

ANDREAS S. KRONFELD

Theoretical Physics Department, Fermi National Accelerator Laboratory, Batavia, IL, USA

Computational and theoretical developments in lattice QCD calculations of B and D mesons are surveyed. Several topical examples are given: new ideas for calculating the HQET parameters $\bar{\Lambda}$ and λ_1 ; form factors needed to determine $|V_{cb}|$ and $|V_{ub}|$; bag parameters for the mass differences of the B mesons; and decay constants. Prospects for removing the quenched approximation are discussed.

1 Introduction

In the standard model, interactions involving the Cabibbo-Kobayashi-Maskawa (CKM) matrix violate CP , with strength proportional to the area of the “unitarity triangle.” Fig. 1 shows a recent summary¹ of the triangle. The dominant uncertainties are theoretical, coming from non-perturbative QCD. Each blob shows experimental uncertainties for fixed theoretical inputs, so the range of blobs illustrates the theoretical uncertainties. If measuring the apex $(\bar{\rho}, \bar{\eta})$ were the only goal, one might conclude from Fig. 1 that the most pressing issue is to reduce the theoretical uncertainties, which would require greater investment in computing for lattice QCD.

Measuring $(\bar{\rho}, \bar{\eta})$ is not the most exciting goal, however. “High-energy physics is exciting and will remain exciting, precisely because it exists in a state of permanent revolution.”² That means we would prefer to discover additional, non-KM sources of CP violation. Indeed, “it is possible, likely, un-

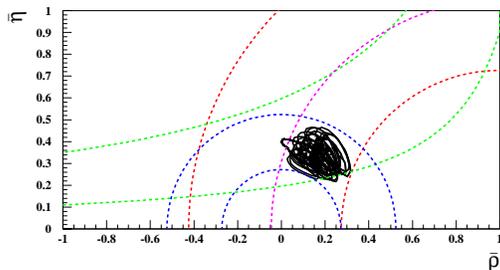


Figure 1. Constraints on the unitarity triangle from CP conserving B decays and indirect CP violation in the kaon. From Plaszczynski and Schune.¹

avoidable, that the standard model’s picture of CP violation is incomplete.”³

Lattice QCD can aid the discovery of new sources of CP violation and may be essential. A lot of information will be necessary to figure out what is going on at short distances. One way to think about this is sketched in Fig. 2. The triangle $B\gamma A$ is determined from (quark-level) tree processes. The side B requires $|V_{ud}|$ from $n \rightarrow pe^- \bar{\nu}$, and $|V_{ub}|$ from $B^- \rightarrow \rho^0 l^- \bar{\nu}$ or $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}$; the angle γ requires the CP asymmetry of $B^\pm \rightarrow D_{CP}^0 K^\pm$ (or $B_s \rightarrow D_s^\pm K^\mp$); the side A requires $|V_{cd}|$ from $D^0 \rightarrow \pi^- l^+ \nu$, and $|V_{cb}|$ from $B^- \rightarrow D^{0(*)} l^- \bar{\nu}$. One could call this the “tree triangle”. The triangle $\alpha C \beta$ is determined from mixing processes (including interference of decays with and without mixing). The angle α requires the asymmetry of $B \rightarrow \rho\pi$; the side C requires $|V_{td}|$ from $\Delta m_{B_d^0}$, and $|V_{tb}|$ from $t \rightarrow W^+ b$; the angle β requires the asymmetry of $B^0 \rightarrow J/\psi K_S$. One could call this the “mixing triangle”.

Checking whether the mixing triangle agrees with the tree triangle tests for new physics in the amplitude of B_d^0 - \bar{B}_d^0 mixing. New physics in the magnitude muddles the

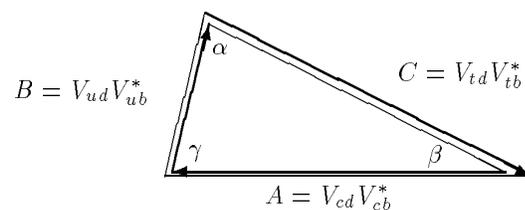


Figure 2. Two different unitarity triangles: the tree triangle $B\gamma A$, and the mixing triangle $\alpha C \beta$.

extraction of $|V_{td}|$, and new physics in the phase muddles the extraction of angles α and β . Similarly, taking $\Delta m_{B_s^0}$, $B_s \rightarrow D_s^\pm K^\mp$, and $B_s \rightarrow J/\psi \eta^{(\prime)}$ (or $J/\psi \phi$) sorts out new physics in B_s^0 - \bar{B}_s^0 mixing.

These tests are impossible without knowing the sides accurately, so hadronic matrix elements are needed. In a few cases a symmetry provides it, e.g., isospin cleanly yields the matrix element for $n \rightarrow pe^- \bar{\nu}$. For the others, we must “solve” non-perturbative QCD and, therefore, *you* need lattice calculations.

You are probably tired of waiting and may ask why results should come any time soon. The fastest of today’s computers are now powerful enough to eliminate the sorest point: the quenched approximation. Also, (lattice) theorists have slowly developed a culture of estimating systematic uncertainties, which is now not bad and would improve if more non-practitioners became sufficiently informed about the methods to make constructive suggestions.

The rest of this talk starts with some theoretical aspects that might make it easier for the outsider to judge the systematic errors of heavy quarks on the lattice. Then I show recent results needed for the sides A , B , and C .

2 Lattice Spacing Effects (Theory)

Lattice QCD calculates matrix elements by computing the functional integral, using a Monte Carlo with importance sampling. Hence, there are statistical errors. This part of the method is well understood and, these days, rarely leads to controversy. When conflicts do arise, they usually originate in the treatment of systematics. The non-expert does not need to know how the Monte Carlo works, but can develop some intuition of how the systematics work. Don’t be put off by lattice jargon: the main tool is familiar to all: it is effective field theory.

Lattice spacing effects can be cataloged with Symanzik’s local effective Lagrangian

(LE \mathcal{L}).⁴ Finite-volume effects can be controlled and exploited with a general, massive quantum field theory.⁵ The computer algorithms work better for the strange quark than for down or up, but the dependence on m_q can be understood and controlled via the chiral Lagrangian.⁶ Finally, discretization effects of the heavy-quark mass m_Q are treated with HQET⁷ or NRQCD.⁸ In each case one can control the extrapolation of artificial, numerical data, if one generates numerical data close enough to the real world.

Volume effects are unimportant in what follows, and chiral perturbation theory is a relatively well-known subject. Therefore, here I will focus on the effective field theories that help us control discretization effects.

2.1 Symanzik’s LE \mathcal{L}

Symanzik’s formalism⁴ describes the lattice theory with continuum QCD:

$$\mathcal{L}_{\text{lat}} \doteq \mathcal{L}_{\text{cont}} + \sum_i a^{s_i} C_i(a; \mu) \mathcal{O}_i(\mu), \quad (1)$$

where the symbol \doteq means “has the same physics as”. The LE \mathcal{L} on the right-hand side is defined in, say, the $\overline{\text{MS}}$ scheme at scale μ . The coefficients C_i describe short-distance physics, so they depend on the lattice spacing a . The operators do not depend on a .

If $\Lambda_{\text{QCD}} a$ is small enough the higher terms can be treated as perturbations. So, the a dependence of the proton mass is

$$m_p(a) = m_p + a C_{\sigma F} \langle p | \bar{\psi} \sigma \cdot F \psi | p \rangle, \quad (2)$$

taking the leading operator for Wilson fermions as an example. To reduce the second term one might try to reduce a greatly, but CPU time goes as $a^{-(5 \text{ or } 6)}$. It is more effective to combine several data sets and extrapolate, with Eq. (2) as a guide. It is even better to adjust things so $C_{\sigma F}$ is $O(\alpha_s^\ell)$ or $O(a)$, which is called Symanzik improvement of the action. For light hadrons, a combination of improvement and extrapolation is best, and you should look for both.

2.2 HQET for large m_Q

The Symanzik theory, as usually applied, assumes $m_q a \ll 1$. The bottom and charm quarks' masses in lattice units are at present large: $m_b a \sim 1$ -2 and $m_c a$ about a third of that. It will not be possible to reduce a enough to make $m_b a \ll 1$ for many, many years. So, other methods are needed to control the lattice spacing effects of heavy quarks. There are several alternatives:

1. static approximation⁹
2. lattice NRQCD¹⁰
3. extrapolation from $m_Q \approx m_c$ up to m_b
- 3'. combine 3 with 1
4. normalize systematically to HQET¹¹

All use HQET in some way. The first two discretize continuum HQET; method 1 stops at the leading term, and method 2 carries the heavy-quark expansion out to the desired order. Methods 3 and 3' keep the heavy quark mass artificially small and appeal to the $1/m_Q$ expansion to extrapolate back up to m_b . Method 4 uses the same lattice action as method 3, but uses the heavy-quark expansion to normalize and improve it. Methods 2 and 4 are able to calculate matrix elements directly at the b -quark mass.

The methods can be compared and contrasted by *describing* the lattice theories with HQET.¹² This is, in a sense, the opposite of *discretizing* HQET. One writes down a (continuum) effective Lagrangian

$$\mathcal{L}_{\text{lat}} \doteq \sum_n \mathcal{C}_{\text{lat}}^{(n)}(m_Q a; \mu) \mathcal{O}_{\text{HQET}}^{(n)}(\mu), \quad (3)$$

with the operators defined exactly as in continuum HQET, so they do not depend on m_Q or a . As long as $m_Q \gg \Lambda_{\text{QCD}}$ this description makes sense. There are two short distances, $1/m_Q$ and the lattice spacing a , so the short-distance coefficients $\mathcal{C}_{\text{lat}}^{(n)}$ depend on $m_Q a$. Since all dependence on $m_Q a$ is isolated into the coefficients, this description

shows that heavy-quark lattice artifacts arise only from the mismatch of the $\mathcal{C}_{\text{lat}}^{(n)}$ and their continuum analogs $\mathcal{C}_{\text{cont}}^{(n)}$.

For methods 1 and 2, Eq. (3) is just a Symanzik $\text{LE}\mathcal{L}$. For lattice NRQCD we recover the well-known result that some of the coefficients have power-law divergences.¹⁰ So, to take the continuum limit one must add more and more terms to the action. This leaves a systematic error, which, in practice, is usually accounted for conservatively.

Eq. (3) is more illuminating for methods 3 and 4, which use Wilson fermions (with an improved action). Wilson fermions have the same degrees of freedom and heavy-quark symmetries as continuum QCD, so the HQET description is admissible for all $m_Q a$. Method 4 matches the coefficients of Eq. (3) term by term, by adjusting the lattice action. In practice, this is possible only to finite order, so there are errors $(\mathcal{C}_{\text{lat}}^{(n)} - \mathcal{C}_{\text{cont}}^{(n)}) \langle \mathcal{O}_{\text{HQET}}^{(n)} \rangle$, starting with some n . Method 3 reduces $m_Q a$ until the mismatch is of order $(m_Q a)^2 \ll 1$ (or $\lesssim 1$). This runs the risk of reducing m_Q until the heavy-quark expansion falls apart.

The non-expert can get a feel for which methods are most appropriate by asking himself what order in Λ_{QCD}/m_b is needed. For zeroth order, method 1 will do. For the first few orders, the others are needed, although with method 3 one should check that the calculation's Λ_{QCD}/m_Q is small enough too.

3 New Results

3.1 $\bar{\Lambda}$ and λ_1

The matching of lattice gauge theory to HQET provides a new way to calculate matrix elements of the heavy-quark expansion.¹³ The spin-averaged B^*-B mass is given by¹⁴

$$\bar{M} = m + \bar{\Lambda} - \lambda_1/2m, \quad (4)$$

where m is the heavy quark mass, and $\bar{M} = \frac{1}{4}(3M_{B^*} + M_B)$. The lattice changes the short-distance definition of the quark mass:¹²

$$\bar{M}_1 - m_1 = \bar{\Lambda}_{\text{lat}} - \lambda_{1\text{lat}}/2m_2. \quad (5)$$

Because the lattice breaks Lorentz symmetry, $m_1 \neq m_2$, but they are still calculable in perturbation theory.¹⁵ The lambdas in Eq. (5) are labeled “lat” because they suffer lattice artifacts from the gluons and light quark.

After fitting a wide range of lattice data to Eq. (5) *and* taking the continuum limit, we find¹³ $\bar{\Lambda} = 0.68_{-0.02}^{+0.02}$ GeV and $\Lambda_1 = -(0.45 \pm 0.12)$ GeV² in the quenched approximation. The lambdas appear also in the heavy-quark expansion of inclusive decays. Although the current analysis is thorough, there are several ways to improve it.¹³ For example, Λ_{lat} has an unexpectedly large a dependence, so the analysis should be repeated with an action for which $C_{\mathcal{O}\mathcal{F}}$ in Eq. (2) is $O(a)$.

3.2 $B \rightarrow \pi l \nu$ form factors and V_{ub}

It is timely to discuss $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}$, because there are three calculations to compare, using lattice NRQCD (method 2),¹⁶ the extrapolation method (method 3),¹⁷ and the HQET matching method (method 4).¹⁸ UKQCD’s work is final,¹⁷ and the other two are preliminary.^{16,18} The decay rate requires a form factor, called $f_+(E)$, which depends on the pion’s energy in the B ’s rest frame, $E = v \cdot p_\pi$. It is related to the matrix element $\langle \pi | V^\mu | B \rangle$, which can be computed in lattice QCD. The systematics are smallest when the pion’s three-momentum is small.

The three recent results are compared in Fig. 3. The error bars shown are statistical only. For NRQCD these are larger than expected.¹⁶ For the other two, the comparison gives a fair idea of systematics that are not common to both, because the extrapolation of heavy quark mass needed with method 3 amplifies the statistical error.¹⁷ The other two works^{16,18} compute directly at the b quark mass and, thus, circumvent this problem. The heavy quark masses of UKQCD¹⁷ are all below 1.3 GeV, and as low as 500 MeV, so one might worry whether the heavy-quark expansion applies.

3.3 $B^- \rightarrow D^{0(*)} l^- \bar{\nu}$ and V_{cb}

The form factors $\mathcal{F}_{B \rightarrow D^{(*)}}$ of the decays $B^- \rightarrow D^{(*)} l^- \bar{\nu}$ are normalized to unity for infinite quark masses. What is needed from lattice QCD, therefore, is the deviation from the unity for physical quark masses. Hashimoto *et al.*^{19,20} have devised methods based on double ratios, in which all the uncertainties cancel in the symmetry limit. Consequently, all errors scale as $\mathcal{F} - 1$, not as \mathcal{F} .

For $B^- \rightarrow D l^- \bar{\nu}$ they find¹⁹ (published)

$$\mathcal{F}_{B \rightarrow D}(1) = 1.058 \pm 0.016 \pm 0.003_{-0.005}^{+0.014} \quad (6)$$

where error bars are from statistics, adjusting the quark masses, and higher-order radiative corrections. For $B^- \rightarrow D^* l^- \bar{\nu}$ find²⁰ (still preliminary)

$$\mathcal{F}_{B \rightarrow D^*}(1) = 0.935 \pm 0.022_{-0.011}^{+0.008} \pm 0.008 \pm 0.020, \quad (7)$$

where now the last uncertainty is from $1/m_q^3$. In both results, an ongoing test of the lattice spacing dependence is not included, but that will probably not be noticeable. More seriously, these results are, once again, in the quenched approximation, but the associated uncertainty are still only a fraction of $\mathcal{F} - 1$. Both results will be updated soon, with calculations at a second lattice spacing and refinements in the radiative corrections.

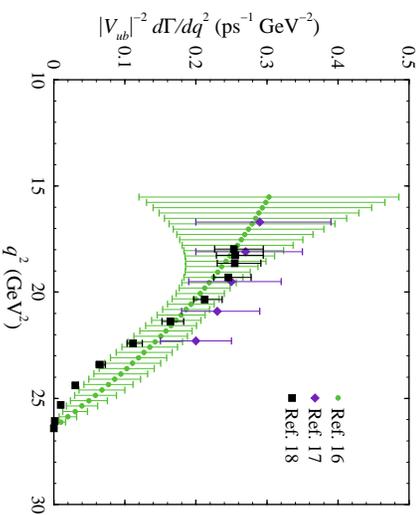


Figure 3. Recent results for the decay $B \rightarrow \pi l \nu$.

group	method	$B_{B_d}(4.8 \text{ GeV})$
JLQCD ²¹	2	$0.85 \pm 0.03 \pm 0.11$
APE ²³	3	$0.93 \pm 0.08^{+0.0}_{-0.6}$
UKQCD ²⁴	3	$0.92 \pm 0.04^{+0.3}_{-0.0}$

Table 1. Recent quenched results for B_B .

3.4 B_q^0 - \bar{B}_q^0 mixing: B_B , f_B , and V_{tq}

The mass difference of CP eigenstates is

$$\Delta m_{B_q^0} = \frac{G_F^2 m_W^2 S_0}{16\pi^2} |V_{tq}^* V_{tb}|^2 \eta_B \langle Q_q^{\Delta B=2} \rangle, \quad (8)$$

where the light quark q is d or s , S_0 is an Inami-Lim function, and $\langle Q_q^{\Delta B=2} \rangle$ is

$$\langle \bar{B}_q^0 | Q_q^{\Delta B=2} | B_q^0 \rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q}. \quad (9)$$

The μ dependence in η_B and $Q_q^{\Delta B=2}$ cancels. New physics could compete with the W and t box diagrams and change Eq. (8).

Lattice QCD gives matrix elements, so the basic results are $\langle Q_q^{\Delta B=2} \rangle$ and f_{B_q} . It is often stated that uncertainties in B_{B_q} and $\xi^2 = f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$ should be small, because they are ratios. Some cancelation should occur, but only if one can show that the errors are under control. At present there are unresolved issues in method 3, so one should be cautious.

With that warning, results for B_B from three groups are in Table 1. Note that JLQCD now includes the short-distance part of the $1/m_Q$ contribution.²² APE²³ (final) and UKQCD²⁴ (preliminary) both extrapolate linearly in $1/m_Q$ from charm (e.g., $1.75 \text{ GeV} < m_{cB} < 2.26 \text{ GeV}$ for APE). It is not clear whether the first term of the heavy-quark expansion is adequate here; everyone working in B physics can and should form his or her own opinion. It is also not clear how the $(m_Q a)^2$ lattice artifacts of method 3 fare through the $1/m_Q$ extrapolation. It is likely that the systematic error is not well controlled and, thus, possibly underestimated in for the last two rows Table 1. At present one should prefer the JLQCD results.

$f_P(n_f)$	MILC ²⁵	CP-PACS ²⁶
$f_B(0)$	$171 \pm 6 \pm 17^{+21}_{-4}$	$190 \pm 3 \pm 9$
(2)	$190 \pm 6^{+20+9}_{-15-0}$	$215 \pm 11 \pm 11$
$f_{B_s}(0)$	$197 \pm 5 \pm 23^{+25}_{-6}$	$224 \pm 2 \pm 15$
(2)	$218 \pm 5^{+26+11}_{-23-0}$	$250 \pm 10 \pm 13$
$f_D(0)$	$199 \pm 6 \pm 12^{+14}_{-0}$	$224 \pm 2 \pm 15$
(2)	$213 \pm 4^{+14+7}_{-13-0}$	$236 \pm 14 \pm 14$
$f_{D_s}(0)$	$222 \pm 5^{+19+15}_{-17-0}$	$252 \pm 1 \pm 18$
(2)	$240 \pm 4^{+25+9}_{-23-0}$	$275 \pm 10 \pm 17$

Table 2. Preliminary unquenched results for f_B , etc., in the continuum limit. All values in MeV.

The MILC²⁵ and CP-PACS²⁶ groups have new, *preliminary* unquenched calculations of the heavy-light decay constants f_B , f_{B_s} , f_D , and f_{D_s} . Both use method 4. Both have results at several lattice spacings, so they can study the continuum limit. The status for Osaka is tabulated in Table 2. The first error is statistical, the second systematic. MILC also provides an estimate of the error from quenching. (With $n_f = 2$ the strange quark is still quenched.) CP-PACS²⁶ also has results with method 2, which agree very well with method 4. One should not, at this time, take the differences between the two groups' central values very seriously. It is more important to understand the different systematics of methods 2, 3, and 4.

4 Prospects

For B physics it is important to remove the quenched approximation, more so than to reduce the lattice spacing much further. To do so, we need more computing. Fermilab, MILC, and Cornell are building a cluster of PCs to tackle the problem.²⁷ Our pilot cluster has 8 nodes with a Myrinet switch. We plan to go up to 48–64 nodes, and then hope to assemble a cluster of thousands of nodes. The large cluster would evolve, by upgrading a third or so of the nodes every year. This is an ambitious plan, but not more ambitious than the experimental effort to understand

flavor mixing and CP violation.

The last few years have seen significant strides in understanding heavy quarks in lattice QCD. The progress has been both computational and theoretical, with one guiding the other. Calculations shown here, for $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}$, $D^0 \rightarrow \pi^- l^+ \nu$, $B^- \rightarrow D^{0(*)} l^- \bar{\nu}$, and B_d^0 - \bar{B}_d^0 mixing are a subset, but in Fig. 2 they are as basic as A , B , C . With the right amount of support from the rest of the community, we hope to obtain the tools needed to resolve the few outstanding problems and to produce excellent unquenched results. Indeed, the example of f_B shows that this is already beginning.

Acknowledgments

I would like to thank Arifa Ali Khan, Claude Bernard, Rüdi Burkhalter, Tetsuya Onogi, Hugh Shanahan and Akira Ukawa for correspondence. I have benefited greatly from collaboration with Shoji Hashimoto, Aida El-Khadra, Paul Mackenzie, Sinéad Ryan, and Jim Simone. Fermilab is operated by Universities Research Association Inc., under contract with the U.S. Department of Energy.

References

1. S. Plaszczynski and M. Schune, "Overall determination of the CKM matrix," hep-ph/9911280.
2. J. D. Lykken, "Physics needs for future accelerators," hep-ph/0001319.
3. Y. Nir, "Future lessons from CP violation," <http://www-theory.fnal.gov/people/ligeti/Brun2/> (Feb., 2000).
4. K. Symanzik, in *Recent Developments in Gauge Theories*, ed. G. 't Hooft *et al.* (Plenum, New York, 1980).
5. M. Lüscher, *Commun. Math. Phys.* **104**, 177 (1986); **105**, 153 (1986).
6. J. Gasser and H. Leutwyler, *Annals Phys.* **158**, 142 (1984).
7. For a popular review, see M. Neubert, *Phys. Rept.* **245**, 259 (1994).
8. W.E. Caswell and G.P. Lepage, *Phys. Lett.* **167B**, 437 (1986).
9. E. Eichten, *Nucl. Phys. B Proc. Suppl.* **4**, 170 (1987); E. Eichten and B. Hill, *Phys. Lett.* **B234**, 511 (1990).
10. G. P. Lepage and B. A. Thacker, *Nucl. Phys. B Proc. Suppl.* **4**, 199 (1987); B. A. Thacker and G. P. Lepage, *Phys. Rev.* **D43**, 196 (1991).
11. A. X. El-Khadra, A. S. Kronfeld, and P. B. Mackenzie, *Phys. Rev.* **D55**, 3933 (1997).
12. A. S. Kronfeld, *Phys. Rev.* **D62**, 014505 (2000).
13. A. S. Kronfeld and J. N. Simone, hep-ph/0006345.
14. A. F. Falk and M. Neubert, *Phys. Rev.* **D47**, 2965 (1993).
15. B. P. Mertens, A. S. Kronfeld and A. X. El-Khadra, *Phys. Rev.* **D58**, 034505 (1998).
16. S. Aoki *et al.* [JLQCD Collaboration], *Nucl. Phys. Proc. Suppl.* **83**, 325 (2000); T. Onogi, private communication.
17. K. C. Bowler *et al.* [UKQCD Collaboration], *Phys. Lett.* **B486**, 111 (2000).
18. S. M. Ryan *et al.*, *Nucl. Phys. Proc. Suppl.* **83**, 328 (2000).
19. S. Hashimoto *et al.*, *Phys. Rev.* **D61**, 014502 (2000).
20. J. N. Simone *et al.*, *Nucl. Phys. Proc. Suppl.* **83**, 334 (2000).
21. S. Hashimoto *et al.*, *Phys. Rev.* **D60**, 094503 (1999).
22. S. Hashimoto *et al.*, hep-lat/0004022.
23. D. Becirevic *et al.*, hep-lat/0002025.
24. L. Lellouch and C. J. Lin [UKQCD Collaboration], hep-ph/9912322.
25. C. Bernard *et al.* [MILC Collaboration], *Nucl. Phys. Proc. Suppl.* **83**, 289 (2000); C. Bernard, private communication.
26. A. Ali Khan, R. Burkhalter, and H. Shanahan, private communication.
27. For more details, visit our web-site at <http://www-theory.fnal.gov/pcqcd/>.



Figure 4. Artist's conception of the author striving to comprehend charm and beauty. (©2000 Mercedes Kronfeld Jordan.)