



## EXPLICIT $SO(10)$ SUPERSYMMETRIC GRAND UNIFIED MODEL\*

Carl H. Albright<sup>1</sup> and S.M. Barr<sup>2</sup>

<sup>1</sup>*Department of Physics, Northern Illinois University, DeKalb, IL 60115*  
and

*Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510*

<sup>2</sup>*Bartol Research Institute, University of Delaware, Newark, DE 19716*

### Abstract

A complete set of Higgs and matter superfields is introduced with well-defined  $SO(10)$  properties and  $U(1) \times Z_2 \times Z_2$  family charges from which the Higgs and Yukawa superpotentials are constructed. The Higgs fields solve the doublet-triplet splitting problem, while the structures of the four Dirac fermion mass matrices obtained involve just six effective Yukawa operators. The right-handed Majorana matrix arises from one Higgs field coupling to several pairs of superheavy conjugate neutrino singlets. In terms of 10 input parameters to the mass matrices, the model accurately yields the 20 masses and mixings of the lightest quarks and leptons, as well as the masses of the 3 heavy right-handed neutrinos. The bimaximal atmospheric and solar neutrino vacuum solutions are favored in this simplest version.

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This conference paper presents a brief summary of recent work [1], [2] carried out by the authors, where it is shown how one can start from a set of Higgs and matter superfields specified by their  $SO(10)$  properties and  $U(1) \times Z_2 \times Z_2$  family charges, construct the Higgs and Yukawa superpotentials, and derive fermion mass and mixing matrices for both quarks and leptons. Previously in several publications [3] - [6], an effective operator approach involving the same symmetries was employed to postulate Dirac mass matrices with similar textures which lead to accurate predictions for the quark and charged lepton masses and CKM mixing matrix. But the right-handed neutrino Majorana matrix remained undetermined. With our more fundamental approach discussed here, the simplest input version favors the bimaximal atmospheric and solar neutrino vacuum solutions.

In the minimal  $SO(10)$  grand unified theory, the two Higgs doublets which break the electroweak symmetry lie completely in one vector representation,  $\mathbf{10}_H$ , which contains a  $\mathbf{5}$  and  $\bar{\mathbf{5}}$  of  $SU(5)$ :  $H_U \subset \mathbf{5}(\mathbf{10}_H)$ ,  $H_D \subset \bar{\mathbf{5}}(\mathbf{10}_H)$ . The colored Higgs triplets also present in the  $\mathbf{10}_H$  can be made superheavy at the GUT scale,  $\Lambda_G$ , by means of the Dimopoulos-Wilczek mechanism [7], in the presence of a Higgs adjoint  $\mathbf{45}_H$  whose VEV points in the  $B - L$  direction and a second  $\mathbf{10}'_H$  which gets massive. Barr and Raby [8] showed that this solution of the doublet-triplet splitting problem can be stabilized with the introduction of two pairs of spinor ( $\mathbf{16}_H, \bar{\mathbf{16}}_H$ )'s plus several Higgs singlets. One pair gets  $SU(5)$ -singlet VEV's at  $\Lambda_G$  and together with the  $\mathbf{45}_H$  breaks  $SO(10)$  down to the standard model. One can then arrange the surviving Higgs doublets to be

$$H_U \subset \mathbf{5}(\mathbf{10}_H), \quad H_D \subset \bar{\mathbf{5}}(\mathbf{10}_H) \cos \gamma + \bar{\mathbf{5}}(\mathbf{16}'_H) \sin \gamma \quad (1)$$

while the combination orthogonal to  $H_D$  gets massive at the GUT scale. With both the  $\bar{\mathbf{5}}(\mathbf{10}_H)$  and the  $\bar{\mathbf{5}}(\mathbf{16}'_H)$  getting a VEV at the EW scale, complete  $t - b - \tau$  Yukawa coupling unification is possible with  $\tan \beta \equiv v_u/v_d \ll 55$ . This minimal  $SO(10)$  Higgs structure can naturally be achieved with a global  $U(1) \times Z_2 \times Z_2$  symmetry.

With the above construction as a natural starting point for our model, we add several more Higgs  $\mathbf{10}$ 's and singlets in order to construct the full Higgs superpotential necessary for the generation of the desired fermion mass matrices. A complete listing of the Higgs superfields is given in Table I.

### Higgs Fields Needed to Solve the 2-3 Problem:

$$\begin{aligned} \mathbf{45}_{B-L}: & A(0)^{+-} \\ \mathbf{16}: & C(\frac{3}{2})^{-+}, C'(\frac{3}{2} - p)^{++} \\ \bar{\mathbf{16}}: & \bar{C}(-\frac{3}{2})^{++}, \bar{C}'(-\frac{3}{2} - p)^{-+} \\ \mathbf{10}: & T_1(1)^{++}, T_2(-1)^{+-} \\ \mathbf{1}: & X(0)^{++}, P(p)^{+-}, Z_1(p)^{++}, Z_2(p)^{++} \end{aligned}$$

### Additional Higgs Fields for the Mass Matrices:

$$\begin{aligned} \mathbf{10}: & T_0(1 + p)^{+-}, T'_0(1 + 2p)^{+-}, \\ & \bar{T}_0(-3 + p)^{-+}, \bar{T}'_0(-1 - 3p)^{-+} \\ \mathbf{1}: & Y(2)^{-+}, Y'(2)^{++}, S(2 - 2p)^{--}, S'(2 - 3p)^{--}, \\ & V_M(4 + 2p)^{++} \end{aligned}$$

**Table I. Higgs superfields in the proposed model.**

The full Higgs superpotential follows from the  $SO(10)$  and  $U(1) \times Z_2 \times Z_2$  assignments:

$$\begin{aligned}
W_{\text{Higgs}} &= W_A + W_{CA} + W_{2/3} + W_{H_D} + W_R \\
W_A &= \text{tr} A^4 / M + M_A \text{tr} A^2 \\
W_{CA} &= X(\overline{C}C)^2 / M_C^2 + F(X) \\
&\quad + \overline{C}'(PA/M_1 + Z_1)C + \overline{C}(PA/M_2 + Z_2)C' \\
W_{2/3} &= T_1 A T_2 + Y' T_2^2 \\
W_{H_D} &= T_1 \overline{C} C Y' / M + \overline{T}_0 C C' + \overline{T}_0 (T_0 S + T_0' S') \\
W_R &= \overline{T}_0 \overline{T}_0' V_M
\end{aligned} \tag{2}$$

The properties described above are obtained from a study of the F-flat constraint conditions, while the role of the extra Higgs fields will become apparent in Eq. (3).

The matter superfields consist of three chiral families together with two vector-like pairs of spinor fields, one pair of vector fields and three pairs of singlets. All but the chiral fields are integrated out to obtain the GUT scale structure for the fermion mass matrices. A complete listing is given in Table II.

$$\begin{array}{lll}
\mathbf{16}_1(-\frac{1}{2} - 2p)^{+-} & \mathbf{16}_2(-\frac{1}{2} + p)^{++} & \mathbf{16}_3(-\frac{1}{2})^{++} \\
\mathbf{16}(-\frac{1}{2} - p)^{-+} & \mathbf{16}'(-\frac{1}{2})^{-+} & \\
\overline{\mathbf{16}}(\frac{1}{2})^{+-} & \overline{\mathbf{16}}'(-\frac{3}{2} + 2p)^{+-} & \\
\mathbf{10}_1(-1 - p)^{-+} & \mathbf{10}_2(-1 + p)^{++} & \\
\mathbf{1}_1(2 + 2p)^{+-} & \mathbf{1}_2(2 - p)^{++} & \mathbf{1}_3(2)^{++} \\
\mathbf{1}_1^c(-2 - 2p)^{+-} & \mathbf{1}_2^c(-2)^{+-} & \mathbf{1}_3^c(-2 - p)^{++}
\end{array}$$

**Table II. Matter superfields in the proposed model.**

The Yukawa superpotential can then be uniquely constructed from the Higgs and matter superfields by taking into account their  $SO(10)$  properties and  $U(1) \times Z_2 \times Z_2$  family charges. One finds

$$\begin{aligned}
W_{Yukawa} &= \mathbf{16}_3 \cdot \mathbf{16}_3 \cdot T_1 + \mathbf{16}_2 \cdot \mathbf{16} \cdot T_1 + \mathbf{16}' \cdot \mathbf{16}' \cdot T_1 \\
&\quad + \mathbf{16}_3 \cdot \mathbf{16}_1 \cdot T_0' + \mathbf{16}_2 \cdot \mathbf{16}_1 \cdot T_0 + \mathbf{16}_3 \cdot \overline{\mathbf{16}} \cdot A \\
&\quad + \mathbf{16}_1 \cdot \overline{\mathbf{16}}' \cdot Y' + \mathbf{16} \cdot \overline{\mathbf{16}} \cdot P + \mathbf{16}' \cdot \overline{\mathbf{16}}' \cdot S \\
&\quad + \mathbf{16}_3 \cdot \mathbf{10}_2 \cdot C' + \mathbf{16}_2 \cdot \mathbf{10}_1 \cdot C + \mathbf{10}_1 \cdot \mathbf{10}_2 \cdot Y \\
&\quad + \mathbf{16}_3 \cdot \mathbf{1}_3 \cdot \overline{C} + \mathbf{16}_2 \cdot \mathbf{1}_2 \cdot \overline{C} + \mathbf{16}_1 \cdot \mathbf{1}_1 \cdot \overline{C} \\
&\quad + \mathbf{1}_3 \cdot \mathbf{1}_3^c \cdot Z + \mathbf{1}_2 \cdot \mathbf{1}_2^c \cdot P + \mathbf{1}_1 \cdot \mathbf{1}_1^c \cdot X \\
&\quad + \mathbf{1}_3^c \cdot \mathbf{1}_3^c \cdot V_M + \mathbf{1}_1^c \cdot \mathbf{1}_2^c \cdot V_M
\end{aligned} \tag{3}$$

Note that the right-handed Majorana matrix elements are all generated through the Majorana couplings of the  $V_M$  Higgs field with the conjugate fermions listed in Table II.

In order to derive the four Dirac mass matrices  $U$ ,  $D$ ,  $N$ ,  $L$ , from Eq. (3) one first finds the three zero mass eigenstates for each type of fermion  $f = u_L$ ,  $u_L^c$ ,  $d_L$ ,  $d_L^c$ ,  $\ell_L^-$ ,  $\ell_L^+$ ,  $\nu_L$  and

$\nu_L^c$  for the superheavy mass matrix connecting  $f$  to  $\bar{f}$  at  $\Lambda_G$  with the electroweak VEV's set equal to zero. The terms in Eq. (3) involving  $\langle T_1 \rangle$ ,  $\langle C' \rangle$ ,  $\langle T_0 \rangle$ , and  $\langle T'_0 \rangle$  then give rise to the  $3 \times 3$  Dirac mass matrices coupling  $u_L$  to  $u_L^c$ , etc. See [1] for details. Just 6 effective Yukawa operators are obtained of the following types connecting the two families indicated:

$$\begin{aligned}
33 &: \mathbf{16}_3 \cdot \mathbf{10}_H \cdot \mathbf{16}_3 \\
23 &: [\mathbf{16}_2 \cdot \mathbf{10}_H]_{16} \cdot \overline{\mathbf{16}} [\mathbf{45}_H \cdot \mathbf{16}_3] / M_G \\
23 &: [\mathbf{16}_2 \cdot \mathbf{16}_H]_{10_1} \cdot \overline{\mathbf{10}_2} [\mathbf{16}'_H \cdot \mathbf{16}_3] / M_G \\
13 &: [\mathbf{16}_1 \cdot \mathbf{16}_3]_{10'_H} \cdot \overline{\mathbf{10}'_H} [\mathbf{16}_H \cdot \mathbf{16}'_H] / M_G \\
12 &: [\mathbf{16}_1 \cdot \mathbf{16}_2]_{10'_H} \cdot \overline{\mathbf{10}'_H} [\mathbf{16}_H \cdot \mathbf{16}'_H] / M_G \\
11 &: [\mathbf{16}_1 \cdot \mathbf{1}_H]_{\overline{\mathbf{16}'}} \cdot \overline{\mathbf{16}'} [\mathbf{10}_H]_{16'} \cdot \overline{\mathbf{16}'} [\mathbf{1}_H \cdot \mathbf{16}_1] / M_G^2
\end{aligned} \tag{4}$$

The subscripts on the bracketed terms indicate the fermion or Higgs fields which are contracted with a Higgs scalar and integrated out. The  $SU(5)$  structure of the above operators will be determined which of the matrix elements actually appear in  $U$ ,  $D$ ,  $N$  and  $L$ . Note that only the 33 operator is renormalizable, while the 23, 13 and 12 operators are dimension-5 and the 11 operator is dimension-6.

The Dirac matrix elements determined by these effective operators are conveniently pictured in terms of Froggatt-Nielsen diagrams [9] in Figs. 1 and 2, where the fields integrated out are explicitly given. The convention is that the  $f_{iL}^c$  lines for the light fermions appear on the left, while the  $f_{iL}$  lines appear on the right. The first 23 operator involving  $\mathbf{45}_H$  contributes in an antisymmetric fashion to the 23 and 32 matrix elements and is proportional to the  $B - L$  quantum number of the fermion line involved. The  $SU(5)$  structure of the second 23 operator reveals a 23 element contribution only to the  $D$  matrix and a 32 element contribution only to the  $L$  matrix.

The textures for the four Dirac mass matrices arise from the above effective operators and diagrammatic structures and can be parametrized as follows:

$$\begin{aligned}
U &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix}, \\
N &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix}, \quad L = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma + \epsilon & 1 \end{pmatrix},
\end{aligned} \tag{5}$$

With  $\sigma \gg \epsilon$ , the Georgi-Jarlskog relations [10] are recovered:

$$m_s/m_b = m_\mu/3m_\tau, \quad m_d/m_s = 3m_e/m_\mu \tag{6}$$

The asymmetrical ‘‘lopsided’’  $\sigma$  entries appearing in  $D$  and  $L$  arise from the  $SU(5)$  structure of the  $\langle \bar{\mathbf{5}}(\mathbf{16}_H) \rangle$  VEV noted above. They can account for the small  $V_{cb}$  quark mixing and the large atmospheric  $\nu_\mu - \nu_\tau$  mixing [11], since a small left-handed rotation in the 23 sector is required to diagonalize  $D$  while a large left-handed rotation is required to diagonalize  $L$  as demonstrated later. A more detailed study given in [2] reveals that the above simple textures are obtained provided  $\tan \beta$  is not too large:  $\tan \beta \leq 5 - 10$ .

The right-handed Majorana mass matrix  $M_R$  can be constructed in a similar fashion and involves the Majorana Higgs  $V_M$  coupling massive singlet conjugate neutrinos. The effective dimension-6 operators involved are

$$(M_R)_{ij} : [\mathbf{16}_i \cdot \overline{\mathbf{16}}_H]_{1_i} \cdot 1_i^c [\mathbf{V}_M]_{1_j^c} \cdot 1_j [\overline{\mathbf{16}}_H \cdot \mathbf{16}_j] / M_G^2, \quad ij = 33, 12 \quad (7)$$

The corresponding Froggatt-Nielsen diagrams for the 33, 12 and 21 elements of  $M_R$  are shown in Fig. 3. The matrix can be parameterized by

$$M_R = \begin{pmatrix} 0 & A\epsilon^3 & 0 \\ A\epsilon^3 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Lambda_R \quad (8)$$

The effective light neutrino mass matrix follows according to

$$M_\nu = N^T M_R^{-1} N = \begin{pmatrix} 0 & 0 & -\eta/(A\epsilon^2) \\ 0 & \epsilon^2 & \epsilon \\ -\eta/(A\epsilon^2) & \epsilon & 1 \end{pmatrix} M_U^2 / \Lambda_R \quad (9)$$

A rotation in the 2-3 plane by an angle  $\epsilon$  followed by a rotation in the 1-3 plane by an angle  $(\eta/A)\epsilon$  brings  $M_\nu$  into an apparent pseudo-Dirac form whereby two of the neutrinos are nearly degenerate, provided  $|\eta/A| \ll 1$ . This result corresponds most readily to the vacuum “just-so” solar neutrino solution [12].

The masses and mixings of the quarks and leptons follow by diagonalization of the matrices in Eqs. (5) and (9), with the mixing matrices at the GUT scale given by

$$V_{CKM} = U_V^\dagger U_D, \quad U_{MNS} = U_V^\dagger U_L \quad (10)$$

In order to obtain numerical results for the predictions at the low scales, the masses and mixings at  $\Lambda_G$  were evolved from the GUT scale to the SUSY scale by use of the 2-loop MSSM beta functions and then to the running mass or 1 GeV scale with the 3-loop QCD and 1-loop QED renormalization group equations. We have used

$$\begin{aligned} \tan \beta &= 5, & \Lambda_G &= 2 \times 10^{16} \text{ GeV}, & \Lambda_{SUSY} &= m_t(m_t) \\ \alpha_s(M_Z) &= 0.118, & \alpha(M_Z) &= 1/127.9, & \sin^2 \theta_W &= 0.2315 \end{aligned} \quad (11)$$

The 10 model mass matrix parameters are chosen to be

$$\begin{aligned} \sigma &= 1.78, & \epsilon &= 0.145, & \eta &= 8 \times 10^{-6} \\ \delta &= 0.0086, & \delta' &= 0.0079, & \phi &= 54^\circ, & A &= 0.05 \\ M_U &= 113 \text{ GeV}, & M_D &= 1 \text{ GeV}, & \Lambda &= 2.4 \times 10^{14} \text{ GeV} \end{aligned} \quad (12)$$

which lead to the following 10 input observables [13]:

$$\begin{aligned} m_t(m_t) &= 165 \text{ GeV}, & m_u(1 \text{ GeV}) &= 4.5 \text{ MeV} \\ m_\tau &= 1.777 \text{ GeV}, & m_\mu &= 105.7 \text{ MeV} \\ m_e &= 0.511 \text{ MeV}, & V_{us} &= 0.220 \\ V_{cb} &= 0.0395, & \delta_{CP} &= 64^\circ \\ m_3 &= 54.3 \text{ meV}, & m_2 &= 59.6 \text{ } \mu\text{eV} \end{aligned} \quad (13)$$

The following 5 predictions are then obtained for the remaining quark observables:

$$\begin{aligned}
m_c(m_c) &= 1.23 \text{ GeV}, & m_b(m_b) &= 4.25 \text{ GeV} \\
m_s(1 \text{ GeV}) &= 148 \text{ MeV}, & m_d(1 \text{ GeV}) &= 7.9 \text{ MeV} \\
|V_{ub}/V_{cb}| &= 0.080
\end{aligned}
\tag{14}$$

The good agreement with the experimental quark data is shown in Fig. 4 for the CKM unitarity triangle in effectively the  $\tilde{\rho} - \tilde{\eta}$  plane.

The 5 additional predictions for the lepton observables are

$$\begin{aligned}
m_1 &= 56.5 \text{ } \mu\text{eV} \\
U_{e2} &= 0.733, & U_{\mu3} &= -0.818 \\
U_{e3} &= 0.047, & \delta'_{CP} &= -0.2^\circ
\end{aligned}
\tag{15}$$

from which one finds the neutrino oscillation solutions

$$\begin{aligned}
\Delta m_{23}^2 &= 3.0 \times 10^{-3} \text{ eV}^2, & \sin^2 2\theta_{atm} &= 0.89 \\
\Delta m_{12}^2 &= 3.6 \times 10^{-10} \text{ eV}^2, & \sin^2 2\theta_{solar} &= 0.99 \\
\Delta m_{13}^2 &= 3.0 \times 10^{-3} \text{ eV}^2, & \sin^2 2\theta_{reac} &= 0.009
\end{aligned}
\tag{16}$$

Finally the 3 heavy right-handed Majorana masses are determined to be

$$M_3 = 2.4 \times 10^{14} \text{ GeV}, \quad M_2 = M_1 = 3.66 \times 10^{10} \text{ GeV}
\tag{17}$$

It is clear that maximal atmospheric neutrino mixing arises from the structure of charged lepton mass matrix  $L$ , while the maximal mixing vacuum “just-so” solar neutrino solution emerges from the very simple form assumed for the coupling of the  $V_M$  Higgs singlet with the conjugate neutrino states in the superpotential. It also depends critically on the appearance of the small parameter  $\eta$  appearing in the Dirac matrix  $N$ , corresponding to the non-zero  $\eta$  entry in  $U$  resulting in a non-zero up quark mass at the GUT scale. The bimaximal large mixing angle MSW solution [14] would require a considerably larger hierarchy in  $M_R$  and is somewhat disfavored. Other more complex textures resulting from a larger set of matter superfields and a more complex Yukawa superpotential are possible but require one or more additional model parameters and are hence less predictive.

The result for  $\delta'_{CP}$  obtained above indicates that CP violation in the leptonic sector is predicted to be negligible. On the other hand, while  $\sin^2 2\theta_{reac} = 0.009$  is considerably below the present CHOOZ reactor bound of 0.1 [15], it should be measurable at a neutrino factory.

Many additional details of the model described are given in references [1] and [2].

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FIGURES

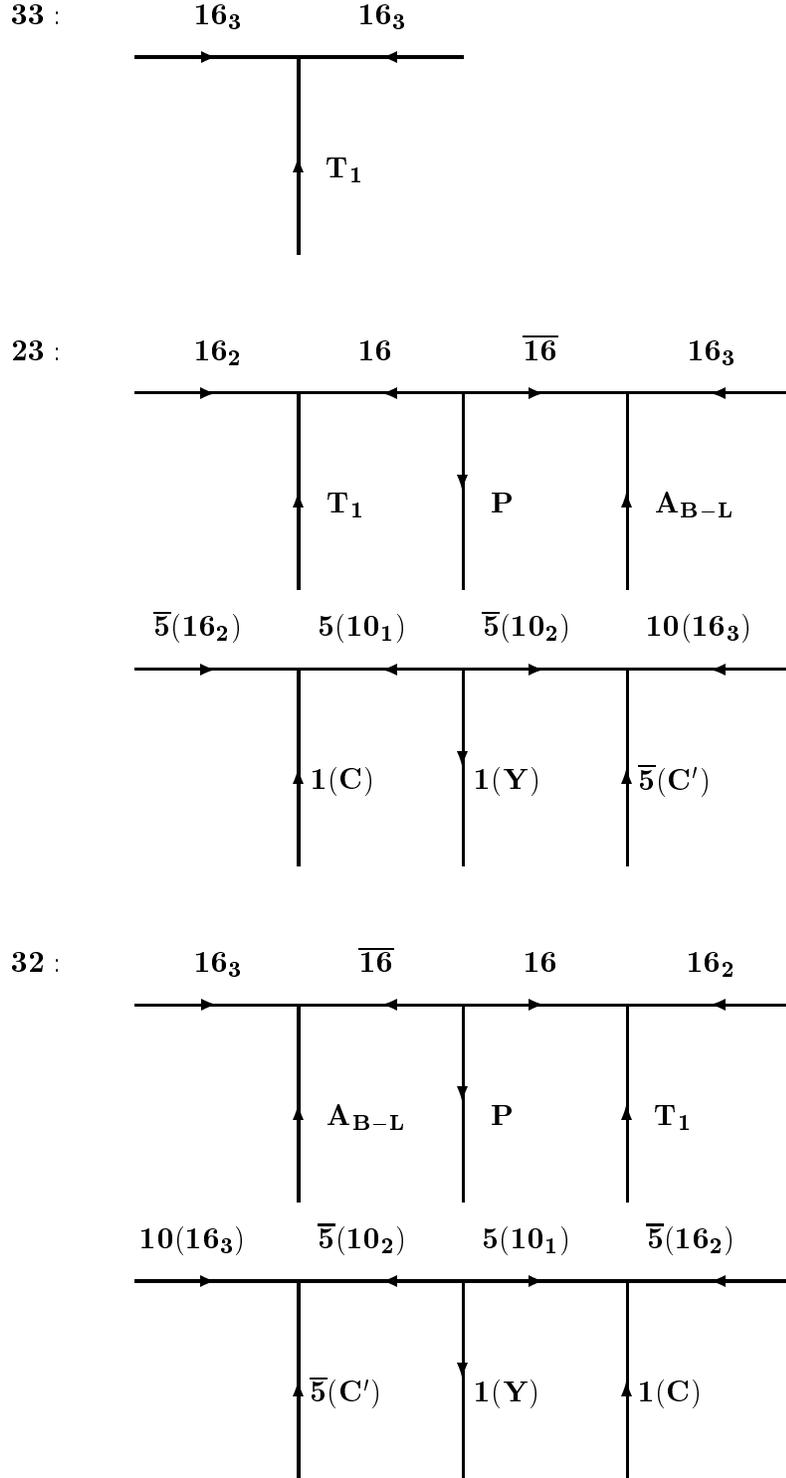


FIG. 1. Froggatt-Nielsen diagrams that generate the 33, 23 and 32 elements of the quark and lepton Dirac mass matrices. The second 23 diagram contributes only to  $D_{23}$ , while the second 32 diagram contributes only to  $L_{32}$ .

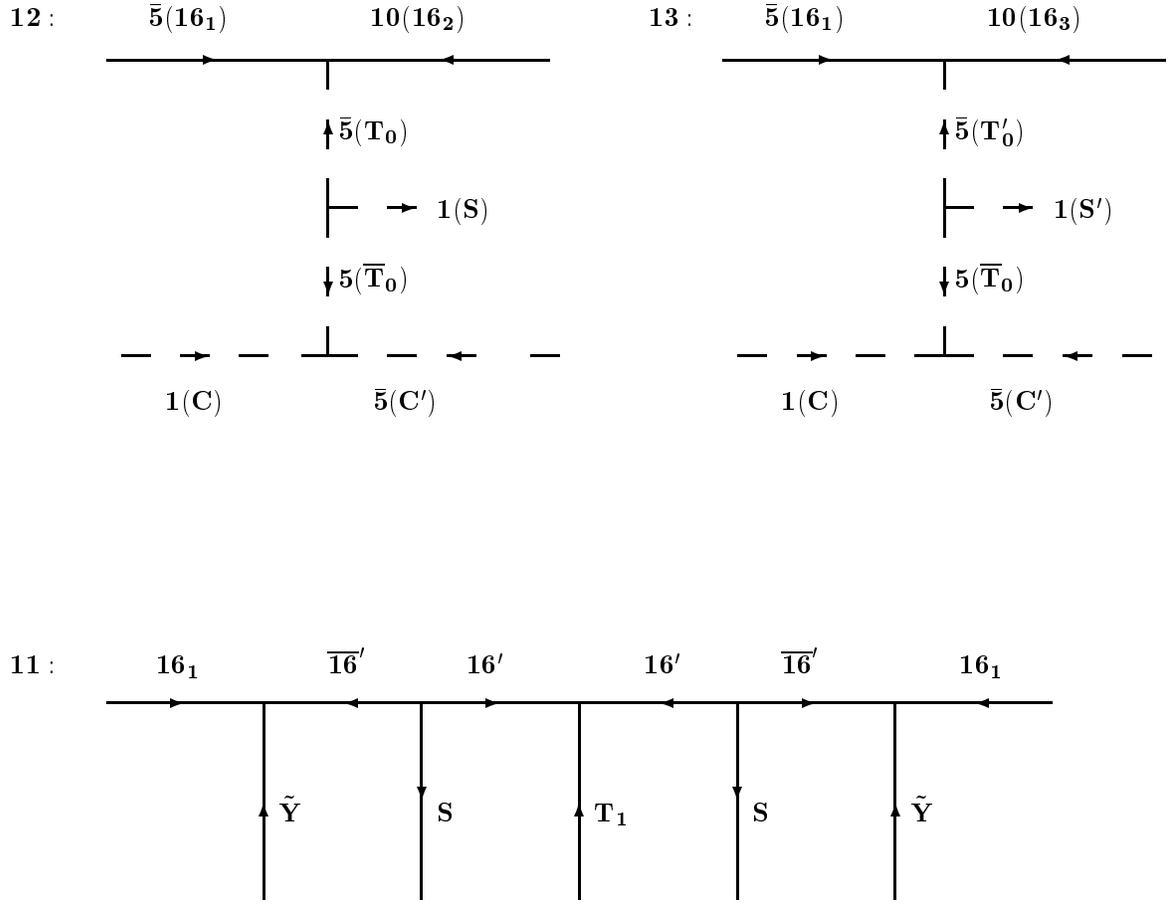


FIG. 2. Additional diagrams for the symmetrical 12 and 21, 13 and 31, and 11 elements of the quark and lepton Dirac mass matrices.

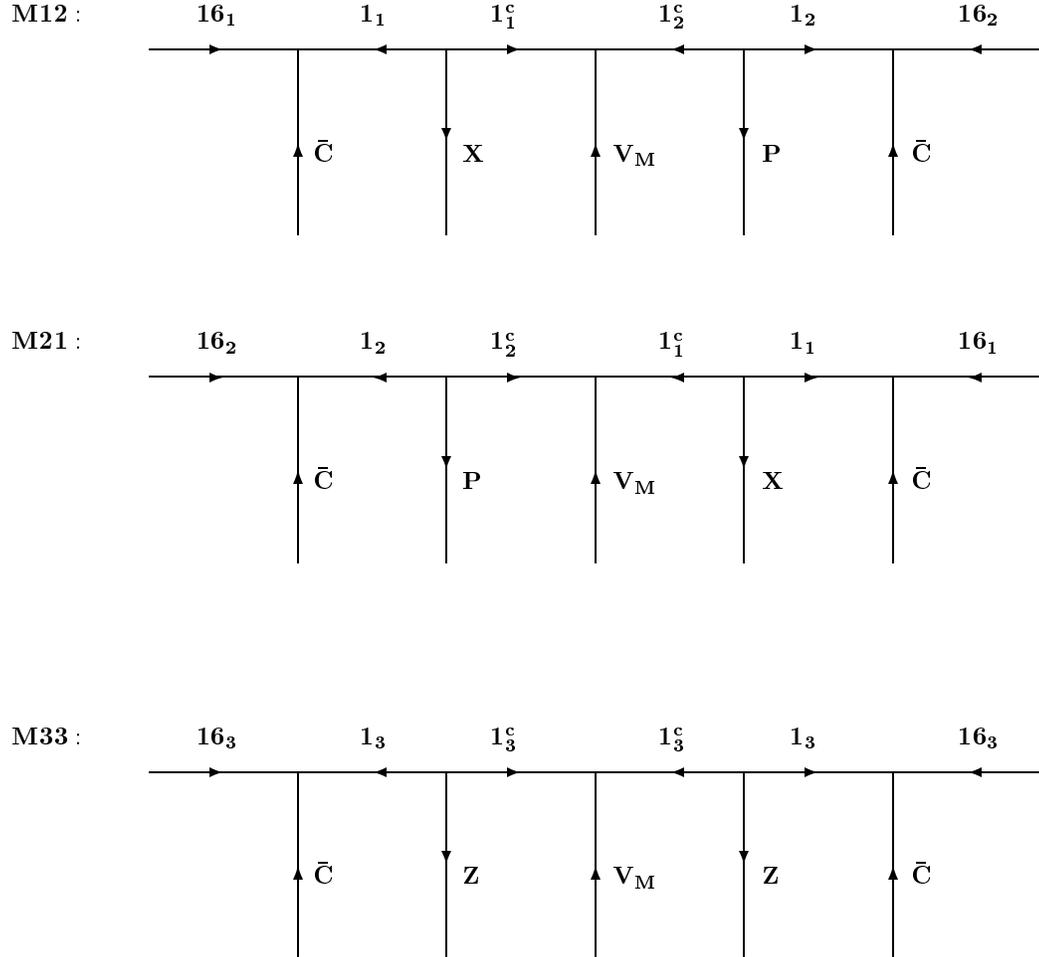


FIG. 3. Diagrams that generate the 33, 12, and 21 elements of the Majorana mass matrix  $M_R$  of the superheavy right-handed neutrinos.

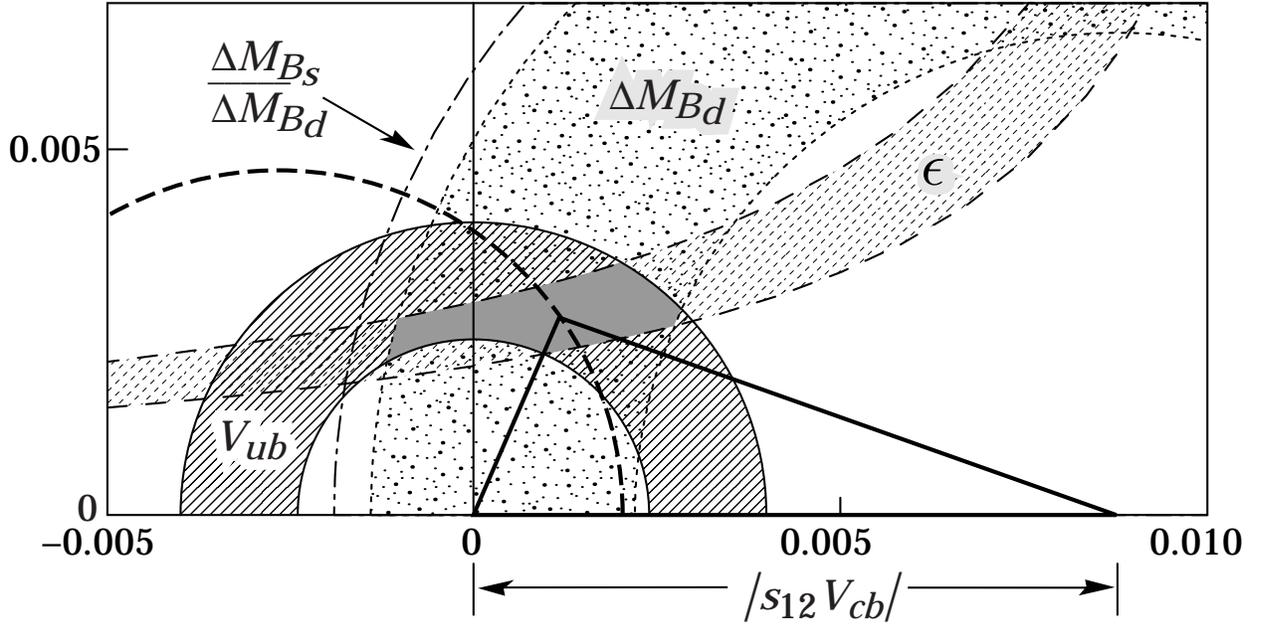


FIG. 4. The unitarity triangle for  $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$  is displayed along with the experimental constraints on  $V_{ud}V_{ub}^*$ , which is the upper vertex in the triangle. The constraints following from  $|V_{ub}|$ , B-mixing and  $\epsilon$  extractions from experimental data are shown in the lightly shaded regions. The experimentally allowed region is indicated by the darkly shaded overlap. The model predicts that  $V_{ud}V_{ub}^*$  will lie on the dashed circle. The particular point on this circle used to draw the triangle shown is obtained from the CP-violating input phase,  $\delta_{CP} = 64^\circ$ .