy is lower) if it goes into the  $n = 2$  orbit. After some analysis, however, it turns out that there would have been plenty of room for more electrons to pile into the  $n = 1$  orbit, as  $Z$  is increased, minimizing the total energy. Thus Helium will have two electrons in the  $n = 1$  orbits. As we increase  $Z$ , then, are all atoms just fatter and

fatter Hydrogen atoms, with all electrons in the  $n = 1$  orbit? For example, does the Iron atom with  $Z = 56$  have 56 electrons squashed into the  $n = 1$  orbit?

The answer is no! We have just seen that here is an “influence” in nature, much stronger than the Coulomb force, which determines where the second, and third, and fourth, .... electrons *cannot* go. It is just the odd property of the wave-function under exchange of two electrons, and it implies the dramatic result:

**No two electrons can co-exist in the same identical quantum state.**

This “influence” is called: *The Pauli Exclusion Principle*, after Wolfgang Pauli, the Austrian genius who did most of his research at the Swiss University, ETH, in Zurich, and helped to build the modern quantum theory in the early 20th Century. This crucial principle determines why The Periodic Table is what it is; it also determines the future of stars and planets and protons and people. Pauli’s discovery ultimately gives rise to the strict set of rules as to how we go from the simplest element (Hydrogen of  $Z = 1$ ), to the complex elements with  $Z = 2, 3, \dots 92, \dots 110\dots$ . In German, it is the word “Aufbauprinzip”; in English translation, it is the “Principle of Building Up (the Elements)”.

In the full quantum theory description of an electron in an atom, the electron is completely described by its four quantum numbers:  $n, \ell, m, s$ . The Pauli Principle then says: “no two electrons in an atom can have the same four quantum numbers.” *This explains the Periodic Table of Elements*. It prevents all the electrons from going to the lowest energy state, as they would in a world ruled by classical physics. It also influences the chemical activity of an atom by determining whether or not electrons can be shared between atoms. Thus, the columns of the Periodic Table of elements, which represent the common chemical properties of the atoms within a given column, is controlled by the exclusion principle.

As the simplest example, the electron in atomic Hydrogen goes to the  $n = 1, \ell = 0, m = 0, s = 1/2$  state. In Helium we need to add another electron. It too can go into the  $n = 1$  state (which forces  $\ell = 0, m = 0$ ) only if its spin is  $s = -1/2$ , i.e. opposite to the first electron. This exhausts all the possibilities of the  $n = 1$  state. We thus have a “closed shell.”

Lithium has three electrons, the first two huddle in the  $n = 1$  state, i.e., the closed shell structure of Helium. The third electron must go into the  $n = 2$  state.

Beyond Lithium, how many electrons can go into the  $n = 2$  state? First we fill the  $\ell = 0$  subshell with two electrons, one with  $s = +1/2$ , one with  $s = -1/2$ . This shell has  $m = 0$ . Then we fill the  $\ell = 1, m = -1$  state with 2 electrons. Next, we can put 2 electrons in the  $\ell = 1, m = 0$ , state and finally 2 electrons in the  $\ell = 1, m = +1$  state. The total number of electrons allowed into the next  $n = 2$  shell is 8. This takes us from Lithium through Beryllium, Boron ... to Neon with  $Z = 10$ . All of this is illustrated in table I.

Not only do we build up all the elements this way, but we get to understand the chemical properties of the elements. Hydrogen has room for one electron in the  $n = 1$  state. It therefore forms compounds with elements that have a spare electron it can share in the  $n = 1$  shell, e.g.,  $H_2$  or  $LiH$ , Lithium Hydride. Once a shell is completely filled, as in  $n = 1$  Helium, or  $n = 2$  Neon, or  $n = 3$  Argon, etc., we have an inert or noble gas.

To follow the "aufbauprinzip" in detail requires a knowledge of the rules which connect the various quantum numbers. These emerge from the solution of the quantum theory equivalent of Newton's Laws, the Schrodinger equation. The rules are not important for one to grasp the central idea, electrons will try to go to the lowest energy state, but they must be consistent with the Pauli rule.

Although we noted that the various  $n$ -states differ from each other in energy, there are smaller energy differences between different  $\ell$ -values and even  $m$ -values. The problems get pretty complex when we are deciding how electrons are shared between atoms in a molecule. Larger molecules require computer solutions and quantum computational chemistry is a hot subject these days.

The exchange forces (which are not real forces) play a major role in the theory of molecular bonds — the heart of chemistry. The Pauli Exclusion Principle also shows up in the detailed construction of atomic nuclei, a crowded little volume of space filled with spin-1/2 neutrons and protons. It shows up in such astronomical processes which govern the life cycle of stars, from supernova, neutron stars to black holes. You did good, Professor Pauli!

It is rather remarkable that the identity of particles gives rise to the 100 or so chemical elements, which in turn give rise to billions of possible molecules and gives our world variety and richness!

As we have seen, the patterns of atoms, the properties of chemistry, indeed much of the stability of matter itself, is governed by the Pauli principle. Where does it come

from? We have only asserted, but not proved, that odd half-integer spins,  $\frac{1}{2}, \frac{3}{2}, \dots$ , (known as “fermions”) obey the odd solution under exchange of position, of eq(3.17), while the integer spins,  $0, 1, 2, \dots$  (known as “bosons”) go with the even solution. Indeed, the Pauli exclusion principle actually follows from deeper symmetries of physics, in particular, the symmetry of “rotational invariance.” Sometimes the Pauli principle is called the “spin and statistics theorem.” We attempt to give a simple heuristic proof at our website, **[www.emmynoether.com](http://www.emmynoether.com)** devoted to symmetry and physics. We encourage you to visit the website, and welcome your comments.

Element	$Z$	$n = 1$ $\ell = 0$ $m = 0$	$n = 2$ $\ell = 0$ $m = 0$	$n = 2$ $\ell = 1$ $m = (-1, 0, +1)$
H	1	$+\frac{1}{2}$	0	0
He	2	$+\frac{1}{2}, -\frac{1}{2}$	0	0
Li	3	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}$	0
Be	4	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}, -\frac{1}{2}$	0
B	5	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}$
C	6	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}, -\frac{1}{2}$
N	7	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}$
O	8	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}$
F	9	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}$
Ne	10	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}$

Table I. We indicate the sequential filling of atomic orbitals by electrons. Each electron is denoted by the value of its spin quantum number,  $+\frac{1}{2}$  or  $-\frac{1}{2}$ . The columns are available orbitals labelled by  $n$ , the principal quantum number,  $\ell$  the angular momentum quantum number, and  $m$  the magnetic, or  $z$ -component of angular momentum, quantum number.

## Bibliography

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