FERMILAB-CONF-00/310-T

TESTS OF QUARK MASS TEXTURES*

ANDREA ROMANINO
Department of Physics, Theoretical Physics, University of Oxford
1 Keble Road, Oxford OX1 3NP, UK
and Theory Group, Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, IL 60510, USA

The classic hints on the structure of the quark mass matrices are shortly reviewed and the possibility of obtaining further information through precise texture analysis is discussed with the aid of a specific example.

In this presentation, we consider the bottom-up approach to the problem of the origin of flavor. In this approach, one looks for indications about the structure of the flavor matrices, the starting point for the quest for a theory of flavor, in the measured values of fermion masses and mixings. After shortly reviewing the well known hints on quark mass textures, we discuss the possibility of obtaining further information through "precision texture physics". This possibility arises on one hand due to the improvement in the experimental fits of the CKM matrix, on the other hand due to an efficient use of the available information on the light quark masses.

In order to pursue a bottom-up approach, one has to assume that the smallness of the light quark masses does not arise from cancellations in the diagonalization of mass matrices whose entries are all of order one, but is rather reflected in the hierarchical structure of those matrices.[a] By using this assumption, one can derive the relations

$$\frac{m_d}{m_s} = \left|\frac{M^D_{11}}{m_s}e^{i\phi_D} - \frac{M^D_{12}}{m_s}\frac{M^D_{21}}{m_s}\right| \qquad \left|\frac{V_{td}}{V_{ts}}\right| = \left|\frac{M^D_{21}}{m_s} - \Delta\right| \tag{1a}$$

$$\frac{m_u}{m_c} = \left|\frac{M^U_{11}}{m_c}e^{i\phi_U} - \frac{M^U_{12}}{m_c}\frac{M^U_{21}}{m_c}\right| \qquad \left|\frac{V_{ub}}{V_{cb}}\right| = \left|\frac{M^U_{21}}{m_c} - \Delta\right|, \tag{1b}$$

where

$$\Delta = \frac{1}{V_{cb}}\left[\frac{M^U_{31}}{m_t} + \frac{\bar{M}^U_{23}M^U_{21}}{m_t^2} - \frac{M^D_{31}}{m_b} - \frac{\bar{M}^D_{23}M^D_{21}}{m_b^2}\right]. \tag{2}$$

The presence of texture zeros in the mass matrices is suggested by the observation

*This work is supported by the TMR Network under the EEC Contract No. ERBFMRX–CT960090.

[a]Light masses can arise in a natural way from the diagonalization of a non-hierarchical mass matrix if the necessary cancellations are guaranteed by a symmetry, as in "democratic" models of quark masses [1].

that the phenomenological relations

$$\left|\frac{V_{td}}{V_{ts}}\right| \sim \sqrt{\frac{m_d}{m_s}} \qquad \left|\frac{V_{ub}}{V_{cb}}\right| \sim \sqrt{\frac{m_u}{m_c}} \tag{3}$$

are exact at the lowest possible order in the diagonalization of M_U, M_D if $M_{11} = M_{13} = M_{31} = 0$ and $|M_{12}| = |M_{21}|$ in both M_U, M_D [2].

It turns out that it is possible to study the next to leading order corrections to eqs. (3) despite the uncertainties in our knowledge of quark masses and mixing angles. Notice that these corrections are sensitive to some matrix elements below the diagonal otherwise unaccessible which, in turn, correspond at leading order to right-handed rotations. In particular, $|V_{ub}/V_{cb}|$ (and, to a lesser extent, $|V_{td}/V_{ts}|$) is sensitive to the right-handed angle $\theta_{23}^{D_R}$, defined by $\tan\theta_{23}^{D_R} = |M^D_{32}/M^D_{33}|$. This angle is particularly interesting since in presence of a minimal stage of grand unification a large mixing angle for atmospheric neutrinos originating from the charged lepton sector would correspond to $\tan\theta_{23}^{D_R} = \mathcal{O}(1)$ [3]. As an example, we will consider the sensitivity to $\theta_{23}^{D_R}$ in the context of U(2) theories [4], that provide a particularly elegant theoretical motivation for the observed pattern of fermion masses and mixings (see ref. [5] for an example of how $\tan\theta_{23}^{D_R} = \mathcal{O}(1)$ can arise in this context). In this case the contribution proportional to $\tan\theta_{23}^{D_R}$ dominates the correction Δ in eqs. (1) and one has

$$\left|\frac{V_{ub}}{V_{cb}}\right| \simeq \left|\sqrt{\frac{m_u}{m_c}} + e^{i\phi_1}\tan\theta_{23}^{D_R}\frac{m_s/m_b}{|V_{cb}|}\sqrt{\frac{m_d}{m_s}}\right| \tag{4a}$$

$$\left|\frac{V_{td}}{V_{ts}}\right| \simeq \left|\sqrt{\frac{m_d}{m_s}} + e^{i(\phi_1+\phi_2)}\tan\theta_{23}^{D_R}\frac{m_s/m_b}{|V_{cb}|}\sqrt{\frac{m_d}{m_s}}\right| \tag{4b}$$

besides $|V_{us}| = |\sqrt{m_d/m_s} - e^{i\phi_2}\sqrt{m_u/m_c}|$, where ϕ_1 and ϕ_2 are generic phases. For a given value of $\theta_{23}^{D_R}$, the relations (4) allow a comparison between the experimental determination of $|V_{ub}/V_{cb}|$ and $|V_{td}/V_{ts}|$ and the determination in terms of the light quark mass rations. Here, "experimental" refers to the measurement of $|V_{ub}/V_{cb}|$ at LEP and CLEO and to the measurement of $|V_{td}/V_{ts}|$ through the mass differences in the B_d and B_s systems. Notice that, being one loop quantities in the standard model, the latter might be sensitive to new physics in B mixing. For our purposes, it is useful not to include ϵ_K and $\sin 2\beta$ (also likely to receive contributions at one loop) in the experimental fit.

Fig. 1 shows such a comparison in the plane of the Wolfenstein corrected $\bar{\rho}$, $\bar{\eta}$ parameters. The fit of the experimental constraints on $\bar{\rho}$, $\bar{\eta}$ is represented by the shaded regions (68% and 95% CL). The dashed circles around the origin correspond to the $|V_{ub}/V_{cb}|$ constraint. The Δm_{B_d} mass difference determines the distance from the point (1,0) with a large hadronic uncertainty (dashed curves). The lower limit on $\Delta m_{B_s}/\Delta m_{B_d}$, less sensitive to hadronic uncertainties, sets an upper bound on that distance (solid curve). The predictions for $\bar{\rho}$, $\bar{\eta}$ in terms of the quark masses through eqs. (4) for $\tan\theta_{23}^{D_R} = 2|V_{cb}|$ and $e^{i\phi_1} = \pm 1$ are instead represented by the empty contours. In obtaining such precise predictions it is crucial

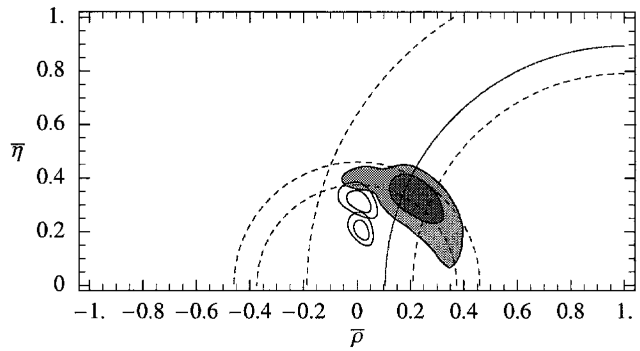

Fig. 1. Comparison between the conservative "experimental" determination of $\bar{\rho}$, $\bar{\eta}$ (shaded regions) and the "theoretical" determination in terms of light quark masses, eqs. (4), for $\tan\theta_{23}^{D_R} = 2|V_{cb}|$ and $e^{i\phi_1} = \pm 1$.

to take advantage of the precisely determined combination of light quark masses $Q = (m_s/m_d)/\sqrt{1-(m_u/m_d)^2} = 22.7 \pm 0.8$ [6]. Notice the good sensitivity to a relatively small variation of $\theta_{23}^{D_R}$. Notice also that, roughly speaking, varying $\theta_{23}^{D_R}$ moves the contours along the $\bar{\eta}$ axis. The combination $\tan\theta_{23}^{D_R}\cos\phi_1$ is thus essentially determined by the agreement with the measured value of $|V_{ub}/V_{cb}|$, which is unlikely to be affected by new physics. A quantitative study shows that largish values of $\tan\theta_{23}^{D_R}\cos\phi_1$ are preferred, thus suggesting a non-negligible contribution to the atmospheric neutrino mixing from the charged lepton sector which might be large enough to saturate it.[b] On the other hand, $\theta_{23}^{D_R}$ does not affect the distance from (1,0), which appears to be different for the theoretical prediction (empty blobs) and the standard model fit. In the present context, such a mismatch can be regarded as a measure of the new physics contributions to B-mixings. The measurement of Δm_{B_s} will allow a more precise determination of these contributions.

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[b]This depends on the value of the unknown phase ϕ_1 and on the Clebsh-Gordan coefficients of SU(5) associated to its breaking in the Yukawa sector.