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Beam-beam studies for the Tevatron

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1 MOTIVATION FOR RUNII BEAM-BEAM STUDIES.

In the first stage of Run II, the Tevatron will be operated with 36 bunches in each beam with bunch separations of 396 nanoseconds. The expected peak luminosity is $\mathcal{L} = 8.6 \times 10^{31}$ cm^{-2}sec^{-1} with an average number of 2.3 interactions per bunch crossing. In the second stage of Run II, the goal is to increase the luminosity to about $1.5{\times}10^{32}$ cm^{-2}sec^{-1}. If the bunch spacing were kept constant, the average number of interactions per bunch crossing would increase to about 4. This is thought to be unacceptably large and might saturate the efficiency of the detectors. This is the main reason for decreasing the bunch spacing at higher luminosities.

One possibility is to reduce the bunch spacing to 132 nanoseconds which lowers the average number of interactions to an acceptable value of 1.4. This shorter bunch spacing though has several consequences on beam dynamics. Collisions between bunches will now occur every 19.78m. This is shorter than the distance of the nearest separators from the main IPs at B0 and D0. Consequently the beams will not be separated at the parasitic collisions nearest to the IPs if the geometry of the orbit is left unchanged. A sketch of this orbit is seen in the top part of Figure 1. This will lead to unacceptably large beam losses and background. Moving the separators closer to the detectors does not separate the beams sufficiently at the locations PC1L and PC1R. The phase advance from the first available position for the separators to these points is too small for the separator strengths that are available [1].

One way to increase the transverse separation between the beams is to make the beams cross at an angle at the IPs. The optimum crossing angle depends upon a number of issues and requires a detailed investigation. The issues include a reduction in the luminosity, change in the beam-beam tune spreads, excitation of synchro-betatron resonances, orbit offset in IR quadrupoles which increases the nonlinear fields seen by the beams, required separation between the beams at the nearest parasitic collisions, the dispersion wave generated by the orbit offset, increase in the strength of the coupling etc. A crossing angle of $\sim \pm 200\mu$rad in the 45 degree plane separates the beams by $\sim 4\sigma$ at the first parasitic collision. A sketch of the orbits with a crossing angle is shown in the bottom part of Figure 1.

The crossing angles that are thought to be necessary have a major impact on the luminosity. If $\mathcal{L}_0$ is the nominal luminosity without a crossing angle, then the luminosity with

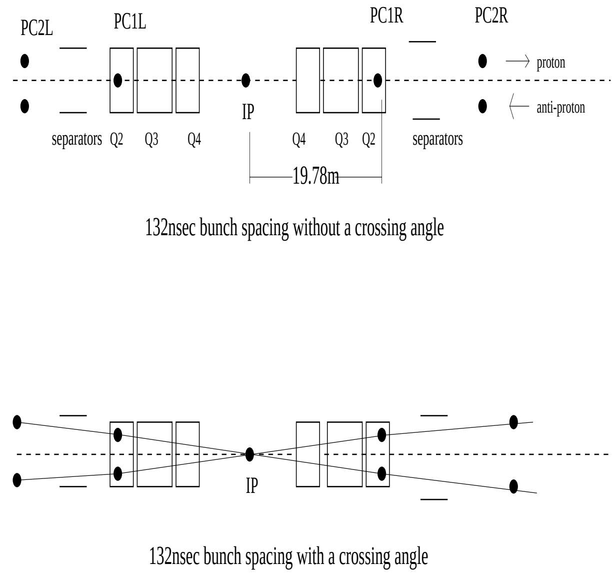


Figure 1: Sketch of the locations of the main beam-beam collisions and the next two parasitic collisions, e.g. PC1R, PR2R on the right, with respect to the triplet quadrupoles and the separators. The top figure shows the geometry without a crossing angle, the bottom figure shows the geometry with a crossing angle.

a total crossing angle of 2ϕ is

$$\mathcal{L} = \frac{1}{\sqrt{1 + (\sigma_s\phi/\sigma_\perp)^2}}\mathcal{L}_0 \equiv \mathcal{R}\mathcal{L}_0 \qquad (1)$$

where $\sigma_\perp$ is the transverse beam size at the IP. Figure 2 shows the relative loss in luminosity as the crossing angle is increased. For example at a half crossing angle of 200μradians, the luminosity is only 38% of its value without a crossing angle. The smaller overlap between the beams which lowers the luminosity also decreases tbe beam-beam tune shift. *If one assumes that we can replace the beam size $\sigma_\perp$ at the IP by $\sigma_\perp\sqrt{1 + (\sigma_s\phi/\sigma_\perp)^2}$* then the head-on tune shift parameter is reduced from its value ξ_0 at zero crossing angle to $\xi = \mathcal{R}^2\xi_0$. Figure 2 shows that with this assumption, the relative tune shift at a half crossing angle of 200μradians is about 28% of its value at zero crossing angle. This hand-waving estimate of the beam-beam tune shift with a crossing angle is useful only as a rough guide. The beam-beam tune shift with a crossing angle depends on the synchrotron oscillation amplitude so it is not enough to specify only the transverse amplitudes when computing the tune shift. However it is true that at any betatron amplitude, the tune shift at all synchrotron amplitudes except zero is smaller than the tune shift without a crossing angle.

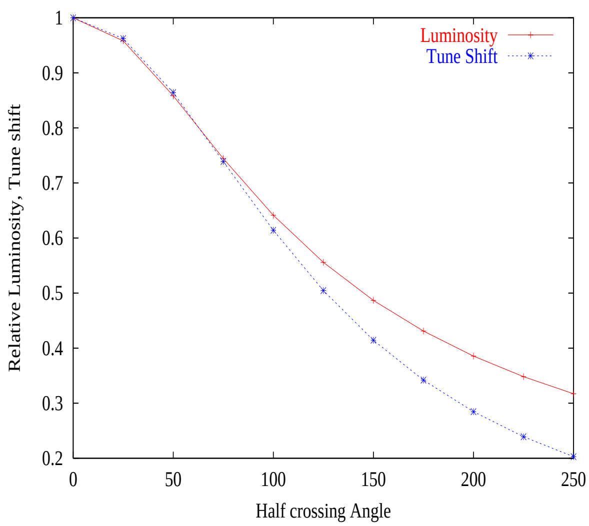


Figure 2: The relative decrease in luminosity and the head-on tune shift parameter as a function of the half crossing angle in the 45° plane.

Once the crossing angles are introduced with more than one hundred bunches in each beam, several beam dynamics issues become imnportant. Some of them are listed here:

- Single beam issues

  Dynamic and physical aperture resulting after off-axis excursion in IR quads. At the first parasitic interaction which occurs within the quadrupole Q2, the beam size is about 2mm. Assuming that a minimum of 4σ separation is necessary, they will be apart by about 8mm. Coupled with the large beam size, this orbit relatively far from the quadrupole axis will make both beams more sensitive to the nonlinear fields of the triplet quadrupoles. In addition, orbit perturbations could lead to larger beam loss due to the tighter physical aperture in these quadrupoles.

- Beam-beam issues

  – Long range interactions at collision. The long-range interactions distort the tune footprint significantly. For example, the zero amplitude tune shift can lie within the interior of the footprint and there can be folds within the footprint. In such cases the tuneshifts at large amplitudes may be greater than at smaller amplitudes. The impact of these folds on the stability needs to be investigated. From studies on the SSC and the LHC [2], it is known that the amplitude in phase space where diffusive motion begins is smaller than the separation between the beams if all the long-range kicks occur at the same phase. This diffusive amplitude r_{diff} can be expressed as

  $$r_{diff} = r_{sep} - \Delta \tag{2}$$

  where r_{sep} is the average separation between the beams and $\Delta \propto \sqrt{N_{PC} N_p}$ where N_{PC} is the number of parasitic collisions and N_p is the intensity of the strong bunch. In the Tevatron the long-range kicks occur at different phases so this expression may not be directly applicable. Nevertheless if there are enough such interactions where the tails of the beams overlap, diffusive motion and eventually particle loss may start at amplitudes less than the average separation.

  – Crossing angle induced synchro-betatron resonances. The strength of these resonances is often characterized by the Piwinski parameter $\chi = \sigma_s \phi / \sigma_\perp$. The typical requirement is that this parameter should be much less than one for these resonances to have negligible effect. This would favour shortening the bunch length. However resonance strengths cannot increase monotonically with χ because at large crossing angles the overlap between the beams decreases and the strength of the beam-beam force and the resonances decrease. Nevertheless, a detailed study of these resonances and how they combine with the long-range interactions to affect growth of particle amplitudes needs to be done.

  – Bunch to bunch variations in orbit. A separator scheme to ensure that collisions of most bunches are well centered will be essential. Dipole kicks due to the long-range beam-beam collisions will also produce significant variations in orbits from bunch to bunch.

  – PACMAN bunches. Bunches which are the furthest away from the center of a train might be in a different tune region and therefore more susceptible to losses.

  – Long-range interactions at injection and during the ramp. As the beams are ramped to top energy, the separation helix changes and the separation is very small at some locations. This could be a problem when there are nearly two hundred interactions. However, the beams are larger during the ramp so beam-beam kicks are smaller.

Figure 3 shows the sequence of collisions for different bunches in a train. The head of the train will meet the head of the opposing train at the IP and all subsequent long-range encounters with the other train will be downstream of the IP. A bunch in the center of the train will experience half of its long-range encounters upstream of the IP and the remaining encounters downstream of the IP. The last bunch in the train will have all long-range encounters upstream of the IP. Figure 4 shows the anti-symmetric optics around the IP. As a consequence of the anti-symmetry, there is no reflection symmetry about the center of the train and the strength of the beam-beam kicks is different for each bunch. In Run IIa where there will be three trains of 12 bunches each, there

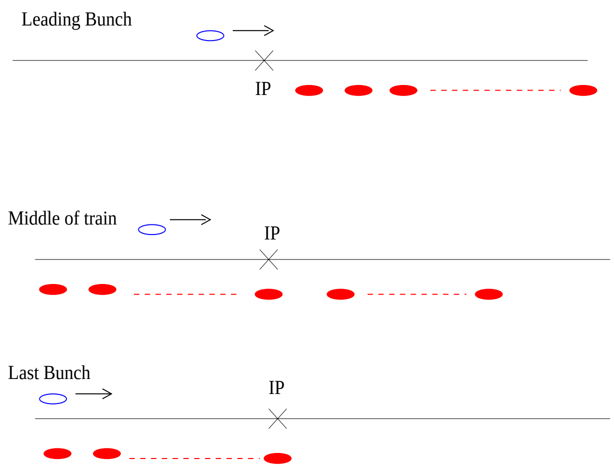


Figure 3: Schematic of the collision scheme for different bunches in a train.

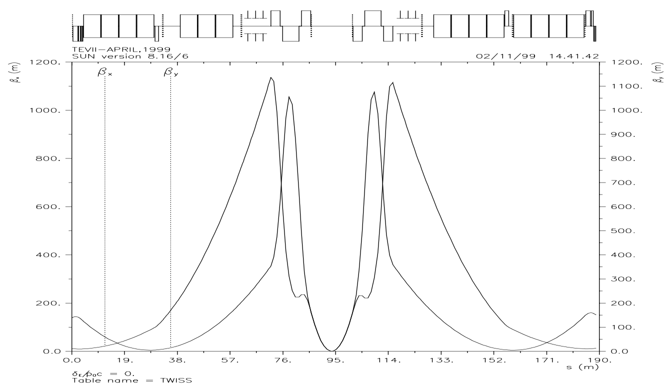


Figure 4: Plot of the beta functions around the IP showing that the optics is anti-symmetric around the IP.

is a three-fold symmetry so there are twelve equivalence classes of beam-beam kicks. In Run IIb with 140×105, there will possibly be one train of proton bunches meeting two trains of anti-proton bunches. This is required so that every anti-proton bunch meets a proton bunch at B0 and D0. There is no symmetry in this scenario so there will be 105 different equivalence classes of beam-beam kicks for the anti-proton beam. Table 1 shows a set of basic parameters for Run IIb. These values are subject to change.

Some of the questions which a study of the beam-beam interactions must answer include:

• Do the beam-beam interactions with crossing angles excite significant synchro-betatron resonances?

• Which of the long-range interactions have an important influence on the beam?

• What is the optimum crossing angle?

• Which of the following effects have an important influence on the beam?

Static: Transverse coupling, bunch to bunch intensity variations, unequal emittances, phase advance errors from IP to

	Run IIb
Luminosity	14.0×10^{31}
Number of bunches ($p \times \bar{p}$)	$\sim 140 \times 105$
Interactions/crossing	1.3
N_p per bunch	2.7×10^{11}
$N_{\bar{p}}$ per bunch	3×10^{10}
Total $\bar{p}$'s	3.15×10^{12}
Bunch separation [nsec]	132
Emittances ($p/\bar{p}$)	20/15
σ^* ($p/\bar{p}$) [μm]	33/29
σ_s ($p/\bar{p}$) [cm]	37/37
Half crossing angle ϕ [μrad]	~ 200
Beam-beam tune shift - 2IPs ($p/\bar{p}$)	$(0.77/6.0)\times10^{-3}$
Transverse tunes	20.581/20.575
Synchrotron tune	7.2×10^{-4}
Piwinski parameter $(\sigma_s/\sigma^*)\phi$ [μrad]	2.1/2.5
No. of long-range interactions	208

Table 1: Basic parameters for Run IIb with a 132 nanosecond bunch spacing. Some of these parameters such as the number of bunches and crossing angle represent best estimates at present.

IP, chromatic variation in β^*, ...

Time dependent: Tune modulations and/or fluctuations, beam offset modulations and/or fluctuations .

• What measures are useful in improving the lifetime? e.g. resonance compensation, reduction of tune shift with amplitude, beam-beam compensation,...

2 BEAM-BEAM INTERACTIONS WITH A CROSSING ANGLE

The impact of all the beam-beam interactions with Run IIb parameters requires a detailed study before we will know if the beams are sufficiently stable. As a start we have begun investigations of the effect of the synchro-betatron resonances excited by the crossing angle at the main IPs. In this section I will report on our simulation studies with a crossing angle.

Figure 5 shows the simulation model for treating the beam-beam interactions at a crossing angle. This model has the following features:

• 6D interactions at B0 and D0. This includes the change in energy from the beam-beam interaction.

• Strong beam bunch (protons) is sliced into 9 disks to account for the crossing angle. The transverse distance of the anti-proton from the center of each disk is used to calculate the beam-beam kick from that disk and then the kicks are summed over all disks. All of these kicks are delivered at the same instant so the anti-proton is not propagated from disk to disk.

• Transverse size of the disks increases away from the IP. This takes into account the hourglass effect.

• Equal crossing angles in both planes - the crossing plane

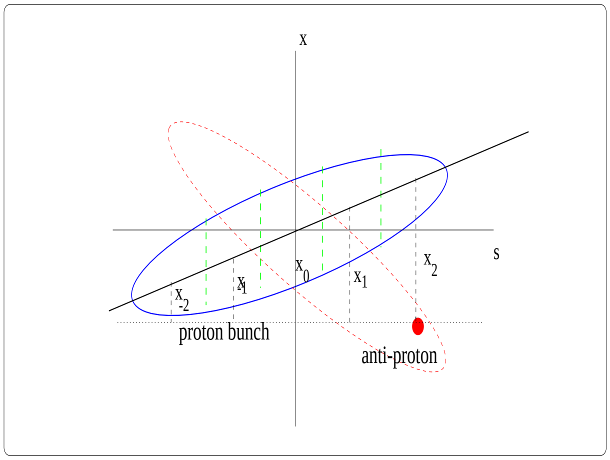


Figure 5: Simulation model for beam-beam interactions

is at 45° to the horizontal plane.
• 6D Linear transport through interaction region and arcs.
• Phase advance between B0 and D0 is taken from a recent lattice model of the Tevatron [3].
• Particles are tracked for 1 million turns (∼21 seconds).

Tune footprints at various crossing angles have been calculated with this model. Figure 6 shows the footprints at zero crossing angle and a total crossing angle of 400μ radians or 283μ radians each in the horizontal and vertical planes. Also shown are the sum and difference resonances up to twelfth order. At the desired tunes, the beam straddles the sum twelfth order resonances with fifth and seventh order sum resonances outside the beam distribution. As mentioned earlier, the tune footprint at the crossing angle of 400μradians is considerably smaller than without a crossing angle because of the smaller overlap between the beams. Without a crossing angle, only resonances of the form $2m_x\nu_x + 2m_y\nu_y = n$ can be excited $(m_x, m_y = 0, \pm 1, \pm 2, \ldots)$ while a crossing angle will excite resonances of the form $m_x\nu_x + m_y\nu_s + m_s\nu_s = n$. We observe that at zero crossing angle, all the twelfth order resonances with even coefficients cross the beam distribution starting at amplitudes around 2.5σ. All the nearby difference resonances have at least one odd coefficient so they are not excited by the beam-beam interactions. The tune footprint with the crossing angle shrinks sufficiently so that the sum resonances $2\nu_x + 10\nu_y, \nu_x + 11\nu_y, 12\nu_y$ do not cross the distribution but all other sum twelfth order resonances are excited and are "seen" by the beam at amplitudes greater than about 3σ. None of the difference resonances are seen by the beam when the crossing angle is 400μradians.

While the footprints are useful in determining the resonances that may cause amplitude growth, long term tracking is essential in order to determine their impact on the beam. Figure 7 shows the results obtained after tracking a beam distribution with and without a crossing angle. At each angle, the initial distribution was composed of two sets of particles: a uniform distribution of 1000 particles between 0 and 4 σ and another uniform distribution of 1000 particles between 4 and 10 σ. Particles within the core are well represented and this choice of distribution also enables us to determine amplitude growth in the tails with a significant number of particles which would not be the case with a Gaussian distribution. During the tracking the maximum and minimum amplitude reached by each particle is recorded and the ratio of these limits is taken as the maximum swing of the particle. Figure 7 shows the maximum swing for each particle in the distribution first at zero crossing angle and then at 400μ radians. At zero crossing angle, the swings are in an absolute sense quite small but are *relatively* large between 5 and 6 σ - the region crossed by the $12\nu_x$ and $10\nu_x + 2\nu_y$ resonances. These resonances are also the twelfth order resonances with the largest widths. Tracking shows that the amplitude swings are large where the resonance widths are large, as they should be. Overall at zero crossing angle, the amplitude swings of all particles

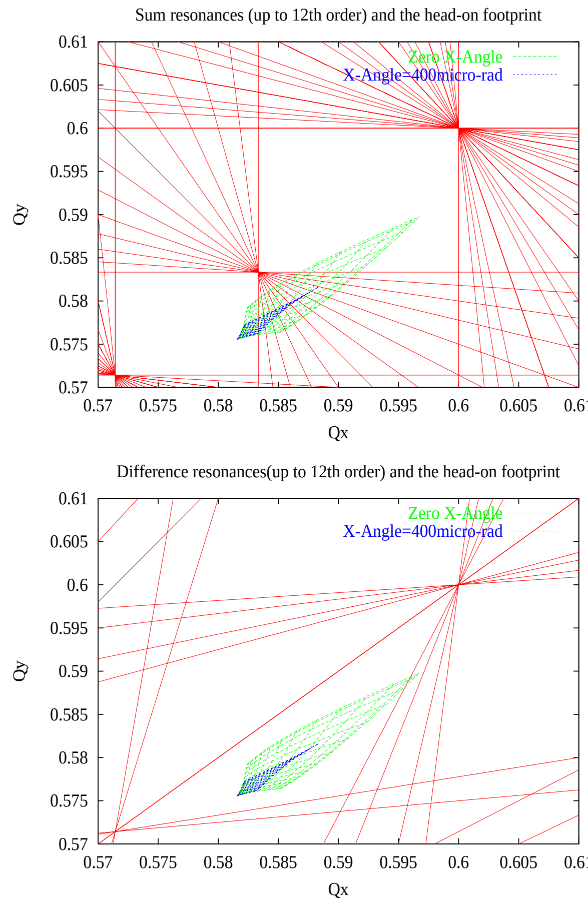


Figure 6: Beam-beam tune footprints with only the interactions at the main IPs. In the top figure, the footprints without a crossing angle and with a total crossing angle of 400 μradians in the 45 degree plane are shown superposed on all the nearby sum resonances up to twelfth order. The bottom figure shows these footprints superposed on the difference resonances up to 12th order.

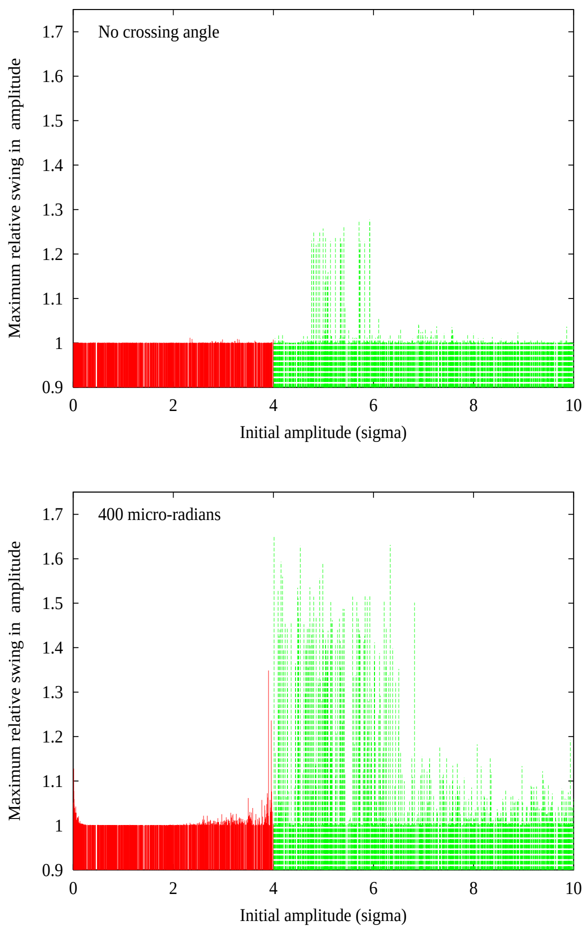


Figure 7: The maximum relative amplitude reached over a million turns as a function of the initial amplitude. Each line represents a particle. The top figure shows the amplitudes without a crossing angle and the bottom figure shows results with a crossing angle of 400μradians. The amplitude swings are relatively large in the region crossed by the twelfth order resonances and their synchrotron sidebands.

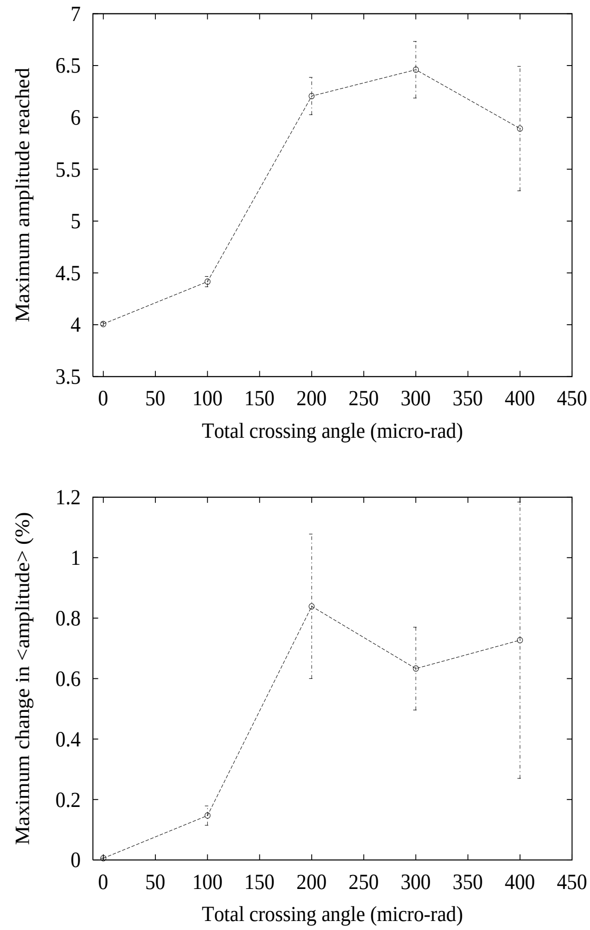


Figure 8: Top: Maximum amplitude reached by any particle within 0 to 4σ averaged over three initial distributions as a function of the crossing angle. The error bars represent rms deviations over the three distributions, each of which had a uniform distribution of 1000 particles between 0 and 4σ. Bottom: The maximum change in the average amplitude of the distribution, also averaged over the three initial distributions.

in the distribution are small enough that there is no increase in the size of the distribution. All particles stay well within the physical aperture ($\sim\ 18\sigma$). The crossing angle generates new betatron resonances and synchrotron sidebands of these resonances leading to a more intricate web of resonances. The bottom part of Figure 7 shows that now there is a greater amplitude swing from $\sim\ 3.5\sigma$ all the way out to 10σ. This region has many more resonances than before. The core however (amplitudes less than 3σ) is relatively unaffected because no resonances cross this region, as seen in Figure 6. Overall even though the amplitude swings are larger in the tails, they are still not large enough for any of the particles in the distribution to reach the physical aperture.

The amplitude growth observed in the simulations is likely to depend on the initial distribution, especially when there are many more resonances in phase space. Figure 8 shows the results of the amplitude growth observed with three initial distributions, each with a uniform distribution of 1000 particles between 0 and 4 σ. The top figure shows the maximum amplitude reached by any particle in the distribution as a function of the crossing angle. At zero crossing angle, there is no growth in the distribution and the rms deviation over the distributions is also negligible. As the crossing angle increases, the average of the maximum amplitude reached increases until a crossing angle of 300μradians before decreasing at 400μ radians. The rms deviations also increase and at 400μ radians, the fluctuations are the largest. This is to be expected since the network of resonances in phase space has a more complicated structure as the crossing angle is increased in this range so some particle distributions may experience the effects of these resonances more than others. Taking into account the error bars, the difference in amplitude growth between 200, 300 and 400 μradians is not statistically significant. The bottom figure shows the maximum change in the sum amplitude averaged over the beam distribution as a function of the crossing angle. The changes are less than 1% in most cases with larger fluctuations between distributions as the crossing angle is increased. The growth of this averaged amplitude with time is not monotonic for any distribution but has more of a "diffusive" nature. The differences in the averaged amplitude between 200, 300 and 400 μradians are also not statistically significant.

Synchro-betatron resonances excited by the crossing angle create synchrotron sidebands around the betatron resonances. Modulation of the betatron tune also creates sidebands around the betatron resonances at the modulation frequency. A natural source of tune modulation occurs when the chromaticity is non-zero (expected to be set to +5 units in Run II to combat head-tail instabilities). Off momentum particles undergoing synchrotron oscillations experience a betatron tune modulation at the synchrotron tune. Particles with the rms energy deviation $\sigma_E/E \simeq 1 \times 10^{-4}$ for example will experience tune modulation at 35Hz with an amplitude 5×10^{-4}. Power supply ripple in quadrupoles causes tune modulation over a whole spectrum of frequencies and with different amplitudes. Since tune modulation will be present, it is useful to compare the relative effects of synchro-betatron resonances excited by the crossing angle and those excited by tune modulation.

Figure 9 shows the maximum amplitude beating with an initial distribution between 0 and 4σ, without a crossing angle and with a crossing angle of 400μradians. The tune modulation increases the amplitude beating range significantly, especially for particles at amplitudes beyond 3.5σ. In this region particles can reach amplitudes nearly three times their initial amplitude. Tune modulation completely dominates the effects due to the crossing angle - the amplitude beating at 400μrad is only slightly different from the case without a crossing angle.

Figure 10 shows the maximum amplitude reached and the maximum change in the averaged amplitude for two tune modulation amplitudes - 5×10^{-4} and 10^{-3} - and averaged over three initial distributions. The maximum amplitude reached is the largest at zero crossing angle and then decreases as the crossing angle is increased. This is easily understood - increasing the crossing angle decreases the overlap of the beams, and hence the beam-beam force, so the nonlinear effects of the beam-beam force and tune modulation are reduced. There is a competition between the resonances excited by the crossing angle and those excited by the tune modulation but at the typical modulation amplitudes considered here, the latter appear to be dominant. The maximum change in the averaged amplitude has a somewhat different behaviour with crossing angle. With the lower modulation amplitude, the change is relatively flat from 100 to 300 μradians while at the larger modulation, the change peaks at 200μradians and falls off steeply on either side. Overall, the growth in the averaged amplitude with tune modulation is significantly greater than without.

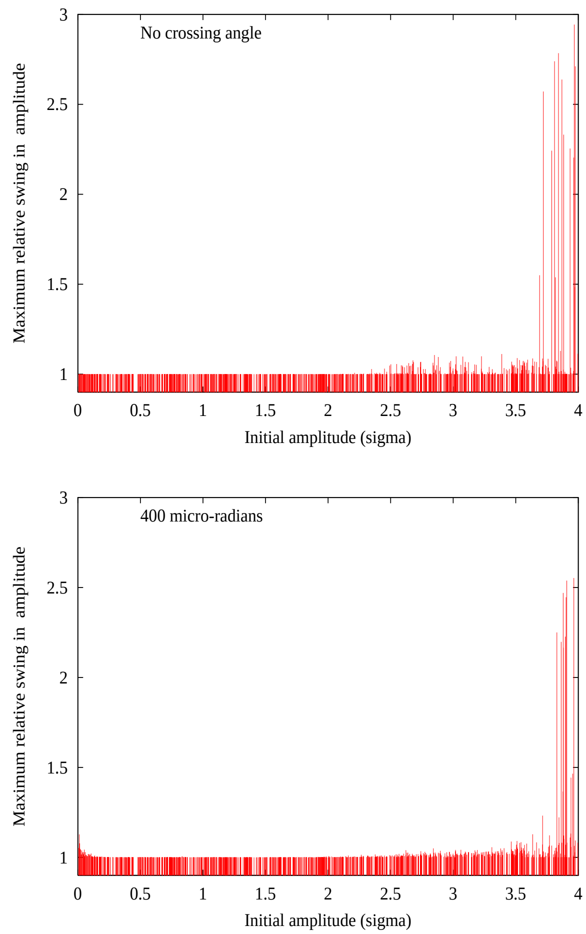
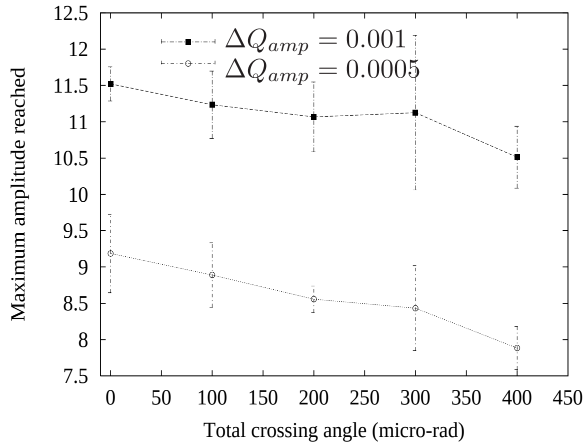


Figure 9: Same as Figure 7 except for two differences. The initial distribution has particles uniformly distributed between 0 and 4 σ and there is tune modulation at an amplitude of 0.001 and frequency 35Hz. Even without a crossing angle, there is much larger amplitude beating for particles at amplitudes beyond 3.5σ compared to the case without tune modulation. The amplitude beating is slightly smaller at 400μradians.



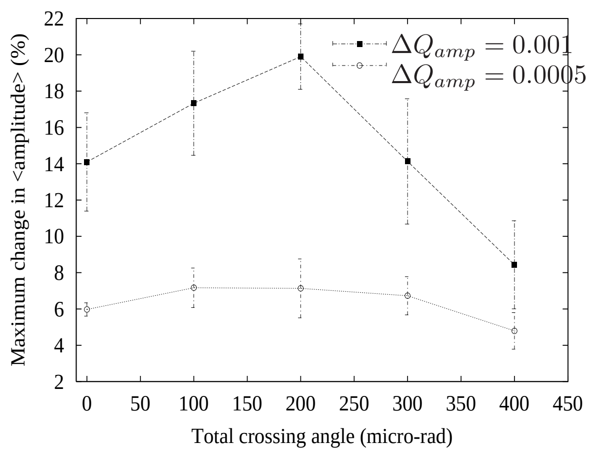


Figure 10: Same as Figure 8 but with added tune modulation at 35Hz and two different amplitudes 5×10^{-4} and 10^{-3}. The amplitude growth with tune modulation is significantly larger than without modulation.

In the simulations done to date, only the main beam-beam interactions have been considered. The long-range interactions, specially the ones nearest to the IPs, will have a significant effect on the particles as will the nonlinearities in the IR quadrupoles. The nearest neighbour long-range interactions will favour larger crossing angles while the magnetic nonlinearities of the IR quadrupoles will favour smaller angles. These effects must be considered before the range of the optimum crossing angle is known.

There is another feature of the main beam-beam interactions which has not been considered until now. The bunches at the Tevatron are long and are comparable in size to the beta function at the IP. This introduces new effects considered in the next section.

3 ANALYTICAL STUDIES OF BUNCH LENGTH EFFECTS

It was pointed out nearly ten years ago by Krishnagopal and Siemann [4] that the phase advance experienced by a particle as it propagates through the opposing bunch can have a strong effect on the strength of the beam-beam interactions. They considered a simplified version of the problem assuming (i) that the beta function stays constant over the interaction length and (ii) one transverse degree of freedom and the longitudinal. Under these assumptions they found that the beam-beam harmonics are of the form

$$V_{m_x m_s} = T_{m_x}(J_x) J_{m_s}\left(\frac{m_x}{2}\frac{a_s \sigma_s}{\beta^*}\right) \exp\left[-\frac{1}{2}\left(\frac{m_x \sigma_s}{2\beta^*}\right)^2\right] \quad (3)$$

where the tunes are close to the resonance $m_x \nu_x + m_s \nu_s = n$. The main point to emphasize here is the exponential decay of the resonance strengths with the square of the bunch length. This rapid fall-off in strength is primarily due to the assumption that the beta function stays constant and therefore the phase advances linearly over the interaction length.

This problem has recently been studied [5] without the major assumptions made in the earlier study. The results show that instead of a monotonic decay with bunch length, the resonance strengths oscillate as a function of the bunch length. Here I present a summary of these results. I will assume that the beams are round over the interaction length, an assumption that is true at the Tevatron and in most hadron colliders.

For infinitely short bunches the Hamiltonian is

$$\begin{aligned} H(J_x, \phi_x, J_y, \phi_y) &= \frac{\nu_x}{R} J_x + \frac{\nu_y}{R} J_y + H_s + \\ & \frac{1}{R} U(J_x, \phi_x, J_y, \phi_y) \delta_P(\theta) \quad (4) \end{aligned}$$

$(J_x, \nu_x), (J_y, \nu_y)$ are the linear actions and tunes in the horizontal and vertical planes respectively, R is the radius of the ring. Here we have assumed that the lattice is completely linear. H_s is the Hamiltonian describing the nonlinear longitudinal motion. U is the beam-beam potential, $\delta_P(\theta)$ is the periodic delta function with period $2\pi/N_{IP}$

when there are N_{IP} equally distant interaction points in the ring.

The beam-beam potential has the Fourier expansion

$$U = \frac{N_p r_p}{\gamma_p} \sum_{m_x=0}^{\infty} \sum_{m_y=0}^{\infty} U_{2m_x,2m_y}(J_x, J_y) \times \cos 2m_x\phi_x \cos 2m_y\phi_y \quad (5)$$

The Fourier coefficients $U_{2m_x,2m_y}$ for a potential due to a Gaussian distribution can be found in a straightforward fashion. This coefficient will be the dominant harmonic in the Fourier expansion if the tunes nearly satisfy the resonance condition

$$2m_x\nu_x + 2m_y\nu_y = n \quad (6)$$

If the bare tunes (ν_{x0}, ν_{y0}) are close enough to this resonance condition, then due to the tune shift with amplitude the resonance condition may be exactly satisfied at an amplitude called the resonant amplitude. The equation for the resonant amplitude can be written as

$$\mathcal{R}(a_x, a_y) \equiv \delta + \Delta\nu_x(a_x, a_y) + \frac{m_y}{m_x}\Delta\nu_y(a_x, a_y) = 0$$
$$\delta = \nu_{x0} + \frac{m_y}{m_x}\nu_{y0} - \frac{n}{2m_x} \quad (7)$$

Here $\Delta\nu_x, \Delta\nu_y$ are the tune shifts with amplitude. For a Gaussian distribution of charge, the resonant amplitudes lie on a one-parameter (r) family of curves determined by the equation

$$\mathcal{R}(a_x, ra_x) = \delta + N_{IP}\xi \int_0^1 \frac{du}{u} \exp[-\frac{(1+r^2)a_x^2 u}{4}] \left\{ [I_0(\frac{a_x^2 u}{4}) - I_1(\frac{a_x^2 u}{4})]I_0(r^2\frac{a_x^2 u}{4}) + \frac{m_y}{m_x}[I_0(r^2\frac{a_x^2 u}{4}) - I_1(r^2\frac{a_x^2 u}{4})]I_0(\frac{a_x^2 u}{4}) \right\} = 0 \quad (8)$$

where $a_y = ra_x$. These resonant amplitudes (a_x, a_y) can be found by numerical integration and are very close to the locus of stable fixed points corresponding to these resonances. The resonance islands are centered on the stable fixed points.

When the bunches are of finite length, the beam-beam potential seen by a particle is

$$V(x, y, s) = \rho_l(s+ct)U(x, y) \equiv \sum_{\vec{m},n} V_{\vec{m},n} \exp[i(\vec{m}\cdot\vec{\psi} - n\theta)] \quad (9)$$

ρ_l is the longitudinal density of the bunch whose center is a distance of $s + ct$ from the particle. Remarkably enough, the Fourier harmonics of the potential for round beams factorize into a product of two terms

$$V_{2m_x,2m_y,m_s,n} = \frac{N_b r_p}{\gamma_p} U_{2m_x,2m_y}(J_x, J_y) L_{2m_x,2m_y,m_s,n}(a_s) \quad (10)$$

where $U_{2m_x,2m_y}$ depends only on the transverse actions and is independent of the longitudinal variables. The dependence on the bunch length σ_s and the synchrotron oscillation amplitude of the particle a_s is all contained in $L_{\vec{m}}$. Assuming that the longitudinal density distribution of the opposing bunch is Gaussian and that the tunes are sufficiently close to a resonance so that

$$\Delta = 2m_x\nu_x + 2m_y\nu_y + m_s\nu_s - n \ll 1 \quad (11)$$

the longitudinal harmonic is of the form

$$L_{\vec{m}} = \frac{1}{(2\pi)^{3/2}} \exp[-\frac{a_s^2}{4}] \sum_{j=-\infty}^{\infty} (-1)^j I_j(\frac{a_s^2}{4}) F_j \quad (12)$$

$$F_j = \int_0^{\infty} du\, e^{-2u^2} \cos[2m_+ \tan^{-1}(\frac{\sigma_s u}{\beta^*})] I_{2j}(2a_s u) \quad (13)$$

where $m_+ = m_x + m_y$. The complicated argument of the cosine in Equation (13) is a consequence of the growth of the beta function as $\beta(s) = \beta^* + s^2/\beta^*$ where s is the distance from the IP. The transverse harmonics U decrease with increasing m_x, m_y as is well known but for finite bunch lengths there is another multiplicative factor $L_{\vec{m}}$ which also decreases as m_+ increases. These expressions can be analytically evaluated to extract the dependence on the bunch length σ_s, synchrotron oscillation amplitude a_s of the particle and the resonance harmonic numbers m_x, m_y. The most useful result is obtained in the limit of high resonance numbers - this is usually the case at most accelerators where tunes are chosen to avoid resonances of order lower than or equal to ten. An asymptotic expansion in the limit that $m_+ \to \infty$ shows that

$$\lim_{m_+\to\infty} L_{\vec{m}} = \frac{1}{2(2\pi)^{3/2}} \frac{1}{\sqrt{m_+\lambda}} \cos[2m_+ \arctan(\lambda)] + O(\frac{1}{m_+}), \qquad \lambda = \frac{a_s\sigma_s}{2\beta^*} \quad (14)$$

This predicts a damped oscillatory dependence on the bunch length. We may define a quasi-wavelength of these oscillations as $(\pi/m_+)(\lambda/\arctan(\lambda))$ which in the limit $\lambda \ll 1$ is π/m_+ while in the opposite limit $\lambda \gg 1$ is $2\lambda/m_+$. Figure 11 shows the behaviour of $L_{\vec{m}}$ in the asymptotic limit for $m_+ = 8, 9, 10$ as a function of λ. At small λ the quasi-periods of the oscillations are short while at large λ, $L_{\vec{m}}$ approaches zero asymptotically. Thus at short bunchlengths, observables such as beam lifetime (due to the beam-beam interactions) are likely to change quickly with bunchlength while at long bunchlengths the lifetime may be somewhat insensitive to the choice of bunch length. This oscillatory behaviour is in contrast to the exponential decay predicted by the earlier analysis [4].

One measure of the influence of the bunch length can be seen in the resonance widths. Assuming, as is usual, that the resonances are isolated the half widths in action are given by the expressions

$$(\Delta J_{x,w}, \Delta J_{y,w}) = (m_x, m_y) \times \left(\frac{|N|}{|D|}\right)^{1/2}$$

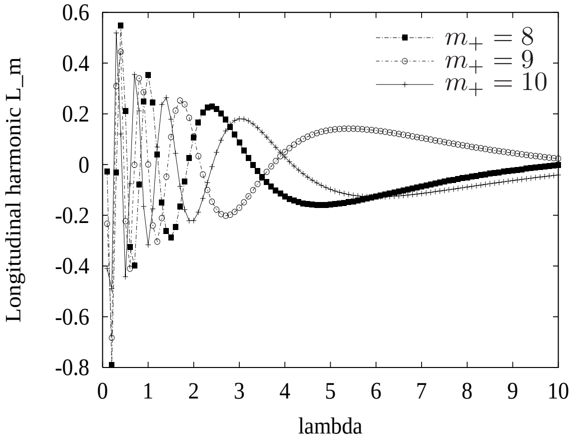
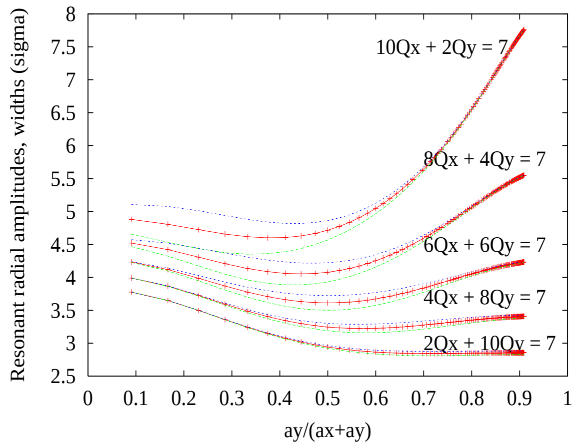


Figure 11: Asymptotic behaviour of the longitudinal part of the beam-beam harmonics, $L_{\vec{m}}$ for large m_+ at $m_+ = 8, 9, 10$ as a function of $\lambda = a_s\sigma_s/(2\beta^*)$.

$$\begin{aligned} N &= 2U_{m_x,m_y}(I_{1,s}, I_2)L_{\vec{m}} \\ D &= L_0[m_x^2\frac{\partial^2}{\partial J_x^2} + 2m_x m_y\frac{\partial^2}{\partial J_x J_y} + m_y^2\frac{\partial^2}{\partial J_y^2}]U_{0,0} \end{aligned} \tag{15}$$

Figure 12 shows the resonant amplitudes and the widths of the islands of the twelfth order sum resonances. It is necessary for the neighbouring islands to touch or intersect in action space in order for the islands to overlap but it does not prove that they do in fact overlap in phase space. Overlapping in action space is therefore a necessary but not sufficient condition for resonance overlap. We observe that for zero length bunches it is possible for the $10\nu_x + 2\nu_y$ and $8\nu_x + 4\nu_y$ resonances to overlap but not for the other sum resonances. The bottom figure shows the resonance widths now calculated for Tevatron bunch lengths and $a_s = 1$. These widths are smaller by an order of magnitude - hence none of these resonances can overlap as is clear from this figure. This is consistent with observations at the Tevatron - in past operations when the working point was chosen to straddle these twelfth order resonances, there was no significant effect on the lifetime. This calculation makes it clear that bunch length effects have a major impact on the beam-beam resonance strengths.

The analytical predictions can be tested by particle tracking. The model to incorporate bunch length effects described here is similar to that in Section 2 but with two additional features. The longitudinal density of each disk falls off as a Gaussian from the center of the bunch and the particle is propagated from the center of each disk to the next by the appropriate transfer matrix. Tracking was done for different bunch lengths, first with all 1000 particles in the distribution at the same initial synchrotron amplitude $a_s = 1$ and then with a Gaussian distribution in a_s with a cutoff at $a_s = 3$. These simulations were done at three different tunes: the Tevatron tunes $\nu_x = 0.581, \nu_y = 0.575$, close to a fourth integer resonance $\nu_x = 0.257, \nu_y = 0.251$,



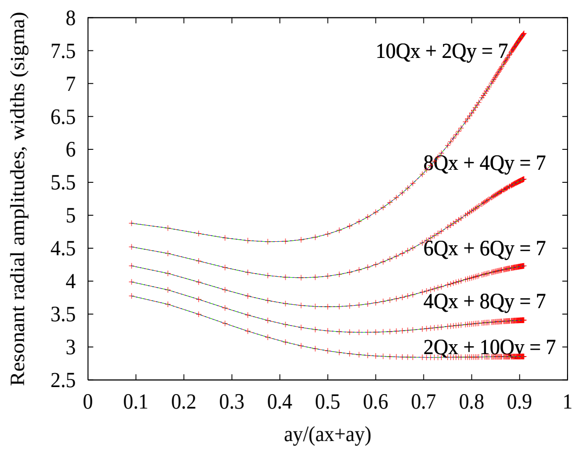


Figure 12: The top figure shows the locations of resonant amplitudes and the widths of sum twelfth order resonances calculated for infinitesimally short bunches, β^*=0.35m as a function of $r = a_y/(a_x+a_y)$. Q_x, Q_y denote the horizontal and vertical tunes respectively. The curves in red show the locations of the resonant amplitude while the curves in blue and green on either side show the width of the resonance. We see that there is the possibility of overlap between the $10Q_x+2Q_y$ and $8Q_x+4Q_y$ resonances for $\sim 0.15 < r < 0.3$. At the bottom we show the same resonances and widths calculated with a bunch length of 36cm and $a_s = 1.0$. The resonance widths are all reduced by an order of magnitude. Now there is no possibility of overlap between any of these resonances.

and close to a sixth order resonance $\nu_x = 0.175, \nu_y = 0.169$. The maximum relative swing of the distribution was recorded for each simulation.

Figures 13 to 15 show the dependence of the swing on the bunch length. At the Tevatron tunes, the maximum swing is close to the value it would be without the beam-beam interaction indicating that the resonances do not have a significant effect. As a function of bunch length, the maximum swing oscillates with decreasing amplitudes. Close to the lower order resonances the swings are much larger as expected and they also oscillate with the bunch length. The results of these simulations at three different tunes are in qualitative agreement with the analytical predictions.

The best test of these predictions would be an experimental measurement. This would require that the bunch length be varied over a range and an observable such as the lifetime be measured at each bunchlength of the strong beam. It would be sufficient to have only a single bunch in each beam. At the Tevatron, it is not possible to shorten the bunch length much below its value of around 36cm. The bunch can be lengthened either by an injection mismatch or with the addition of RF noise. In order to have a clear signature that the observed effects are due to the change in bunchlength, it will be desirable to have other parameters such as bunch intensity, emittance, tunes etc. constant. With careful preparation, it should be possible to carry out such a test.

4 PROPOSED EXPERIMENTS

In RunII the performance limitations may well arise due to the several long-range interactions. This is also true for the LHC where there will be about 60 long-range interactions and almost all at the same phase. In addition, the LHC will be the first hadron collider where both beams will be of the same intensity so strong-strong effects (about which not much is known) might also be important. There are a number of experiments that would address questions relevant to the weak-strong regime (appropriate to the Tevatron) and the strong-strong regime. I will focus here on weak-strong experiments.

- Impact of synchro-betatron resonances.

  It would be useful to measure their impact without the complications of the long-range interactions. The only published observations with crossing angles at hadron colliders were at the SPS [6]. There experiments with two colliding bunches found no significant differences in background losses up to crossing angles of 600μrad. Compared to the Tevatron however, the Piwinski parameter χ was substantially smaller (χ_{max} = 0.7) due to the shorter bunch lengths. At the Tevatron the experiments can be done with one anti-proton bunch and two proton bunches so the anti-protons collide with a bunch at B0 and D0. At the least one would measure the lifetime, and background losses at different crossing angles. Orbit effects due to the crossing angles will need to be eliminated, thus it would be use-

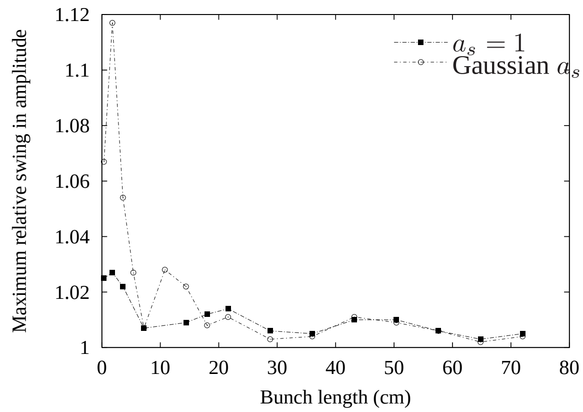


Figure 13: Maximum relative swing amongst 1000 particles tracked for 100,000 turns at each bunchlength with the Tevatron tunes $\nu_{=}0.581, \nu_y = 0.575$. Bunch length effects such as phase advance over the bunch and the longitudinal Gaussian density ditribution of the disks are included in these simulations.

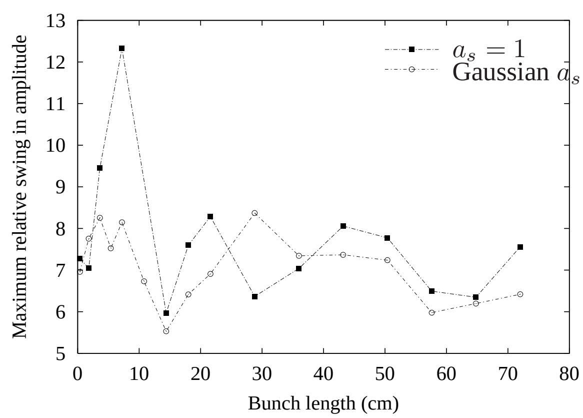


Figure 14: Same as above but close to fourth integer resonances, $\nu_x = 0.257, \nu_y = 0.251$.

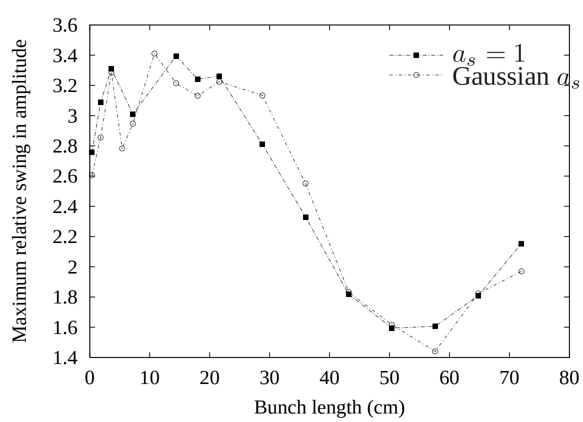


Figure 15: Same as above but close to sixth integer resonances, $\nu_x = 0.175, \nu_y = 0.169$.

ful to first measure the single beam lifetime without and with crossing angles. Limitations due to physical aperture can be determined this way. The lifetime with colliding beams may depend on the relative signs of the crossing angles at the two IPs. Of all the possible combinations of signs, some may be ruled out because they would not separate the beams by the required distances when there are 100 or more bunches in each beam. It would be useful to determine the lifetime for each of the useful sign combinations. These measurements may reveal that there is a crossing angle beyond which the effects due to the nonlinear fields of the IR quadrupoles and the synchro-betatron resonances lead to unacceptably large losses. This can be compared with the results of simulations and would determine if the important physics is contained in the models.

- Impact of long-range interactions.

  Tune footprints are severely distorted when the long-range interactions are included and the footprint changes from bunch to bunch. Preliminary tracking results with 36$\times$36 bunches indicate that these interactions reduce the dynamic aperture by a significant amount. The interactions closest to the IP on either side are at the smallest separations and have the largest effect. As a first experimental test it would be desirable to have a few bunches (say four) in the proton beam and spaced so that each anti-proton bunch experiences only the nearest neighbour interactions in each IR but not the head-on interactions. The lifetime could be measured as a function of the proton intensity and also as a function of the beam separation at these nearest neighbour points. The dependence on separation will be a useful input towards determining the minimum crossing angle while the dependence on intensity may be useful in determining the maximum useful luminosity. This set of experiments will be very useful in testing the predictive power of the simulations with long-range interactions. If the observations are close to the simulation results, then simulations may be used with more confidence in predicting the outcome with 100 or more bunches in each beam. With the bunch spacing at 396 nanoseconds, perhaps the most useful experiment to determine the feasibility of shortening the spacing to 132 nanoseconds would be to collide an anti-proton bunch with 36 proton bunches with crossing angles at B0 and D0. This can be accomplished with the present set of separators. In this experiment the impact of both the synchro-betatron resonances and the long-range interactions will be felt. Observations over a range of crossing angles will go a long way towards our understanding of these phenomena.

- Tune footprint due to the beam-beam interactions.

  Measurement of the footprint is the most basic test of the nonlinearity of the beam-beam force and the machine lattice. A comparison with the theoretical footprint will reveal if all important effects have been included in the theoretical model. The tune as a function of amplitude could be measured with a pencil antiproton bunch which can be kicked to different amplitudes in both transverse planes. If this pencil bunch is sufficiently narrow, it will probe the force within a small region of phase space where the tune is nearly constant. Following the kick this probe bunch will decohere due to the nonlinear beam-beam force and its emittance will grow as it fills out phase space by shearing. Figure 16 shows an example of the decoherence of the beam centroid following an initial kick which placed it at a distance of about 5σ from the center of the opposing bunch. Some of the issues which must be addressed in such an experiment include:
  - The time to measure the tune should be less than the decoherence time.
  - The decoherence time will depend on the kick amplitude and the machine chromaticity.
  - The minimum size of the pencil bunch may depend on the minimum intensity required to trigger the beam position monitors if turn by turn data is used to measure the tunes.
  - If scraping is used to reduce the beam size, then it might be useful to scrape in regions of high dispersion to remove some of the momentum spread. It may also take some time to learn how to scrape efficiently without losing the beam.

  If the bunch decoheres significantly following a tune measurement at a particular amplitude, it may be unusable for a subsequent measurement. In that case we may want a train of pencil bunches, each of which will be kicked to a different amplitude, to obtain the tune footprint. An alternative possibility could be to use an AC dipole, as suggested for other measurements at RHIC, to kick the beam adiabatically and thereby avoid the emittance growth. If this works in practice, then each pencil bunch could be used to measure the tune at more than one amplitude.

5 CONCLUSIONS

The beam-beam interactions will have a major impact on beam stability in Run II. Crossing angles at the main interaction points and the nearly two hundred long-range interactions will be new sources of lifetime limitations. This will be further complicated by the fact that the effects will vary from bunch to bunch. Detailed theoretical and experimental studies are required to know whether this mode of operation will be feasible.

The working point of the Tevatron has been chosen so that the tune footprint does not cross resonances of order less than twelve. When crossing angles are introduced, the footprint shrinks in size. Some twelfth order betatron resonances now do not cross the beam distribution and new resonances are excited. In addition synchrotron sideband res-

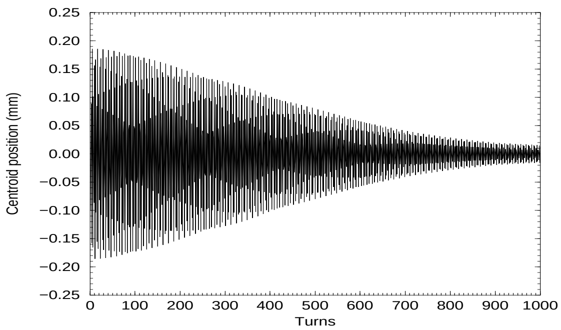


Figure 16: Decoherence of the beam centroid due to the nonlinear beam-beam force between two beams that are initially offset from each other by about 5σ.

onances develop around the betatron resonances and these are a source of concern.

The simulations reported here have studied the effects of the crossing angles but not those of the long-range interactions. These show that that the synchro-betatron resonances induced by the crossing angles do not appear to affect the core of the beam up to crossing angles of 400μ radians. The amplitude growth found at crossing angles between 200μrad to 400μrad are statistically about the same. These simulations also show that tune modulation at typical modulation depths causes large amplitude growth and dominates the effects due to the crossinng angles. Analytical and simulation studies have shown that the long length of the bunches in the Tevatron have a major impact on the strength of the beam-beam resonances. The analytical studies predict that the resonance strengths oscillate as a function of the bunch length. This has been confirmed with simulations. Resonance widths calculated for the Tevatron bunches are about an order of magnitude smaller than those calculated for zero length bunches. These results suggest that it would be very worthwhile to conduct a beam-beam experiment where the bunch length is varied to the extent possible. At longer bunch lengths there is a loss of luminosity due to the hour-glass effect but it may turn out that the gain in lifetime is sufficiently high that the integrated luminosity is larger. In any event, the phase averaging effect due to the long bunch is significant and needs to be taken into account in all theoretical models.

The amplitude growth within the beam distribution may change qualitatively when the long-range interactions are included. The footprint changes and the changes are different from bunch to bunch. The transverse core of some bunches may be excited by resonances. This is now under study.

In the near term, experimental observations with crossing angles appear feasible during the machine studies period at the Tevatron in the fall of 2000. The first stage of Run II will operate with 36 bunches in each beam. This will give us an opportunity to observe the effects of the several long-range interactions. When the faster kickers are available, operation with the shorter bunch spacing of 132 nanoseconds will be tested. It will also be desirable to conduct basic tests of beam-beam models by measuring the tune footprint and perhaps further out, measure the dynamic aperture with beam-beam interactions. These experiments can just as well be conducted at other colliders, especially RHIC when the AC dipoles are available.

6 REFERENCES

[1] J. Johnstone, FNAL Internal note (1994)

[2] J. Irwin, SSC preprint SSC-233 (1989)

[3] MAD lattice from P. Bagley, ca. April 1999.

[4] S. Krishnagopal and R. Siemann, *Phys. Rev. D*, **41**, 2312 (1990)

[5] T. Sen, *The beam-beam interaction of finite length bunches in hadron colliders*, FNAL preprint Pub-00/093-T (2000)

[6] K. Cornelis, W. Herr, M. Meddahi, CERN preprint CERN SL/91-20 (1991)